Lecture 15 – Equipment models

Purdue ME 200, Thermodynamics I

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Outline

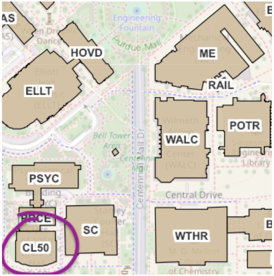
Exam reminders

Nozzles and diffusers

Turbines, compressors and pumps

When and where is exam #1?

- 6:30–7:30 PM in CL-1950 room 224 on Thursday, February 16



- no class Friday, February 17
- homework 14-16 due 11:59 PM Monday, February 20

What does exam #1 cover?

- lectures 1-12 and homework 1-13
- 1 ‘concepts’ problem, 2 homework-style problems
- closed book, closed notes
- we’ll provide equation sheet and any necessary tables
- don’t interpolate tables; just use closest data point

Other exam #1 logistics

- practice problems & exam are on Brightspace
- arrive 10-15 minutes early
- bring pencils, eraser, scientific calculator
    - Texas Instruments: TI-30X, TI-36X
    - Cassio: fx-115 or fx-991
- exams 1, 2 and 3 are each 20% of course grade
- final exam is 25%

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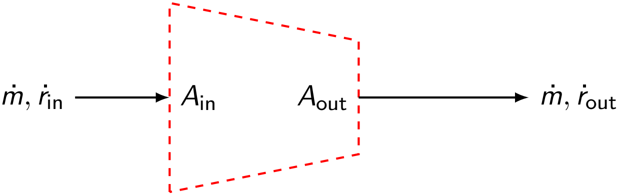
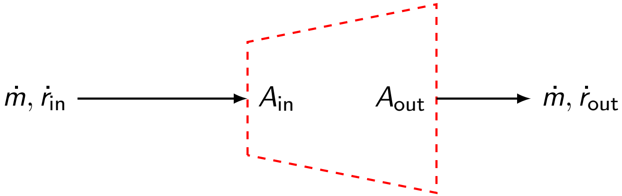
Nozzles



- nozzles increase flow velocity: $\dot{r}_{out} > \dot{r}_{in}$
- and decrease pressure: $p_{out} < p_{in}$

Diffusers



- diffusers are ‘backwards nozzles’
- they decrease flow velocity: $\dot{r}_{out} < \dot{r}_{in}$
- and increase pressure: $p_{out} > p_{in}$

Typical assumptions for nozzles and diffusers

- steady state
- one-dimensional flow
- no change in PE
- no boundary/shaft/electrical/etc. work
- under these assumptions, diffusers and nozzles satisfy

$$\dot{m}\left[\frac{1}{2}(\dot{r}_{\text{in}}^2 - \dot{r}_{\text{out}}^2) + h_{\text{in}} - h_{\text{out}}\right] = -\dot{Q}$$

- if $\dot{Q} = 0$ (good insulation or small ΔT across boundary), then

$$\frac{1}{2}(\dot{r}_{\text{in}}^2 - \dot{r}_{\text{out}}^2) = h_{\text{out}} - h_{\text{in}}$$

Example

Consider a well-insulated diffuser with 0.02 m^2 inlet cross-sectional area. Steam enters at 300 m/s, 5 bar and 180 °C. Steam exits at 7 bar and 200 °C.

(a) What is the exit velocity?

(b) What is the outlet cross-sectional area?

Given and find

- **given:**
  - $A_{\text{in}} = 0.02 \text{ m}^2$, $\dot{r}_{\text{in}} = 300 \text{ m/s}$
  - $p_{\text{in}} = 5 \text{ bar}$, $T_{\text{in}} = 180\ ^\circ\text{C}$
  - $p_{\text{out}} = 7 \text{ bar}$, $T_{\text{out}} = 200\ ^\circ\text{C}$
- **find:**
  (a) $\dot{r}_{\text{out}}$
  (b) A_{out}

Assumptions and basic equations

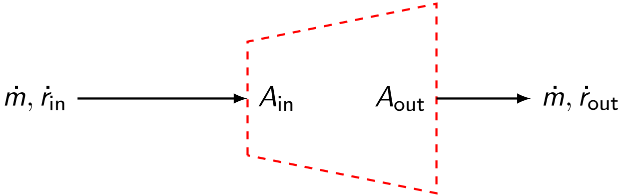
- **assume:**
  - ◇ steady state
  - ◇ one-dimensional flow ($\dot{m} = \rho A \dot{r}$)
  - ◇ no change in PE ($z_{\text{out}} = z_{\text{in}}$)
  - ◇ no boundary/shaft/electrical/etc. work ($\dot{W} = 0$)
  - ◇ well-insulated ($\dot{Q} = 0$)
- **basic equations:**
  - ◇ steady-state, single-input single-output CoM:

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}}$$

  - ◇ steady-state, single-input single-output 1st law:

$$\dot{m}\left[\frac{1}{2}(\dot{r}_{\text{in}}^2 - \dot{r}_{\text{out}}^2) + g(z_{\text{in}} - z_{\text{out}}) + h_{\text{in}} - h_{\text{out}}\right] = \dot{W} - \dot{Q}$$

System diagram



Solution to part (a)

- from the 1st law for open systems in steady state,

$$\frac{1}{2}(\dot{r}_{\text{in}}^2 - \dot{r}_{\text{out}}^2) = h_{\text{out}} - h_{\text{in}}$$

$$\implies \dot{r}_{\text{out}} = \sqrt{\dot{r}_{\text{in}}^2 - 2(h_{\text{out}} - h_{\text{in}})}$$

- superheated steam tables give specific enthalpies:

Temp. (C)	Volume (m^3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kg/K)	Volume (m^3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kg/K)
	p = 5.0 bar = 0.50 MPa, T_{sat} = 151.83°C				p = 7.0 bar = 0.70 MPa, T_{sat} = 164.95°C			
Sat.	0.37481	2560.7	2748.1	6.8207	0.27277	2571.8	2762.8	6.7071
180	0.40466	2610.1	2812.4	6.9673	0.28476	2600.0	2799.4	6.7893
200	0.42503	2643.3	2855.8	7.0610	0.30000	2635.3	2845.3	6.8884

Solution to part (a) (continued)

- plugging in numbers,

$$\begin{aligned}\dot{r}_{\text{out}} &= \sqrt{\dot{r}_{\text{in}}^2 - 2(h_{\text{out}} - h_{\text{in}})} \\ &= \sqrt{(300\text{m/s})^2 - 2(2845\text{kJ/kg} - 2812\text{kJ/kg})(1000\text{J/kJ})} \\ &= 155.6\text{m/s}\end{aligned}$$

- units check: $\text{J/kg} = \text{N m/kg} = (\text{kg m/s}^2)\text{m/kg} = \text{m}^2/\text{s}^2$

Solution to part (b)

- mass inflow and outflow are equal and one-dimensional, so

$$\dot{m} = \rho_{\text{in}} A_{\text{in}} \dot{r}_{\text{in}} = \rho_{\text{out}} A_{\text{out}} \dot{r}_{\text{out}}$$

$$\Longrightarrow A_{\text{out}} = \frac{\rho_{\text{in}} A_{\text{in}} \dot{r}_{\text{in}}}{\rho_{\text{out}} \dot{r}_{\text{out}}} = \frac{v_{\text{out}} A_{\text{in}} \dot{r}_{\text{in}}}{v_{\text{in}} \dot{r}_{\text{out}}}$$

- superheated steam tables give specific volumes, so

$$A_{\text{out}} = \frac{(0.3\text{m}^3/\text{kg})(0.02\text{m}^2)(300\text{m/s})}{(0.405\text{m}^3/\text{kg})(155.6\text{m/s})} = 0.0286\text{m}^2$$

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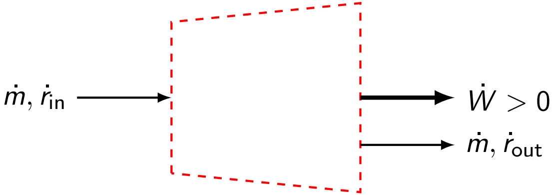
Turbines



- turbines change the state of a working fluid to produce power

Typical assumptions for turbines

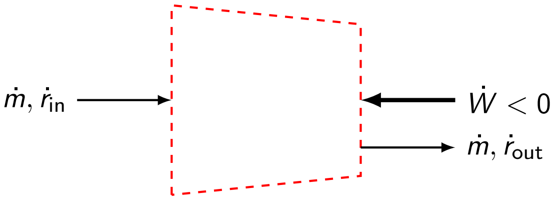
- steady state
- one-dimensional flow
- no change in PE (except for some hydro turbines)
- no change in KE (except for hydro and wind turbines)
- under these assumptions, steam and gas turbines satisfy

$$\dot{W} = \dot{Q} + \dot{m}(h_{\text{in}} - h_{\text{out}})$$

- if heat transfer is negligible ($\dot{Q} = 0$), then

$$\dot{W} = \dot{m}(h_{\text{in}} - h_{\text{out}})$$

Compressors and pumps



- compressors and pumps are ‘backwards turbines’
- they consume power to change the state of a working fluid
- working fluid is a gas in compressors, liquid in pumps

Typical assumptions for compressors

- steady state
- one-dimensional flow
- no change in PE
- no change in KE
- under these assumptions, compressors satisfy

$$\dot{W} = \dot{Q} + \dot{m}(h_{\text{in}} - h_{\text{out}})$$

- if heat transfer is negligible ($\dot{Q} = 0$), then

$$\dot{W} = \dot{m}(h_{\text{in}} - h_{\text{out}})$$

Typical assumptions for pumps

- steady state
- one-dimensional flow
- under these assumptions, pumps satisfy

$$\dot{m}\left[\frac{1}{2}(\dot{r}_{\text{in}}^2 - \dot{r}_{\text{out}}^2) + g(z_{\text{in}} - z_{\text{out}}) + h_{\text{in}} - h_{\text{out}}\right] = \dot{W} - \dot{Q}$$

- if heat transfer is negligible ($\dot{Q} = 0$), then

$$\dot{m}\left[\frac{1}{2}(\dot{r}_{\text{in}}^2 - \dot{r}_{\text{out}}^2) + g(z_{\text{in}} - z_{\text{out}}) + h_{\text{in}} - h_{\text{out}}\right] = \dot{W}$$

Example

Air enters a compressor through 0.1 m^2 at 290 K, 1 bar and 6 m/s. Air exits at 450 K. The compressor loses 3 kW to the surroundings via heat transfer. Find the compressor power.

Given and find

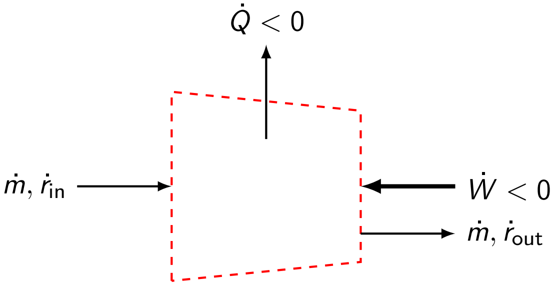
- **given:**
  - $A_{\text{in}} = 0.1\ \text{m}^2$, $\dot{r}_{\text{in}} = 6\ \text{m/s}$
  - $p_{\text{in}} = 1\ \text{bar}$, $T_{\text{in}} = 290\ \text{K}$
  - $T_{\text{out}} = 450\ \text{K}$
  - $\dot{Q} = -3\ \text{kW}$
- **find:**
  - $\dot{W}$

Assumptions and basic equations

- **assume:**
  - ◇ steady state
  - ◇ one-dimensional flow ($\dot{m} = \rho A \dot{r}$)
  - ◇ no change in PE ($z_{\text{out}} = z_{\text{in}}$)
  - ◇ no change in KE ($\dot{r}_{\text{out}} = \dot{r}_{\text{in}}$)
  - ◇ ideal gas ($pv = RT$)
- **basic equations:**
  - ◇ steady-state, single-inlet single-outlet 1st law:

$$\dot{m}\left[\frac{1}{2}(\dot{r}_{\text{in}}^2 - \dot{r}_{\text{out}}^2) + g(z_{\text{in}} - z_{\text{out}}) + h_{\text{in}} - h_{\text{out}}\right] = \dot{W} - \dot{Q}$$

System diagram



Solution

- from the 1st law in steady state,

$$\dot{W} = \dot{Q} + \dot{m}(h_{\text{in}} - h_{\text{out}})$$

- because flow is one-dimensional,

$$\dot{m} = \frac{A_{\text{in}} \dot{r}_{\text{in}}}{v_{\text{in}}}$$

- from the ideal gas law, $v_{\text{in}} = RT_{\text{in}}/p_{\text{in}}$, so

$$\dot{W} = \dot{Q} + \frac{p_{\text{in}} A_{\text{in}} \dot{r}_{\text{in}} (h_{\text{in}} - h_{\text{out}})}{RT_{\text{in}}}$$

Solution (continued)

- ideal gas table for air gives specific enthalpies:

Temp. [K]	h [kJ/kg]
290	290.1
450	452.0

- individual gas constant for air is $R = 0.287$ kJ/(kg K), so

$$\begin{aligned}\dot{W} &= \dot{Q} + \frac{p_{\text{in}} A_{\text{in}} \dot{r}_{\text{in}} (h_{\text{in}} - h_{\text{out}})}{R T_{\text{in}}} \\ &= -3\text{kW} + \frac{(101\text{kPa})(0.1\text{m}^2)(6\text{m/s})(290\text{kJ/kg} - 452\text{kJ/kg})}{(0.287\text{kJ/(kg K)})(290\text{K})} \\ &= -118.0\text{kW}\end{aligned}$$

- since $\dot{W} < 0$, surroundings do work on compressor