Lecture 10 – Modeling liquids and solids

Purdue ME 200, Thermodynamics I

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Outline

Partial derivatives

- consider a function f of two variables, x and y
- the partial derivative of f with respect to x,

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x},$$

  $\diamond$ is the change in the output variable $z = f(x, y)$
  $\diamond$ per unit change in the input variable x
  $\diamond$ with y held constant

- in general, the partial derivative is a function of x and y

Notation for functions

- in math classes, we write $z = f(x, y)$
  - ◊ z denotes the variable (a number)
  - ◊ f denotes the function (a mapping from $\mathbf{R}^2$ to $\mathbf{R}$)
- in this class, we sometimes (sloppily) write

$$z = z(x, y)$$

to emphasize that z is a function only of x and y

Notation for partial derivatives

- in math classes, we write partial derivatives as

$$f_x(x, y) \text{ or } \frac{\partial f}{\partial x}$$

- in this class, we write partial derivatives as

$$\left(\frac{\partial z}{\partial x} \right)_y$$

to emphasize that
  - z is a function of x and y only
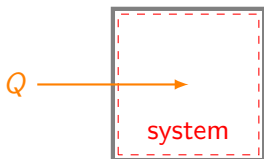 - y is held constant when taking the derivative

Outline

Specific heat at constant volume, c_v

- consider a simple compressible system in state (T, v)
- how much does its specific internal energy $u(T, v)$ change
  - ◇ per unit change in T
  - ◇ with v held constant?
- the answer is the **specific heat at constant volume**,

$$c_v(T, v) = \left(\frac{\partial u}{\partial T} \right)_v$$

How to measure c_v?

- we can't measure internal energy U directly
- so place system of mass m in rigid tank
  - ⋄ no bulk motion, so $\Delta KE = \Delta PE = 0$
  - ⋄ constant volume, so $W = 0$
  - ⋄ from 1st law for closed systems, $Q = \Delta U$
- add small, measured energy Q via heat transfer
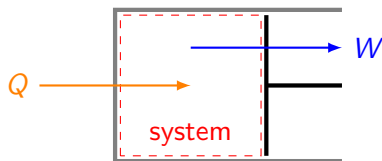- measure temperature change ΔT; then

$$\frac{Q}{m\Delta T} = \frac{\Delta U}{m\Delta T} = \frac{\Delta u}{\Delta T} \approx \left(\frac{\partial u}{\partial T}\right)_v = c_v(T, v)$$

Specific heat at constant pressure, c_p

- consider a simple compressible system in state (T, p)
- how much does its specific enthalpy $h(T, p)$ change
  - ◇ per unit change in T
  - ◇ with p held constant?
- the answer is the **specific heat at constant pressure**,

$$c_p(T, p) = \left(\frac{\partial h}{\partial T} \right)_p$$

How to measure c_p?

- we can't measure enthalpy $H = U + pV$ directly
- so place system of mass m in cylinder with free piston
  - ◇ no bulk motion, so $\Delta KE = \Delta PE = 0$
  - ◇ constant pressure, so $W = p\Delta V$ and $\Delta H = \Delta U + p\Delta V$
  - ◇ from 1st law for closed systems,

$$Q = \Delta U + W = \Delta U + p\Delta V = \Delta H$$

- add small, measured energy Q via heat transfer
- measure temperature change ΔT; then

$$\frac{Q}{m\Delta T} = \frac{\Delta H}{m\Delta T} = \frac{\Delta h}{\Delta T} \approx \left(\frac{\partial h}{\partial T}\right)_p = c_p(T, p)$$

Specific heats and state transformations

- for simple compressible systems,
  - ⋄ any two independent intensive properties define the state
  - ⋄ it doesn't matter *which* two properties we choose
  - ⋄ for example, sometimes we choose (T, v); other times (T, p)
- equations of state, like $pv = RT$, allow state transformations
- for example, suppose we
  - ⋄ choose state (T, v)
  - ⋄ have equation of state $p = f(T, v)$
- then $c_p(T, p) = c_p(T, f(T, v)) =$ function of (T, v) only
- similarly, we can write $c_v(T, v)$ as a function of (T, p) only

Notes on specific heats

- specific heats are intensive properties
- they're only defined in single-phase regions
- $c_p \geq c_v$ since there's no boundary work when v is constant
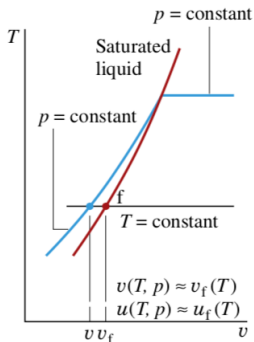- $k = c_p/c_v$ is called the **specific heat ratio**

Outline

Approximating compressed liquids as saturated liquids

- we only have compressed liquid tables for a few substances
- for other compressed liquids, use saturated liquid properties

Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

Saturated liquid approximation

- for a compressed liquid in state (T, p),

$$v(T, p) \approx v_{\text{liq}}(T)$$
$$u(T, p) \approx u_{\text{liq}}(T)$$
$$h(T, p) \approx u_{\text{liq}}(T) + p v_{\text{liq}}(T)$$

- but $h_{\text{liq}}(T) = u_{\text{liq}}(T) + p_{\text{sat}}(T) v_{\text{liq}}(T)$, so

$$h(T, p) \approx [h_{\text{liq}}(T) - p_{\text{sat}}(T) v_{\text{liq}}(T)] + p v_{\text{liq}}(T)$$
$$\implies h(T, p) \approx h_{\text{liq}}(T) + \underbrace{(p - p_{\text{sat}}(T)) v_{\text{liq}}(T)}_{\text{usually negligible}}$$

- so enthalpy can usually be approximated by

$$h(T, p) \approx h_{\text{liq}}(T)$$

The incompressible substance model

- most liquids and solids are ~incompressible:
  - ◇ v is ~constant
  - ◇ u ~depends on T only
- for incompressible substances, specific heats are equal:

$$c_v(T) = \left(\frac{\partial}{\partial T} u(T)\right)_v = \frac{\mathrm{d}u}{\mathrm{d}T}$$

$$c_p(T) = \left(\frac{\partial}{\partial T}(u(T) + pv)\right)_p = \frac{\mathrm{d}u}{\mathrm{d}T} = c_v(T)$$

- so we can write the (one and only) specific heat as $c(T)$

Δu and Δh for incompressible substances

- from the fundamental theorem of calculus,

$$\Delta u = u_2 - u_1 = \int_{T_1}^{T_2} \frac{\mathrm{d}u}{\mathrm{d}T} \mathrm{d}T$$
$$= \int_{T_1}^{T_2} c(T)\mathrm{d}T$$

- also, $\Delta h = h_2 - h_1 = u_2 + p_2 v - (u_1 + p_1 v)$, so

$$\Delta h = \Delta u + v\Delta p$$

- if c is approximately constant with respect to T, then

$$\Delta u \approx c\Delta T$$
$$\Delta h \approx c\Delta T + v\Delta p$$

Summary: finding properties of liquids and solids

- 3 ways to find properties of subcooled (compressed) liquids:
    1. compressed liquid table
    2. saturated liquid approximation
    3. incompressible substance model
- for solids, use the incompressible substance model

Outline

Problem statement

Inside an insulated box, a 20 kg block of steel at 100 °C comes into contact with a block of copper at 4 °C. The blocks reach thermal equilibrium at 37 °C. What is the mass of the copper block? Use 0.49 and 0.39 kJ/kg/K for the specific heats of steel and copper.

Given and find

- **given:**
  - $m_s = 20$ kg, $T_s = 100$ °C, $c_s = 0.49$ kJ/kg/K
  - $T_c = 100$ °C, $c_c = 0.39$ kJ/kg/K
  - $T_2 = 37$ °C
- **find:**
  - m_c

Assumptions and basic equations
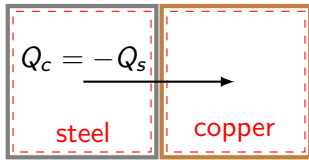
- **assume:**
  - ⋄ closed system
  - ⋄ no bulk motion ($\Delta KE_s = \Delta PE_s = \Delta KE_c = \Delta PE_c = 0$)
  - ⋄ adiabatic container (so $Q_s = -Q_c$)
  - ⋄ incompressible substances, constant specific heats
    - ▶ $\Delta u_s = (T_2 - T_s)c_s$
    - ▶ $\Delta u_c = (T_2 - T_c)c_c$
    - ▶ $W_s = W_c = 0$

- **basic equations:**

$$\Delta KE_s + \Delta PE_s + \Delta U_s = Q_s - W_s$$
$$\Delta KE_c + \Delta PE_c + \Delta U_c = Q_c - W_c$$

System diagrams

$Q_c = -Q_s$

steel

copper

Solution

- energy balances simplify to $\Delta U_s = Q_s$ and $\Delta U_c = Q_c$
- since $Q_c = -Q_s$, we have

$$\Delta U_c = -\Delta U_s$$

- from incompressibility/constant specific heat assumptions,

$$\Delta U_s = m_s \Delta u_s = m_s(T_2 - T_s)c_s$$
$$\Delta U_c = m_c \Delta u_c = m_c(T_2 - T_c)c_c$$

- therefore,

$$\begin{aligned}
m_c &= -\frac{m_s(T_2 - T_s)c_s}{(T_2 - T_c)c_c} \\
&= -\frac{(20\text{kg})(37^\circ\text{C} - 100^\circ\text{C})(0.49\text{kJ/kg/K})}{(37^\circ\text{C} - 4^\circ\text{C})(0.39\text{kJ/kg/K})} \\
&= 47.97\text{kg}
\end{aligned}$$