Lecture 12 – Polytropic processes

Purdue ME 200, Thermodynamics I

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Outline

Polytropic processes

- a polytropic process is a quasi-equilibrium process with

$$pV^n = C$$

throughout the process

- the constant can be written as $C = p_1 V_1^n = p_2 V_2^n$, so

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^n$$

- for closed systems (constant m), pv^n is also constant:

$$pV^n = C \iff p(mv)^n = C \iff pv^n = \frac{C}{m^n}$$

Special cases of polytropic processes

- if $n = 0$, pressure is constant:

$$pV^0 = C \iff p = C$$

- as $|n| \to \infty$, volume becomes constant: for $n \neq 0$,

$$\frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{1/n} \to 1 \text{ as } |n| \to \infty$$

  ($V_2/V_1 \to 1$ if and only if $V_2 \to V_1$)

- if $n = 1$ in a closed ideal gas system, temperature is constant:

$$pV^1 = C \iff mRT = C \iff T = \frac{C}{mR}$$

Polytropic boundary work for closed ideal gas systems

- from lecture 5, polytropic boundary work for any system is

$$\int_{V_1}^{V_2} p\,\mathrm{d}V = \begin{cases} (p_2 V_2 - p_1 V_1)/(1-n) & \text{if } n \neq 1 \\ p_1 V_1 \ln(V_2/V_1) & \text{if } n = 1 \end{cases}$$

- for closed ideal gas systems, $p_1 V_1 = mRT_1$ and $p_2 V_2 = mRT_2$
- so the polytropic boundary work formula simplifies to

$$\int_{V_1}^{V_2} p\,\mathrm{d}V = \begin{cases} mR(T_2 - T_1)/(1-n) & \text{if } n \neq 1 \\ mRT \ln(V_2/V_1) & \text{if } n = 1 \end{cases}$$

($n = 1$ means constant T for closed ideal gas systems)

Polytropic processes for ideal gases

- for a closed ideal gas system in any process,

$$mR = \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \implies \frac{p_2}{p_1} = \frac{V_1 T_2}{V_2 T_1}$$

- but for polytropic processes,

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2} \right)^n$$

- so for a closed ideal gas system in a polytropic process,

$$\frac{p_2}{p_1} = \frac{V_1 T_2}{V_2 T_1} = \left(\frac{V_1}{V_2} \right)^n$$

Polytropic processes for ideal gases (continued)

- multiplying the last line by $(V_1/V_2)^{-1}$ gives

$$\frac{p_2}{p_1} \left(\frac{V_1}{V_2} \right)^{-1} = \frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{n-1}$$

- if $n \neq 0$, then

$$\left(\frac{V_1}{V_2} \right)^{-1} = \left[\left(\frac{V_1}{V_2} \right)^n \right]^{-1/n} = \left(\frac{p_2}{p_1} \right)^{-1/n}$$

- so

$$\frac{p_2}{p_1} \left(\frac{V_1}{V_2} \right)^{-1} = \frac{p_2}{p_1} \left(\frac{p_2}{p_1} \right)^{-1/n} = \left(\frac{p_2}{p_1} \right)^{1-1/n}$$

Polytropic processes for ideal gases (continued)

- in summary, for any closed ideal gas system,

$$\left(\frac{p_2}{p_1}\right)^{1-1/n} = \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

over any polytropic process with $n \neq 0$

- these are 2 equations in 7 variables: n, p_1, V_1, T_1, p_2, V_2, T_2

$\implies$ given any 5 of those variables, we can resolve the other 2

Outline

Ideal gas property tables

- Brightspace has a spreadsheet of ideal gas tables
- tabs for air, water, carbon dioxide, oxygen, nitrogen, ammonia
- all tables have $\bar{h}$ and $\bar{u}$, indexed by T

Temp. [K]	h [kJ/kmol]	u [kJ/kmol]
285	8875	6506
290	9058	6648
295	9242	6791
300	9428	6935
305	9615	7080
310	9802	7226
315	9991	7373

- these data are **molar** (per kmol), not **specific** (per kg)
- to get u (kJ/kg), divide $\bar{u}$ (kJ/kmol) by M (kg/kmol)

Other ideal gas data

- Brightspace also has a PDF containing
  - ◇ molecular weight M (kg/kmol)
  - ◇ individual gas constant R (kJ/(kg K))
  - ◇ specific heats c_v and c_p (kJ/(kg K)) at 300 K
  - ◇ critical temperature T_c (K) and pressure p_c (MPa)

Substance	Formula	MW	R [kJ/(kgK)]	c_P (300 K) [kJ/(kgK)]	c_v (300 K) [kJ/(kgK)]	T_{crit} [K]	P_{crit} [MPa]
Air	-	28.97	0.28700	1.005	0.7177	132.5	3.786
Ammonia	NH_3	17.031	0.48817	2.095	1.607	405.4	11.333
Carbon dioxide	CO_2	44.010	0.18892	0.8435	0.6546	304.1	7.377
Nitrogen	N_2	28.013	0.29680	1.038	0.7409	126.2	3.396
Oxygen	O_2	31.999	0.25983	0.9143	0.6544	154.6	5.043
Propane	C_3H_8	44.094	0.18855	1.666	1.478	369.8	4.247
R134a	CH_2F_4	102.03	0.08149	0.8367	0.7552	374.2	4.059
Water	H_2O	18.015	0.46152	1.868	1.407	647.1	22.064

- the same PDF has c_p vs. T data for air and water

Outline

Problem statement

A piston-cylinder device contains $1\ m^3$ of CO_2 at $12\ ^\circ C$ and atmospheric pressure. The CO_2 undergoes a polytropic process with $n = 1.25$ and ends at 165 kPa.

(a) Calculate the change in internal energy (i) assuming constant specific heats, and (ii) using the ideal gas table.

(b) Calculate the heat transfer over the process.

(c) Plot p vs. v and T vs. v over the process.

Given and find

- **given:**
  - $T_1 = 12\ ^\circ C$, $p_1 = 101$ kPa, $V_1 = 1$ m^3
  - $p_2 = 165$ kPa
  - $n = 1.25$
- **find:**
  - ΔU
  - Q

Assumptions and basic equations

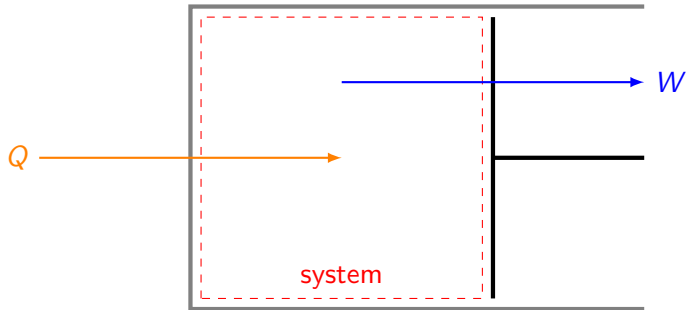- **assume:**
  - ⋄ closed system (constant m)
  - ⋄ ideal gas ($pV = mRT$)
  - ⋄ polytropic process (constant $pV^{1.25}$)
  - ⋄ stationary ($\Delta\text{KE} = \Delta\text{PE} = 0$)
- **basic equation:**

$$\Delta\text{KE} + \Delta\text{PE} + \Delta U = Q - W$$

System diagram

Solution to part (a), method (i)

- the change internal energy is $\Delta U = m\Delta u$
- if specific heats are constant, then $\Delta u = c_v(T_{av})\Delta T$
- this is a closed ideal gas system in a polytropic process, so

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{1-1/n} = 285\text{K} \left(\frac{165\text{kPa}}{101\text{kPa}}\right)^{1-1/1.25} = 314.4\text{K}$$

- average temperature is $T_{av} = (285 \text{ K} + 314.4 \text{ K})/2 = 300 \text{ K}$
- for CO_2 at 300 K, $c_v(T_{av}) = 0.655$ kJ/(kg K)

Solution to part (a), method (i) (continued)

- since $R = 0.189$ kJ/(kg K) for CO_2, the ideal gas law gives

$$m = \frac{p_1 V_1}{R T_1} = \frac{(101\text{kPa})(1\text{m}^3)}{(0.189\text{kJ/(kg K)})(285\text{K})} = 1.88\text{kg}$$

- so change in internal energy is

$$\begin{aligned}
\Delta U = m\Delta u &= m c_v(T_{\text{av}})(T_2 - T_1) \\
&= (1.88\text{kg})(0.655\text{kJ/(kg K)})(314.4\text{K} - 285\text{K}) \\
&= 36.1\text{kJ}
\end{aligned}$$

Solution to part (a), method (ii)

- from ideal gas table, internal energies per mole are
  - $\bar{u}_1 = 6506$ kJ/kmol at $T_1 = 285$ K
  - $\bar{u}_2 = 7373$ kJ/kmol at $T_2 \approx 315$ K
- from molecular weight PDF, $M = 44$ kg/kmol CO_2
- since $u = \bar{u}/M$, change in internal energy is

$$\Delta U = m\Delta u = \frac{m(\bar{u}_2 - \bar{u}_1)}{M}$$
$$= \frac{(1.88\text{kg})[7373\text{kJ/kmol} - 6506\text{kJ/kmol}]}{44\text{kg/kmol}}$$
$$= 37.0\text{kJ}$$

- so methods (i) and (ii) agree to within 2.5% error

Solution to part (b)

- from the 1st law with $\Delta KE = \Delta PE = 0$,
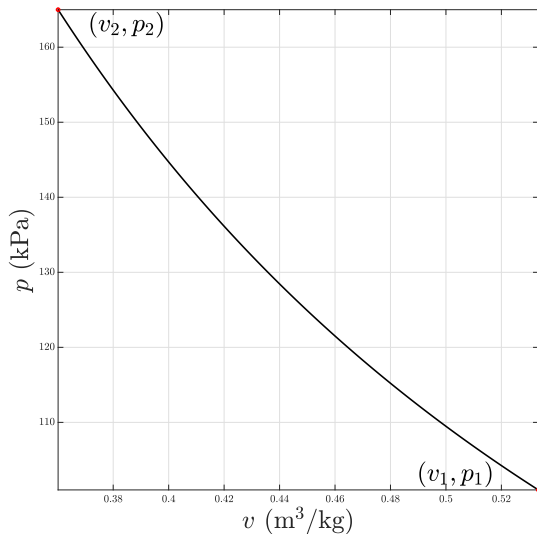
$$Q = \Delta U + W$$

- for closed ideal gas system in $n \neq 1$ polytropic process,

$$
\begin{aligned}
W &= \frac{mR(T_2 - T_1)}{1 - n} \\
&= \frac{(1.88\text{kg})(0.189\text{kJ}/(\text{kg K}))(314.4\text{K} - 285\text{K})}{1 - 1.25} \\
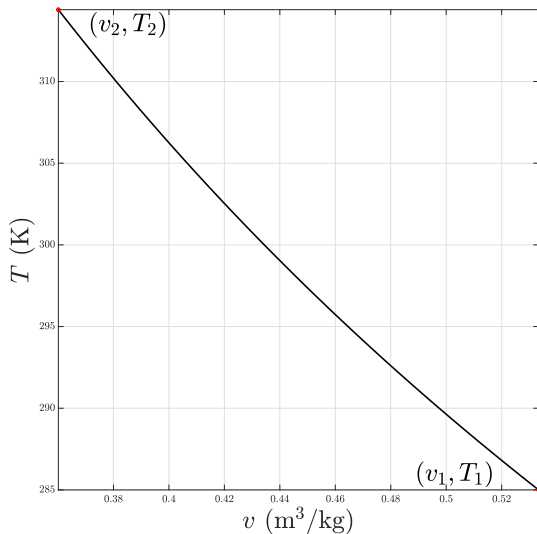&= -41.7\text{kJ}
\end{aligned}
$$

- so heat transfer is

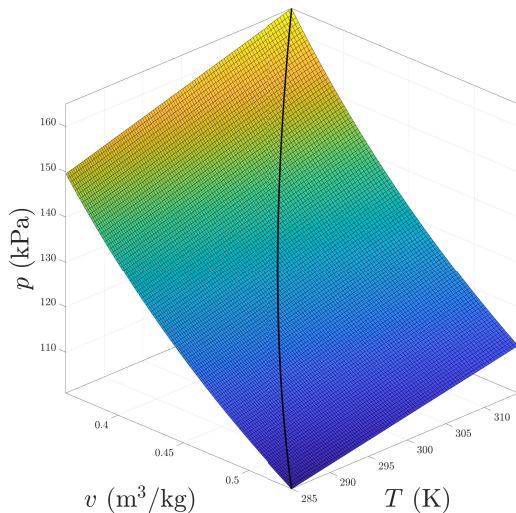$$Q = \Delta U + W = 37.0\text{kJ} - 41.7\text{kJ} = -4.7\text{kJ}$$

(c) Pressure vs. specific volume plot

(c) Temperature vs. specific volume plot

Bonus: path along *p*-*v*-*T* surface

What did we learn?

- surroundings squished CO_2 in a polytropic process
- system volume decreased by $\sim 33\%$
- system (absolute) temperature increased by $\sim 10\%$
- system (absolute) pressure increased by $\sim 63\%$
- surroundings gave ~ 42 kJ to system via work
- system gave back ~ 5 kJ to surroundings via heat transfer
- remaining ~ 37 kJ increased system's internal energy