Lecture 5 – Other types of work

Purdue ME 200, Thermodynamics I

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Outline

Boundary work

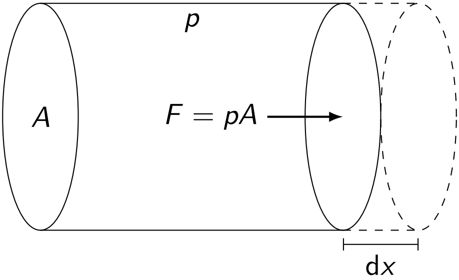
Shaft work

Spring work

Electric work

What is boundary work?

- the boundary of a system may expand during a process
- the system exerts pressure at the (moving) boundary
- this can be viewed as a force acting over a displacement
- in other words, the system does work on the surroundings
- if the system contracts, the surroundings do work on it

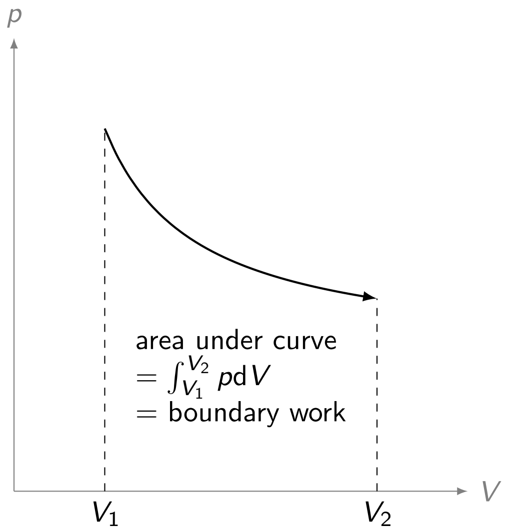


- system exerts force $F = pA$ on moving section of boundary
- differential work done by system over displacement $\mathrm{d}x$ is

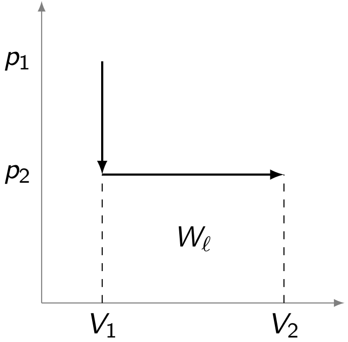
$$\delta W = F\mathrm{d}x = (pA)\mathrm{d}x = p\mathrm{d}V$$

- total work done by system is $W = \int_1^2 \delta W = \int_{V_1}^{V_2} p\mathrm{d}V$
- if p is a constant p_0 throughout the process, then $W = p_0 \Delta V$

Graphical interpretation



Boundary work is path dependent



- same initial and final states on left and right
- but different paths
- boundary work is also different:
  - $W_\ell = 0 + p_2(V_2 - V_1)$
  - $W_r = p_1(V_2 - V_1) + 0$

Example

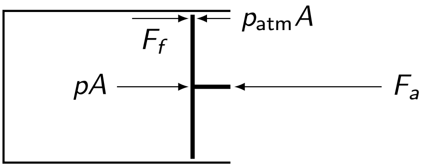
A gas occupies 0.001 m^3 inside a horizontal piston-cylinder device with piston area 0.01 m^2. A force pushes the piston, compressing the gas to 0.0004 m^3. The gas pressure is 500 kPa throughout this process. Friction between the piston and cylinder wall is 200 N.

(a) How much boundary work does the gas do on the piston?

(b) How much work does the applied force do on the piston?

Assumptions, system diagram and basic equations

- **assumptions:**
  - closed system
  - quasi-equilibrium process (so forces are balanced)
  - isobaric (constant pressure) process
- **basic equations:**
  - boundary work: $W = \int_{V_1}^{V_2} p \mathrm{d}V$
- **system diagram:**



Solution to (a)

- pressure is constant, $p = 500$ kPa, so gas does boundary work

$$W_b = \int_{V_1}^{V_2} p\mathrm{d}V = p\Delta V = (500\mathrm{kPa})[(0.0004\mathrm{m}^3) - (0.001\mathrm{m}^3)]$$
$$= -0.3\mathrm{kJ}$$

- $W_b < 0$ because piston does work on gas

Solution to (b)

- from force balance $F_f + pA + F_a - p_{\text{atm}} A = 0$, applied force is

$$
\begin{aligned}
F_a &= -[F_f + (p - p_{\text{atm}})A] \\
&= -[0.2\text{kN} + (500\text{kPa} - 101\text{kPa})(0.01\text{m}^2)] \\
&= -4.19\text{kN}
\end{aligned}
$$

- (constant) applied force acts over displacement

$$\Delta x = \Delta V/A = [(0.0004\text{m}^3) - (0.001\text{m}^3)]/(0.01\text{m}^2) = -0.06\text{m}$$

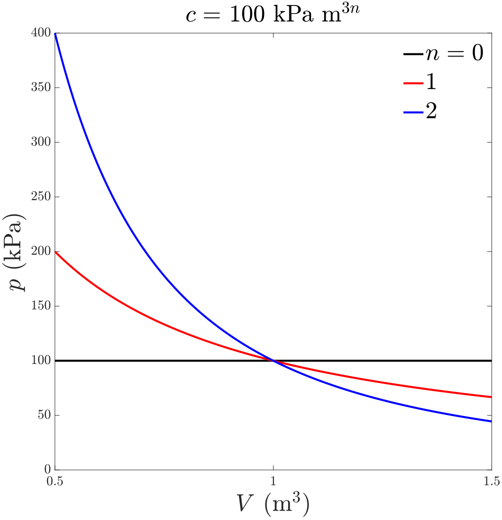
- so applied force does work

$$W_a = F_a \Delta x = (-4.19\text{kN})(-0.06\text{m}) = 0.251\text{kJ}$$

- $W_a > 0$ because applied force acts in direction of displacement

Polytropic processes

- a quasi-equilibrium process with $pV^n = c$ is **polytropic**
- n is the **polytropic coefficient**, c is a constant
- an **isobaric** (constant p) process is polytropic with $n = 0$
- for an ideal gas, an **isothermal** (constant T) process
  - is polytropic with $n = 1$
  - has $c = mRT$ (since $pV = mRT$)



$c = 100$ kPa m^{3n}
$n = 0$
1
2
400
350
300
250
200
150
100
50
0
0.5
1
1.5
p (kPa)
V (m^3)

Boundary work in polytropic processes

- in a polytropic process, system does boundary work

$$W = \int_{V_1}^{V_2} p\mathrm{d}V = \int_{V_1}^{V_2} \frac{c}{V^n}\mathrm{d}V = c\int_{V_1}^{V_2} V^{-n}\mathrm{d}V$$

- if $n = 1$, then (since $c = p_1 V_1 = p_2 V_2$)

$$W = c\ln(V_2/V_1) = p_1 V_1 \ln(V_2/V_1) = p_2 V_2 \ln(V_2/V_1)$$

- if $n \neq 1$, then (since $c = p_1 V_1^n = p_2 V_2^n$)

$$W = \frac{c\left(V_2^{1-n} - V_1^{1-n}\right)}{1-n} = \frac{(p_2 V_2^n)V_2^{1-n} - (p_1 V_1^n)V_1^{1-n}}{1-n}$$

$$= \frac{p_2 V_2 - p_1 V_1}{1-n}$$

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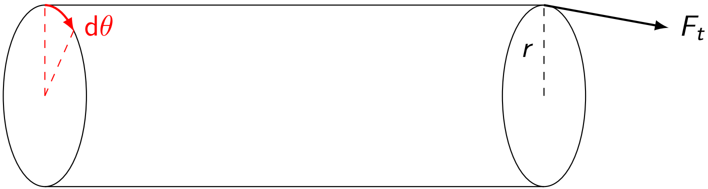
Shaft work



- shaft of radius r rotates at angular velocity ω
- shaft exerts tangential force F_t on surroundings
- differential displacement is $r\mathrm{d}\theta$
- shaft does differential work $\delta W = F_t r \mathrm{d}\theta = \tau \mathrm{d}\theta$
- shaft power is $\dot{W} = F_t v_t = (\tau / r)(r\omega) = \tau\omega$

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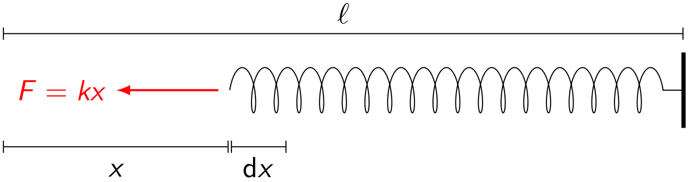
Spring work



- consider a linear spring with spring constant k
- when relaxed, the spring has length ℓ
- suppose the surroundings compress it to a shorter length $\ell - x$
- if the spring compresses further by differential length dx, then
  - the spring resists with force kx
  - the surroundings do differential work

$$\delta W = kx\mathrm{d}x$$

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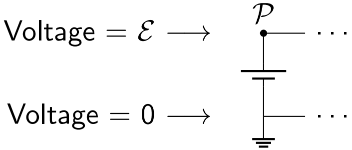
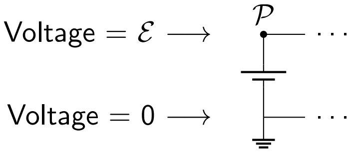
Voltage in electric circuits



- consider a point $\mathcal{P}$ at voltage $\mathcal{E}$ in an electric circuit
- the voltage (potential difference) $\mathcal{E}$ is defined as
  - ◇ the work per unit charge (1 Volt = 1 Joule/Coulomb)
  - ◇ needed to move a test charge from ground to $\mathcal{P}$

Electric work



- suppose current $i = \mathrm{d}q/\mathrm{d}t$ flows from ground to $\mathcal{P}$
- then over differential duration $\mathrm{d}t$,
  - ◇ differential charge $\mathrm{d}q = i\mathrm{d}t$ moves from ground to $\mathcal{P}$
  - ◇ electric field does differential work $\delta W = \mathcal{E}\mathrm{d}q$ on electrons
  - ◇ electric field exerts power

$$\dot{W} = \frac{\delta W}{\mathrm{d}t} = \frac{\mathcal{E}\mathrm{d}q}{\mathrm{d}t} = i\mathcal{E}$$

Example

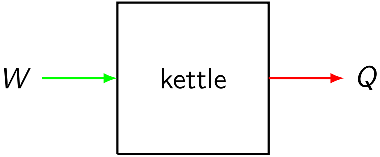
It takes about 80 kJ of energy to bring one cup of water from room temperature to a near-boil at atmospheric pressure. Electric kettles use electric work to supply energy via heat transfer. If an electric kettle draws 12 A of current, how long does it take to make a cup of tea…

(a) in the United States, where most kitchen outlets are 120 V?

(b) in the United Kingdom, where most kitchen outlets are 230 V?

Assumptions, system diagram and basic equations

- **assumptions:**
  - ◇ kettle is perfectly efficient
    - ▶ consider energy transferred to kettle via electric work
    - ▶ kettle transfers all this energy to water via heat transfer
  - ◇ constant electric power, so $\dot{W} = W/\Delta t$
- **basic equations:**
  - ◇ first law: $Q = W$
  - ◇ electric work: $\dot{W} = i\mathcal{E}$
- **system diagram:**



Solution

$$\Delta t = \frac{W}{\dot{W}} = \frac{Q}{i\mathcal{E}} = \frac{80000\text{J}}{(12\text{A})\mathcal{E}} = \begin{cases} 55.6\text{s} & \text{(a) in the US} \\ 30.0\text{s} & \text{(b) in the UK} \end{cases}$$

- units check: since 1 A $=$ 1 C/s and 1 V $=$ 1 J/C,

$$\frac{\text{J}}{(\text{A})(\text{V})} = \frac{\text{J}}{(\text{C/s})(\text{J/C})} = \frac{1}{1/\text{s}} = \text{s}$$