# Lecture 16 – More equipment models

Purdue ME 200, Thermodynamics I

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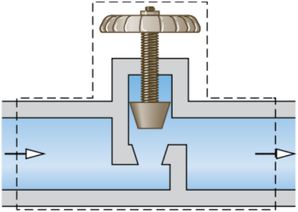
# Outline

Throttles

Heat exchangers

# Throttles



- throttles reduce a fluid's pressure by restricting flow
- often (but not always), throttles also reduce temperature

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Moran et al., *Fundamentals of Engineering Thermodynamics* (2019)

# Typical assumptions for throttles

- steady state
- one-dimensional flow
- no change in PE
- no boundary/shaft/electrical/etc. work
- no heat transfer
- under these assumptions, throttles satisfy

$$\frac{1}{2}(\dot{r}_{\text{in}}^2 - \dot{r}_{\text{out}}^2) = h_{\text{out}} - h_{\text{in}}$$

- often velocities are ∼equal well upstream and downstream, so

$$h_{\text{out}} = h_{\text{in}}$$

# Example

Saturated liquid R-134a enters a throttle at 8 bar and exits at 1.2 bar.

(a) Find the saturated liquid-vapor mixture quality at the exit.

(b) Find the temperature change across the device.

# Given and find

- **given:**
  - ◇ saturated liquid at inlet
  - ◇ $p_{\text{in}} = 8$ bar
  - ◇ $p_{\text{out}} = 1.2$ bar
- **find:**
  - (a) $x_{\text{out}}$
  - (b) $T_{\text{out}} - T_{\text{in}}$

# Assumptions and basic equations

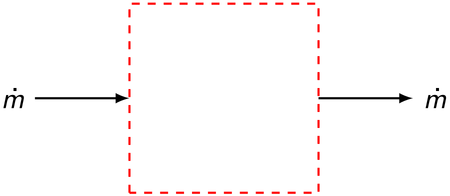
- **assumptions:**
  - ◇ steady state
  - ◇ no change in PE ($z_{\text{out}} = z_{\text{in}}$)
  - ◇ no change in KE ($\dot{r}_{\text{out}} = \dot{r}_{\text{in}}$)
  - ◇ no boundary/shaft/electrical/etc. work ($\dot{W} = 0$)
  - ◇ well-insulated ($\dot{Q} = 0$)
- **basic equations:**
  - ◇ steady-state 1st law for open systems with 1 inlet/outlet,

$$\dot{m}\left[\frac{1}{2}(\dot{r}_{\text{in}}^2 - \dot{r}_{\text{out}}^2) + g(z_{\text{in}} - z_{\text{out}}) + h_{\text{in}} - h_{\text{out}}\right] = \dot{W} - \dot{Q}$$

  - ◇ relationship between specific enthalpy and quality,

$$h_{\text{out}} = h_{\text{liq}} + x_{\text{out}}(h_{\text{vap}} - h_{\text{liq}})$$

# System diagram



# Solution to part (a)

- from relationship between specific enthalpy and quality,

$$x_{\text{out}} = \frac{h_{\text{out}} - h_{\text{liq}}}{h_{\text{vap}} - h_{\text{liq}}}$$

- from 1st law and assumptions, $h_{\text{out}} = h_{\text{in}}$
- from R-134a saturation table,
  - ◇ $h_{\text{in}} = 95.5$ kJ/kg at $p_{\text{in}} = 8$ bar
  - ◇ $h_{\text{liq}} = 22.5$ kJ/kg and $h_{\text{vap}} = 237$ kJ/kg at $p_{\text{out}} = 1.2$ bar
- so

$$x_{\text{out}} = \frac{95.5\text{kJ/kg} - 22.5\text{kJ/kg}}{237\text{kJ/kg} - 22.5\text{kJ/kg}} = 0.34$$

# Solution to part (b)

- from R-134a saturation table,
  - ◇ $T_{\text{in}} = 31.3$ °C at $p_{\text{in}} = 8$ bar
  - ◇ $T_{\text{out}} = -22.3$ °C at $p_{\text{out}} = 1.2$ bar
- so temperature change across throttle is

$$T_{\text{out}} - T_{\text{in}} = -22.3^{\circ}\text{C} - 31.3^{\circ}\text{C} = -53.6^{\circ}\text{C}$$

- refrigerant got much colder just by flowing past a restriction!

# Outline

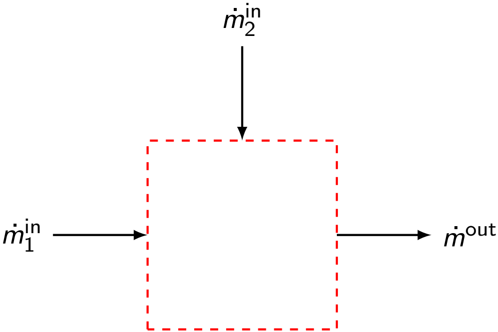
Throttles

Heat exchangers

# Heat exchangers

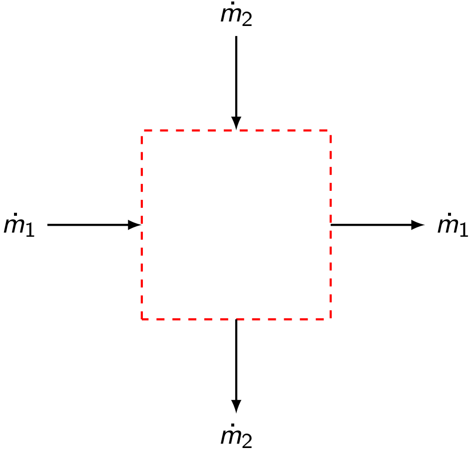
- heat exchangers transfer heat between substances
- typically, the substances are fluids
- there are several common configurations
- all involve multiple inflows and/or outflows
- in **mixing chambers**, fluids come into direct contact and mix
- in **recuperators**, a conductive wall separates the fluids

# Direct contact heat exchanger (mixing chamber)



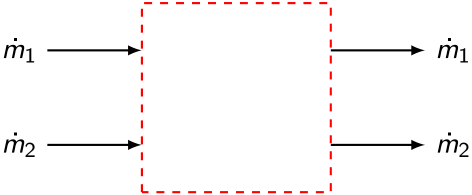
- fluids come into contact and mix

# Cross-flow heat exchanger (recuperator)



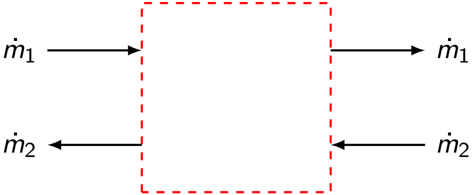
- internally, a conductive wall separates the fluids

# Parallel-flow heat exchanger (recuperator)



- internally, a conductive wall separates the fluids

# Counterflow heat exchanger (recuperator)



- internally, a conductive wall separates the fluids

# Typical assumptions for heat exchangers

- steady state
- one-dimensional flow
- no change in PE
- no change in KE
- no boundary/shaft/electrical/etc. work
- under these assumptions, heat exchangers satisfy

$$\sum_{j=1}^{N^{\text{out}}} \dot{m}_j h_j^{\text{out}} - \sum_{j=i}^{N^{\text{in}}} \dot{m}_i h_i^{\text{in}} = \dot{Q}$$

- if heat transfer across boundary is negligible ($\dot{Q} = 0$), then

$$\sum_{j=1}^{N^{\text{out}}} \dot{m}_j h_j^{\text{out}} = \sum_{j=i}^{N^{\text{in}}} \dot{m}_i h_i^{\text{in}}$$

# Example

R-134a enters a cross-flow heat exchanger at 0.1 kg/s, 1 MPa and 70 °C and exits at 35 °C. Water enters at 300 kPa and 15 °C and exits at 25 °C. Assuming constant pressures, find the water mass flow rate.

# Given and find

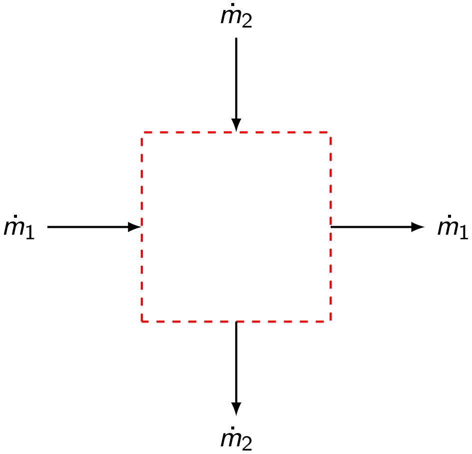
- **given:**
  - ◇ for R-134a,
    - ▶ $\dot{m}_1 = 0.1$ kg/s
    - ▶ $p_1^{\text{in}} = 1$ MPa, $T_1^{\text{in}} = 70\ ^\circ\text{C}$
    - ▶ $p_1^{\text{out}} = 1$ MPa, $T_1^{\text{out}} = 35\ ^\circ\text{C}$
  - ◇ for water,
    - ▶ $p_2^{\text{in}} = 300$ kPa, $T_2^{\text{in}} = 15\ ^\circ\text{C}$
    - ▶ $p_2^{\text{out}} = 300$ kPa, $T_2^{\text{out}} = 25\ ^\circ\text{C}$
- **find:**
  - ◇ $\dot{m}_2$

# Assumptions and basic equations

- **assumptions:**
  - ◇ steady state
  - ◇ no change in PE ($z_{\text{out}} = z_{\text{in}}$)
  - ◇ no change in KE ($\dot{r}_{\text{out}} = \dot{r}_{\text{in}}$)
  - ◇ no boundary/shaft/electrical/etc. work ($\dot{W} = 0$)
  - ◇ heat exchanger is well-insulated ($\dot{Q} = 0$)
- **basic equations:**
  - ◇ steady-state 1st law for open systems,

$$\dot{Q} + \sum_{i=1}^{N^{\text{in}}} \dot{m}_i^{\text{in}} \left[ \frac{1}{2} (\dot{r}_i^{\text{in}})^2 + g z_i^{\text{in}} + h_i^{\text{in}} \right] = \dot{W} + \sum_{j=1}^{N^{\text{out}}} \dot{m}_j^{\text{out}} \left[ \frac{1}{2} (\dot{r}_j^{\text{out}})^2 + g z_j^{\text{out}} + h_j^{\text{out}} \right]$$

# System diagram



# Solution

- after our simplifying assumptions, 1st law becomes

$$
\begin{aligned}
& \sum_{j=1}^{N^{\text{out}}} \dot{m}_j h_j^{\text{out}} = \sum_{j=i}^{N^{\text{in}}} \dot{m}_i h_i^{\text{in}} \\
\Longleftrightarrow\ & \dot{m}_1 h_1^{\text{out}} + \dot{m}_2 h_2^{\text{out}} = \dot{m}_1 h_1^{\text{in}} + \dot{m}_2 h_2^{\text{in}} \\
\Longleftrightarrow\ & \dot{m}_2 (h_2^{\text{out}} - h_2^{\text{in}}) = \dot{m}_1 (h_1^{\text{in}} - h_1^{\text{out}}) \\
\Longleftrightarrow\ & \dot{m}_2 = \dot{m}_1 \frac{h_1^{\text{in}} - h_1^{\text{out}}}{h_2^{\text{out}} - h_2^{\text{in}}}
\end{aligned}
$$

# Solution (continued)

- for R-134a,
  - ◇ $h_1^{\text{in}} = 304$ kJ/kg (superheated vapor table)
  - ◇ $h_1^{\text{out}} = 101$ kJ/kg (compressed liquid $\approx$ saturated liquid)
- for water,
  - ◇ $h_2^{\text{in}} = 63$ kJ/kg (compressed liquid $\approx$ saturated liquid)
  - ◇ $h_2^{\text{out}} = 105$ kJ/kg (compressed liquid $\approx$ saturated liquid)
- so

$$\begin{aligned}\dot{m}_2 &= \dot{m}_1 \frac{h_1^{\text{in}} - h_1^{\text{out}}}{h_2^{\text{out}} - h_2^{\text{in}}} \\ &= (0.1\text{kg/s}) \frac{304\text{kJ/kg} - 101\text{kJ/kg}}{105\text{kJ/kg} - 63\text{kJ/kg}} \\ &= 0.48\text{kg/s}\end{aligned}$$