Lecture 9 – 1st law with property tables

Purdue ME 200, Thermodynamics I

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Outline

1st law for closed systems

- for closed systems (no mass transfer),

$$\Delta \mathrm{KE} + \Delta \mathrm{PE} + \Delta U = Q - W$$

- KE is system's center-of-mass kinetic energy
- PE is system's center-of-mass gravitational potential energy
- U is system's internal energy
- Q is heat transfer from surroundings to system
- W is work done by system on surroundings

How to evaluate ΔU?

- once in a while, we have a mathematical formula for ΔU
- for example,
  - $\sim$incompressible solids and liquids with $\sim$constant specific heats
  - $\sim$ideal gases with $\sim$constant specific heats
- but usually, we just have empirical data in property tables

Evaluating ΔU with property tables

- for any process moving a closed system from state 1 to 2,

$$\Delta U = U_2 - U_1 = m(u_2 - u_1)$$

- if system is a single phase in state i,
  - ◇ can find u_i in compressed liquid or superheated vapor table
- if system is a two-phase liquid-vapor mixture in state i,
  - ◇ $u_i = u_{\mathrm{liq}} + x_i(u_{\mathrm{vap}} - u_{\mathrm{liq}})$
  - ◇ can find u_{liq} and u_{vap} in saturation table
  - ◇ quality x_i must be given or calculated

Outline

Problem statement

A piston compresses 10 kg of CO_2 gas from 1 MPa and 0.5 m^3 to 0.2 m^3 and 50 $^\circ$C. The process is polytropic with $n = 1.2$. How much energy transfers via

(a) work?

(b) heat transfer?

Given and find

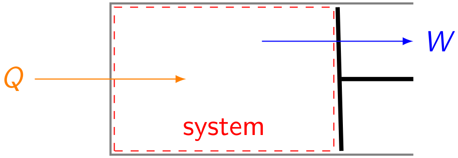
- **given:**
  - ◇ $m = 10$ kg
  - ◇ $p_1 = 1$ MPa $= 1000$ kPa, $V_1 = 0.5$ m^3
  - ◇ $V_2 = 0.2$ m^3, $T_2 = 50$ °C
- **find:**
  (a) W
  (b) Q

Assumptions

- closed system ($\Delta m = 0$)
- no center-of-mass motion ($\Delta \text{KE} = \Delta \text{PE} = 0$)
- quasi-equilibrium process (equilibrium state is well-defined)
- polytropic process ($p_1 V_1^n = p_2 V_2^n$) with $n = 1.2$

System diagram and basic equations

- **system diagram:**



- **basic equations:**

$$\Delta \mathrm{KE} + \Delta \mathrm{PE} + \Delta U = Q - W$$

$$W = \int_{V_1}^{V_2} p \, \mathrm{d}V$$

Solution to part (a)

- process is polytropic, so $pV^n = c$ (some constant)
- work is therefore

$$W = \int_{V_1}^{V_2} p\mathrm{d}V = c\int_{V_1}^{V_2} V^{-n}\mathrm{d}V = \frac{c\left(V_2^{1-n} - V_1^{1-n}\right)}{1-n}$$

- but $c = p_1 V_1^n = p_2 V_2^n$, so

$$W = \frac{p_2 V_2^n V_2^{1-n} - p_1 V_1^n V_1^{1-n}}{1-n} = \frac{p_2 V_2 - p_1 V_1}{1-n}$$

- from polytropic assumption, $p_2 V_2^n = p_1 V_1^n$, so

$$p_2 = \frac{p_1 V_1^n}{V_2^n} = \frac{(1000\mathrm{kPa})(0.5\mathrm{m}^3)^{1.2}}{(0.2\mathrm{m}^3)^{1.2}} = 3000\mathrm{kPa}$$

Solution to part (a) (continued)

- plugging numbers into the expression for work,

$$\begin{aligned} W &= \frac{p_2 V_2 - p_1 V_1}{1-n} \\ &= \frac{(3000\text{kPa})(0.2\text{m}^3) - (1000\text{kPa})(0.5\text{m}^3)}{1-(1.2)} \\ &= -500\text{kJ} \end{aligned}$$

- since W is negative, surroundings do work on system

Solution to part (b)

- since $\Delta \text{KE} = \Delta \text{PE} = 0$, 1st law simplifies to $\Delta U = Q - W$
- change in internal energy is $\Delta U = m(u_2 - u_1)$
- can find u_1 and u_2 in superheated vapor table for CO_2
- at $T_2 = 50\ ^\circ\text{C}$ and $p_2 = 3$ MPa, $u_2 = 335.94$ kJ/kg

Temp. (C)	Volume (m^3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kg/K)
	p = 30.0 bar = 3.0 MPa, T_{sat} = -5.55°C			
Sat.	0.012207	284.09	320.71	1.2098
0	0.012931	290.52	329.32	1.2416
5	0.013525	295.83	336.41	1.2673
10	0.014082	300.83	343.08	1.2911
15	0.014610	305.60	349.44	1.3134
20	0.015116	310.21	355.56	1.3344
30	0.016074	319.07	367.30	1.3738
40	0.016980	327.61	378.55	1.4104
50	0.017847	335.94	389.48	1.4447

Solution to part (b) (continued)

- need to interpolate to find u_1 at $p_1 = 1$ MPa
  - specific volume in state 1 is $v_1 = V_1/m = 0.05$ m^3/kg
  - nearby, $u_a = 316.68$ kJ/kg at $v_a = 0.0491$ m^3/kg
  - and $u_b = 320.21$ kJ/kg at $v_b = 0.050196$

Temp. (C)	Volume (m^3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kg/K)
	p = 10.0 bar = 1.0 MPa, T_{sat} = -40.12°C			
Sat.	0.038453	283.94	322.39	1.3835
-40	0.038485	284.03	322.52	1.3841
-35	0.039766	287.85	327.61	1.4057
-30	0.041012	291.59	332.60	1.4264
-25	0.042228	295.26	337.49	1.4463
-20	0.043418	298.89	342.31	1.4655
-15	0.044587	302.49	347.08	1.4842
-10	0.045738	306.06	351.80	1.5023
-5	0.046872	309.61	356.48	1.5199
0	0.047992	313.15	361.14	1.5371
5	0.049100	316.68	365.78	1.5540
10	0.050196	320.21	370.40	1.5704

  - so interpolated specific internal energy is

$$u_1 = u_a + \frac{u_b - u_a}{v_b - v_a}(v_1 - v_a) = 319.58 \text{kJ/kg}$$

Solution to part (b) (continued)

- rearranging 1st law and plugging in ΔU definition gives

$$\begin{aligned} Q &= \Delta U + W = m(u_2 - u_1) + W \\ &= (10\text{kg})[(335.94\text{kJ/kg}) - (319.58\text{kJ/kg})] + (-500\text{kJ}) \\ &= -336.3\text{kJ} \end{aligned}$$

- since $Q < 0$, heat transfers from system to surroundings

What did we learn?

- superheated CO_2 gas got squished from 0.5 to 0.2 m^3
- gas temperature rose from $\sim$10 to 50 $^\circ$C
- gas pressure rose from 10 to 30 bar
- system gained 500 kJ from surroundings via work
- system lost 336 kJ to surroundings via heat transfer
- remaining 164 kJ ended up as internal energy

Outline

Applying the 1st law with property tables

Example #1

Example #2

Problem statement

A rigid tank has two compartments. The left-hand compartment contains 0.005 m^3 of saturated liquid water at 80 $^\circ$C. The right-hand compartment contains 10 m^3 of water at 200 $^\circ$C and 70 kPa. The divider is removed and the water mixes. The water is then heated to saturated vapor.

(a) What is the internal energy after mixing but before heating?

(b) How much energy is added via heat transfer?

Given and find

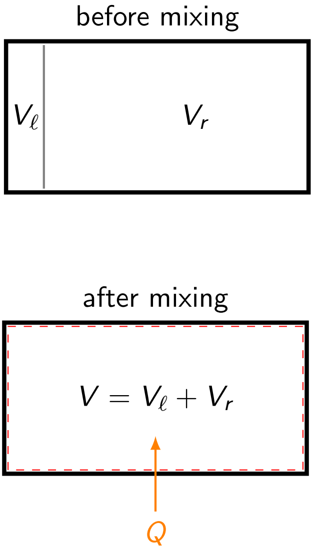
- **given:**
  - ◇ left: saturated liquid with $V_\ell = 0.005\ \text{m}^3$, $T_\ell = 80\ ^\circ\text{C}$
  - ◇ right: $V_r = 10\ \text{m}^3$, $T_r = 200\ ^\circ\text{C}$, $p_r = 70$ kPa
  - ◇ from saturation table and superheated vapor table,
    - ▶ $v_\ell = 0.001029\ \text{m}^3/\text{kg}$, $u_\ell = 334.96$ kJ/kg
    - ▶ $v_r = 3.108\ \text{m}^3/\text{kg}$, $u_r = 2659.3$ kJ/kg
- **find:**
  - ◇ U_1 (internal energy after mixing but before heating)
  - ◇ Q

Assumptions and basic equation

- **assumptions:**
  - ◇ closed system (no mass transfer into tank)
  - ◇ rigid tank (volume is constant, $W = 0$)
  - ◇ no center-of-mass motion ($\Delta\text{KE} = \Delta\text{PE} = 0$)
- **basic equation:**

$$\Delta\text{KE} + \Delta\text{PE} + \Delta U = Q - W$$

System diagram



Solution to part (a)

- internal energy after mixing is

$$U_1 = U_\ell + U_r = m_\ell u_\ell + m_r u_r = \frac{V_\ell u_\ell}{v_\ell} + \frac{V_r u_r}{v_r}$$

- from saturation table and superheated vapor table,
  - ◇ $v_\ell = 0.001029$ m^3/kg, $u_\ell = 334.96$ kJ/kg
  - ◇ $v_r = 3.108$ m^3/kg, $u_r = 2659.3$ kJ/kg
- so internal energy is

$$U_1 = \frac{(0.005\text{m}^3)(334.96\text{kJ/kg})}{0.001029\text{m}^3/\text{kg}} + \frac{(10\text{m}^3)(2659.3\text{kJ/kg})}{3.108\text{m}^3/\text{kg}}$$
$$= 1.628\text{MJ} + 8.556\text{MJ} = 10.18\text{MJ}$$

Solution to part (b)

- since ΔKE = ΔPE = W = 0, 1st law simplifies to

$$Q = \Delta U = U_2 - U_1 = mu_2 - U_1$$

- can find u_2 by interpolating saturation table
- mass is $m = m_\ell + m_r = V_\ell / v_\ell + V_r / v_r = 8.08$ kg
- so $v_2 = (V_\ell + V_r)/m = 1.239$ m^3/kg

Solution to part (b) (continued)

Temp. (C)	Press. (bar)	Volume (v_f, m^3/kg)	Internal Energy (u_f, kJ/kg)	Enthalpy (h_f, kJ/kg)	Entropy (s_f, kJ/kg/K)	Volume (v_g, m^3/kg)	Internal Energy (u_g, kJ/kg)	Enthalpy (h_g, kJ/kg)	Entropy (s_g, kJ/kg/K)
100	1.0142	0.0010435	419.06	419.17	1.3072	1.6718	2506.0	2675.6	7.3541
110	1.4338	0.0010516	461.26	461.42	1.4188	1.2093	2517.7	2691.1	7.2381

- interpolating saturation table with $v_2 = 1.239$ m^3/kg gives

$$u_2 = u_a + \frac{u_b - u_a}{v_b - v_a}(v_2 - v_a) = 2517.0\text{kJ/kg}$$

- so heat transfer is

$$Q = mu_2 - U_1 = (8.08\text{kg})(2517.0\text{kJ/kg}) - 10180\text{kJ}$$
$$= 10.16\text{MJ}$$