Lecture 17 – Integrating equipment into machines

Purdue ME 200, Thermodynamics I

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Outline

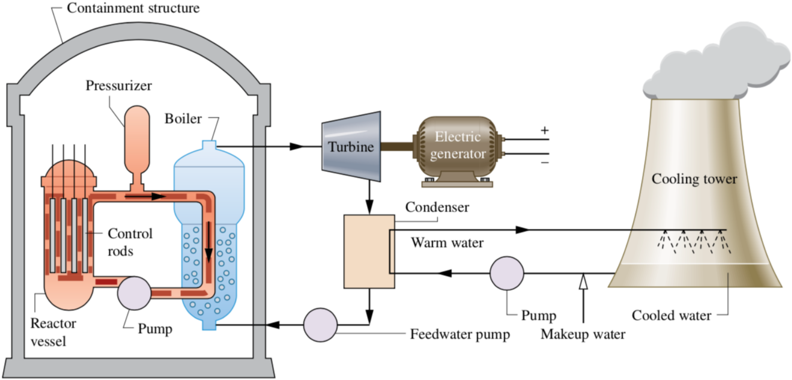
Integrating equipment into machines

Example

Equipment integration

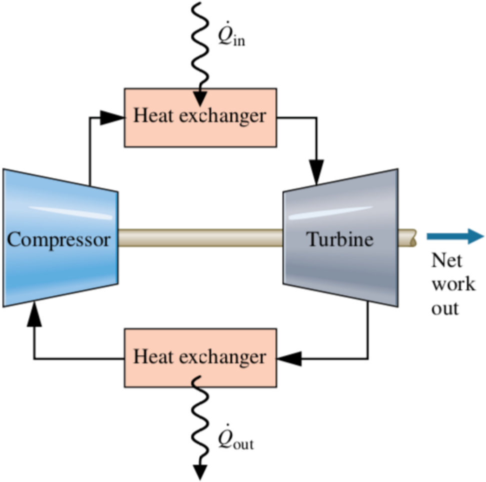
- real machines usually combine many pieces of equipment
  - nozzles and diffusers (backward nozzles)
  - turbines and pumps/compressors (backward turbines)
  - throttles
  - heat exchangers
- we can analyze machines by breaking them into subsystems
- choosing appropriate system boundaries can simplify analysis

A nuclear power plant



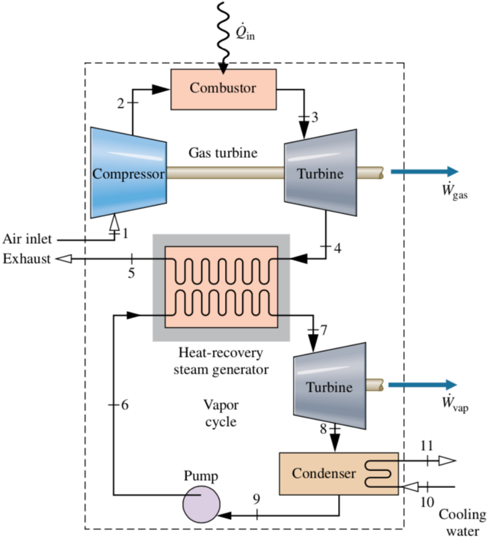
Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

A simple-cycle gas turbine



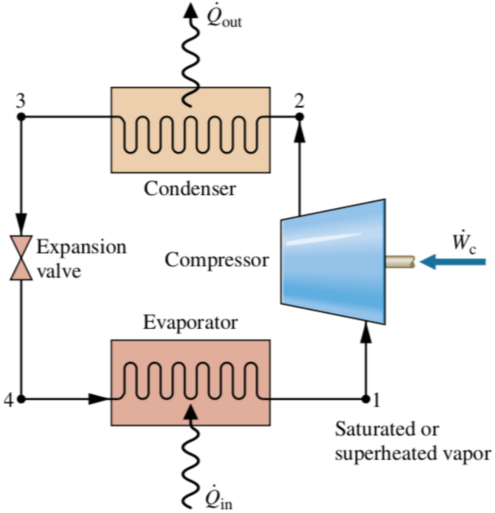
Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

A combined-cycle gas turbine



Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

A vapor-compression refrigerator



Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

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Example

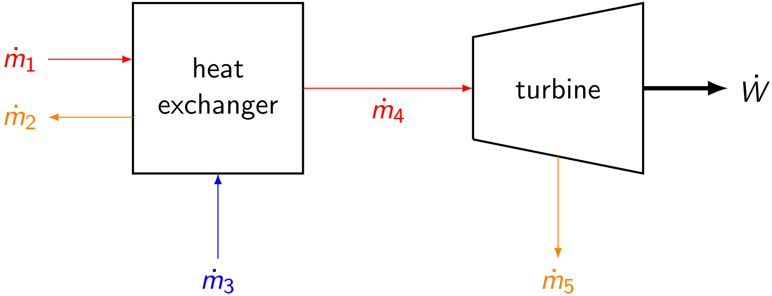
Problem statement

Water from a geothermal well enters a heat exchanger at 200 °C, 20 bar and 100 kg/s and exits at 140 °C. A second stream of water enters the heat exchanger at 40 °C, 3 bar and 10 kg/s, flows through a turbine, and exits at 7 kPa and 90% quality.

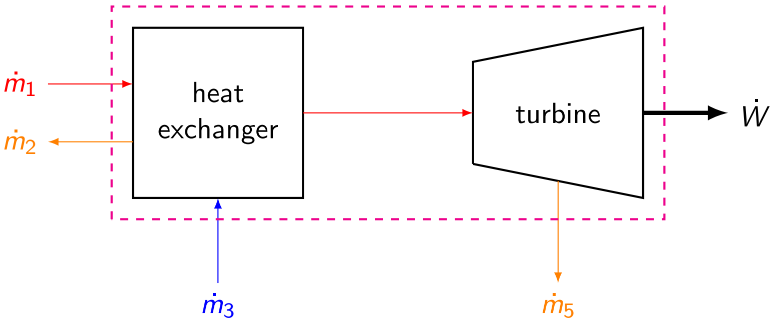
(a) Find the turbine power.

(b) Find the water temperature at the turbine inlet.

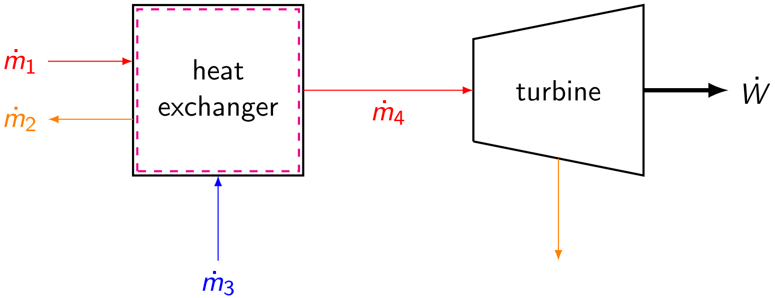
System diagram



System boundary for part (a)



System boundary for part (b)



Given and find

- **given:**
  - ◇ $p_1 = 20$ bar, $T_1 = 200$ °C, $\dot{m}_1 = 100$ kg/s
  - ◇ $p_3 = 3$ bar, $T_3 = 40$ °C, $\dot{m}_3 = 10$ kg/s
  - ◇ $p_5 = 7$ kPa, $x_5 = 0.9$
- **find:**
  - (a) $\dot{W}$
  - (b) T_4

Assumptions

- steady state
- no changes in KE or PE
- no mixing inside heat exchanger
- no pressure changes across heat exchanger
- no heat transfer across boundary of heat exchanger or turbine

Basic equations

- steady-state conservation of mass:

$$\sum_{\text{in}} \dot{m}_{\text{in}} = \sum_{\text{out}} \dot{m}_{\text{out}}$$

- steady-state 1st law with no ΔKE, ΔPE or $\dot{Q}$:

$$\sum_{\text{in}} \dot{m}_{\text{in}} h_{\text{in}} = \dot{W} + \sum_{\text{out}} \dot{m}_{\text{out}} h_{\text{out}}$$

Solution to part (a)

- steady-state 1st law on combined system:

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{W} + \dot{m}_2 h_2 + \dot{m}_5 h_5$$

- steady-state CoM on first heat exchanger pipe: $\dot{m}_2 = \dot{m}_1$
- steady-state CoM on second heat exchanger pipe: $\dot{m}_4 = \dot{m}_3$
- steady-state CoM on turbine: $\dot{m}_5 = \dot{m}_4$
- so steady-state 1st law on combined system simplifies to

$$\begin{aligned}\dot{W} &= \dot{m}_1 h_1 + \dot{m}_3 h_3 - \dot{m}_2 h_2 - \dot{m}_5 h_5 \\ &= \dot{m}_1 (h_1 - h_2) + \dot{m}_3 (h_3 - h_5)\end{aligned}$$

$\Longrightarrow$ need specific enthalpies in states 1, 2, 3 and 5

Solution to part (a) (continued)

- in state 1, $h_1 \approx h_{\text{liq}}(T_1) = 852.3$ kJ/kg
- in state 2, $h_2 \approx h_{\text{liq}}(T_2) = 589.2$ kJ/kg
- in state 3, $h_3 \approx h_{\text{liq}}(T_3) = 167.5$ kJ/kg
- in state 5 ($p_5 = 7$ kPa, $x_5 = 0.9$),
  - ◇ $h_{\text{liq}} = 697$ kJ/kg and $h_{\text{vap}} = 2763$ kJ/kg
  - ◇ so $h_5 = h_{\text{liq}} + x_5(h_{\text{vap}} - h_{\text{liq}}) = 2556$ kJ/kg
- mass flow rates $\dot{m}_1$ and $\dot{m}_3$ are given, so

$$\begin{aligned}
\dot{W} &= \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_5) \\
&= (100\text{kg/s})(852.3\text{kJ/kg} - 589.2\text{kJ/kg}) \\
&\quad + (10\text{kg/s})(167.5\text{kJ/kg} - 2556\text{kJ/kg}) \\
&= 2395\text{kW}
\end{aligned}$$

Solution to part (b)

- steady-state 1st law on heat exchanger:

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4$$

- but $\dot{m}_2 = \dot{m}_1$ and $\dot{m}_4 = \dot{m}_3$, so

$$\begin{aligned} & \dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_1 h_2 + \dot{m}_3 h_4 \\ \iff & \dot{m}_1(h_1 - h_2) = \dot{m}_3(h_4 - h_3) \\ \iff & h_4 = h_3 + \frac{\dot{m}_1(h_1 - h_2)}{\dot{m}_3} \end{aligned}$$

Solution to part (b) (continued)

- plugging in numbers,

$$\begin{aligned} h_4 &= h_3 + \frac{\dot{m}_1(h_1 - h_2)}{\dot{m}_3} \\ &= 167.2\text{kJ/kg} + \frac{(100\text{kg/s})(852.3\text{kJ/kg} - 589.2\text{kJ/kg})}{10\text{kg/s}} \\ &= 2798\text{kJ/kg} \end{aligned}$$

- pressure is constant across heat exchanger, so $p_4 = p_3 = 3$ bar
- interpolating saturated vapor table at 3 bar and 2798 kJ/kg,

$$T_4 = 167.2^\circ\text{C}$$

Solution to part (b) (continued)

- we can check the h_4 calculation via 1st law on turbine:

$$\dot{W} = \dot{m}_4 h_4 - \dot{m}_5 h_5$$

- but $\dot{m}_5 = \dot{m}_4 = \dot{m}_3$, so

$$\begin{aligned} \dot{W} &= \dot{m}_3(h_4 - h_5) \\ \implies h_4 &= h_5 + \frac{\dot{W}}{\dot{m}_3} = 2556\text{kJ/kg} + \frac{2395\text{kW}}{10\text{kg/s}} \\ &= 2796\text{kJ/kg} \end{aligned}$$