Lecture 8 – Property tables

Purdue ME 200, Thermodynamics I

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Outline

The state principle

- for pure simple compressible systems without CoM motion,
  - any 2 independent, intensive properties determine the state
  - and therefore determine all other properties
- property tables use this principle to streamline data look-up
- given e.g. T and p, tables contain v, u, h, s

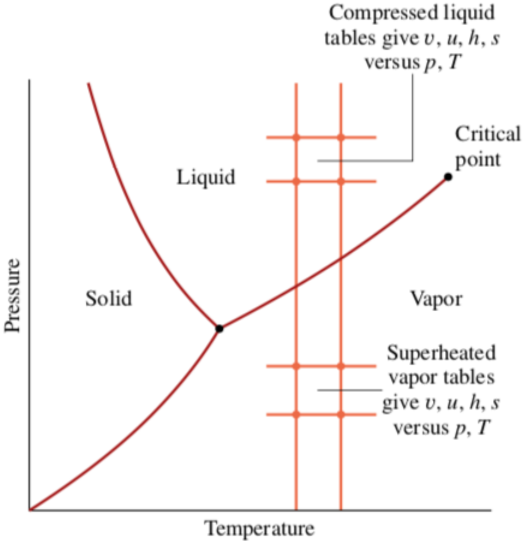
Intensive properties

- intensive properties don't change if we clone the system
- examples: T, p, v, u, h, s (but not V, U, H or S)
  - $v = V/m$ is specific volume
  - $u = U/m$ is specific internal energy
  - $h = H/m$ is specific enthalpy ($H = U + pV$)
  - $s = S/m$ is specific entropy

When are two properties independent?

- if we can change one without changing the other
- T, p and v are all independent in single-phase regions
- but in two-phase regions,
  - T and p are not independent, so can't determine system state
  - but T and v are independent, as are p and v

Outline



Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

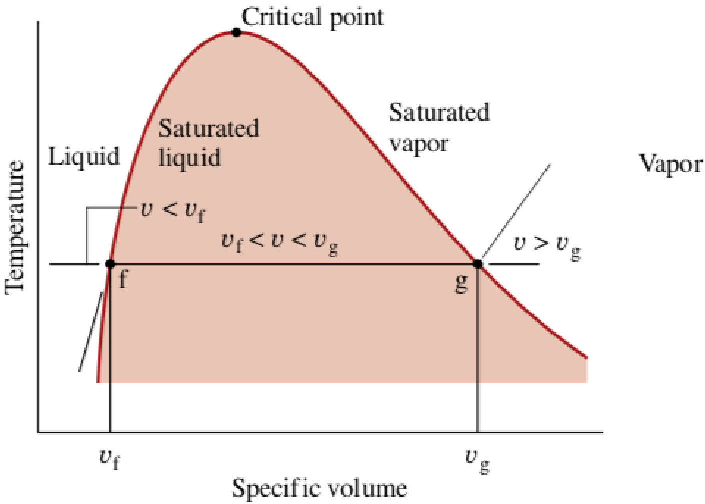
Compressed (subcooled) liquids

Temp. (C)	Volume (m^3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kg/K)	Volume (m^3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kg/K)
	p = 25 bar, T_{sat} = 223.95°C				p = 50 bar = 5.0 MPa, T_{sat} = 263.94°C			
20	1.0007E-03	83.76	86.26	0.29596	9.9956E-04	83.61	88.61	0.29543
40	1.0068E-03	167.22	169.74	0.57143	1.0057E-03	166.92	171.95	0.57046
80	1.0279E-03	334.39	336.96	1.0740	1.0267E-03	333.82	338.95	1.0723
100	1.0422E-03	418.36	420.97	1.3053	1.0410E-03	417.64	422.85	1.3034
140	1.0784E-03	587.85	590.55	1.7370	1.0769E-03	586.79	592.18	1.7344
180	1.1261E-03	760.99	763.81	2.1372	1.1240E-03	759.46	765.08	2.1338
200	1.1556E-03	849.76	852.65	2.3290	1.1531E-03	847.91	853.68	2.3251
220	1.1899E-03	940.65	943.63	2.5173	1.1868E-03	938.39	944.32	2.5127
Sat.	1.1974E-03	958.91	961.91	2.5543	1.2864E-03	1148.20	1154.60	2.9210

Superheated vapors

Temp. (C)	Volume (m^3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kg/K)	Volume (m^3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kg/K)
	p = 0.7 bar = 0.07 MPa, T_{sat} = 89.93°C				p = 1.0 bar = 0.10 MPa, T_{sat} = 99.61°C			
Sat.	2.3648	2493.9	2659.4	7.4790	1.6939	2505.6	2674.9	7.3588
100	2.4343	2509.4	2679.8	7.5344	1.6959	2506.2	2675.8	7.3610
120	2.5710	2539.7	2719.7	7.6385	1.7932	2537.3	2716.6	7.4678
160	2.8409	2599.5	2798.4	7.8292	1.9841	2598.0	2796.4	7.6610
200	3.1083	2659.3	2876.8	8.0024	2.1724	2658.2	2875.5	7.8356
240	3.3745	2719.5	2955.7	8.1624	2.3595	2718.7	2954.6	7.9962
280	3.6400	2780.4	3035.2	8.3116	2.5459	2779.8	3034.4	8.1459
320	3.9050	2842.2	3115.6	8.4518	2.7317	2841.7	3114.9	8.2864
360	4.1697	2904.9	3196.8	8.5844	2.9173	2904.5	3196.3	8.4191
400	4.4341	2968.6	3279.0	8.7103	3.1027	2968.3	3278.6	8.5452
440	4.6985	3033.4	3362.3	8.8304	3.2879	3033.1	3361.9	8.6653
500	5.0948	3132.4	3489.1	9.0011	3.5655	3132.2	3488.7	8.8361

Outline



Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

Saturated liquids and vapors (indexed by temperature)

		Liquid				Vapor			
Temp. (C)	Press. (bar)	Volume (v_f, m^3/kg)	Internal Energy (u_f, kJ/kg)	Enthalpy (h_f, kJ/kg)	Entropy (s_f, kJ/kg/K)	Volume (v_g, m^3/kg)	Internal Energy (u_g, kJ/kg)	Enthalpy (h_g, kJ/kg)	Entropy (s_g, kJ/kg/K)
20	0.023393	0.0010018	83.912	83.914	0.296480	57.757	2402.3	2537.4	8.6660
21	0.024882	0.0010021	88.096	88.098	0.310730	54.483	2403.7	2539.3	8.6437
22	0.026453	0.0010023	92.279	92.282	0.324930	51.418	2405.0	2541.1	8.6217
23	0.028111	0.0010025	96.462	96.465	0.339080	48.548	2406.4	2542.9	8.5998
24	0.029858	0.0010028	100.64	100.65	0.353180	45.858	2407.8	2544.7	8.5781
25	0.031699	0.0010030	104.83	104.83	0.367220	43.337	2409.1	2546.5	8.5566
26	0.033639	0.0010033	109.01	109.01	0.381230	40.973	2410.5	2548.3	8.5353
27	0.035681	0.0010035	113.19	113.19	0.395180	38.754	2411.8	2550.1	8.5142
28	0.037831	0.0010038	117.37	117.37	0.409080	36.672	2413.2	2551.9	8.4933
29	0.040092	0.0010041	121.55	121.55	0.422940	34.716	2414.6	2553.7	8.4725
30	0.042470	0.0010044	125.73	125.73	0.436750	32.878	2415.9	2555.5	8.4520
31	0.044969	0.0010047	129.91	129.91	0.450520	31.151	2417.3	2557.3	8.4316
32	0.047596	0.0010050	134.09	134.09	0.464240	29.526	2418.6	2559.2	8.4113
33	0.050354	0.0010054	138.27	138.27	0.477920	27.998	2420.0	2561.0	8.3913
34	0.053251	0.0010057	142.45	142.45	0.491550	26.560	2421.3	2562.8	8.3714
35	0.056290	0.0010060	146.63	146.63	0.505130	25.205	2422.7	2564.5	8.3517
36	0.059479	0.0010064	150.81	150.81	0.518670	23.929	2424.0	2566.3	8.3321
37	0.062823	0.0010068	154.99	154.99	0.532170	22.727	2425.4	2568.1	8.3127
38	0.066328	0.0010071	159.17	159.17	0.545620	21.593	2426.7	2569.9	8.2935
39	0.070002	0.0010075	163.35	163.35	0.559030	20.524	2428.0	2571.7	8.2745
40	0.073849	0.0010079	167.53	167.53	0.572400	19.515	2429.4	2573.5	8.2555

Saturated liquids and vapors (indexed by pressure)

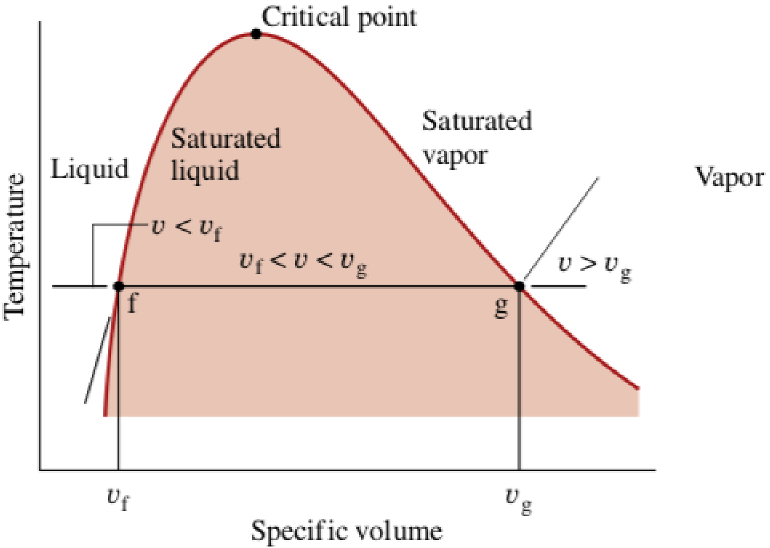
		Liquid				Vapor			
Press. (bar)	Temp. (C)	Volume (v_f, m^3/kg)	Internal Energy (u_f, kJ/kg)	Enthalpy (h_f, kJ/kg)	Entropy (s_f, kJ/kg/K)	Volume (v_g, m^3/kg)	Internal Energy (u_g, kJ/kg)	Enthalpy (h_g, kJ/kg)	Entropy (s_g, kJ/kg/K)
0.01	6.970	0.0010001	29.298	29.299	0.10591	129.18	2384.5	2513.7	8.9749
0.02	17.50	0.0010014	73.426	73.428	0.26056	66.987	2398.9	2532.9	8.7226
0.03	24.08	0.0010028	100.97	100.98	0.35429	45.653	2407.9	2544.8	8.5764
0.04	28.96	0.0010041	121.38	121.39	0.42239	34.791	2414.5	2553.7	8.4734
0.05	32.87	0.0010053	137.74	137.75	0.47620	28.185	2419.8	2560.7	8.3938
0.06	36.16	0.0010065	151.47	151.48	0.52082	23.733	2424.2	2566.6	8.3290
0.07	39.00	0.0010075	163.34	163.35	0.55903	20.524	2428.0	2571.7	8.2745
0.08	41.51	0.0010085	173.83	173.84	0.59249	18.099	2431.4	2576.2	8.2273
0.09	43.76	0.0010094	183.24	183.25	0.62230	16.199	2434.4	2580.2	8.1858
0.1	45.81	0.0010103	191.80	191.81	0.64920	14.670	2437.2	2583.9	8.1488
0.2	60.06	0.0010172	251.40	251.42	0.83202	7.6480	2456.0	2608.9	7.9072
0.3	69.10	0.0010222	289.24	289.27	0.94407	5.2284	2467.7	2624.5	7.7675
0.4	75.86	0.0010264	317.58	317.62	1.0261	3.9930	2476.3	2636.1	7.6690
0.5	81.32	0.0010299	340.49	340.54	1.0912	3.2400	2483.2	2645.2	7.5930
0.6	85.93	0.0010331	359.84	359.91	1.1454	2.7317	2489.0	2652.9	7.5311
0.7	89.93	0.0010359	376.68	376.75	1.1921	2.3648	2493.9	2659.4	7.4790
0.8	93.49	0.0010385	391.63	391.71	1.2330	2.0871	2498.2	2665.2	7.4339
0.9	96.69	0.0010409	405.10	405.20	1.2696	1.8694	2502.1	2670.3	7.3943
1.0	99.61	0.0010432	417.40	417.50	1.3028	1.6939	2505.6	2674.9	7.3588
1.5	111.35	0.0010527	466.97	467.13	1.4337	1.1593	2519.2	2693.1	7.2230
2.0	120.21	0.0010605	504.49	504.70	1.5302	0.88568	2529.1	2706.2	7.1269

Example: Boiling water at elevation

A camper on a mountainside at 10,000 feet, where the air pressure is 0.7 bar, makes pasta. When the water begins to boil, what will be its temperature and specific internal energy?

		Liquid			
Press. (bar)	Temp. (C)	Volume (v_f, m^3/kg)	Internal Energy (u_f, kJ/kg)	Enthalpy (h_f, kJ/kg)	Entropy (s_f, kJ/kg/K)
0.7	89.93	0.0010359	376.68	376.75	1.1921

Outline



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Quality

- any two-phase liquid-vapor mixture has a **quality**,

$$x = \frac{m_{\text{vap}}}{m} = 1 - \frac{m_{\text{liq}}}{m}$$

  $(m = m_{\text{vap}} + m_{\text{liq}})$

- given quality x, the mixture's specific volume is

$$\begin{aligned} v &= \frac{V}{m} = \frac{V_{\text{liq}} + V_{\text{vap}}}{m} = \frac{m_{\text{liq}} v_{\text{liq}} + m_{\text{vap}} v_{\text{vap}}}{m} \\ &= (1 - x) v_{\text{liq}} + x v_{\text{vap}} \\ &= v_{\text{liq}} + x(v_{\text{vap}} - v_{\text{liq}}) \end{aligned}$$

- these formulas also work for u, h and s

Example

A pressure cooker contains 250 g of water in a two-phase liquid-vapor mixture at 2 bar. By mass, the mixture is 20% liquid and 80% vapor. What is its internal energy?

- quality is $x = m_{\text{vap}}/m = 0.8$

Press. (bar)	Temp. (C)	Volume (v_f, m^3/kg)	Internal Energy (u_f, kJ/kg)	Enthalpy (h_f, kJ/kg)	Entropy (s_f, kJ/kg/K)	Volume (v_g, m^3/kg)	Internal Energy (u_g, kJ/kg)	Enthalpy (h_g, kJ/kg)	Entropy (s_g, kJ/kg/K)
2.0	120.21	0.0010605	504.49	504.70	1.5302	0.88568	2529.1	2706.2	7.1269

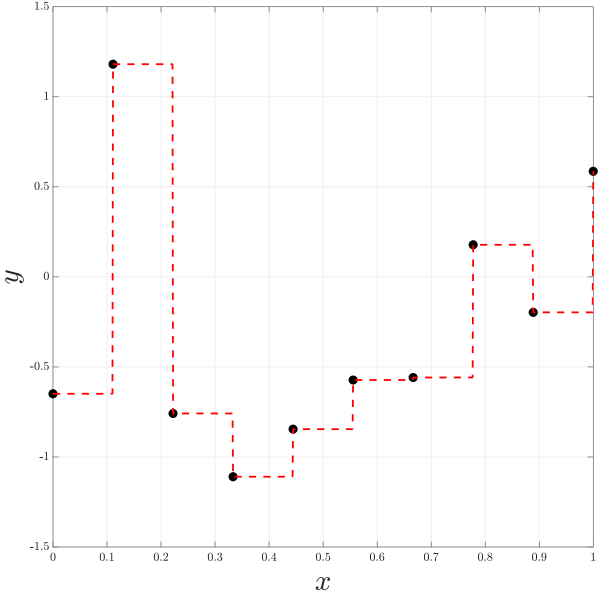
- at 2 bar, $u_{\text{liq}} = 505$ kJ/kg and $u_{\text{vap}} = 2530$ kJ/kg
- so $u = 0.2u_{\text{liq}} + 0.8u_{\text{vap}} = 2125$ kJ/kg
- and $U = mu = (0.25\text{ kg})(2125\text{ kJ/kg}) = 531$ kJ

Outline

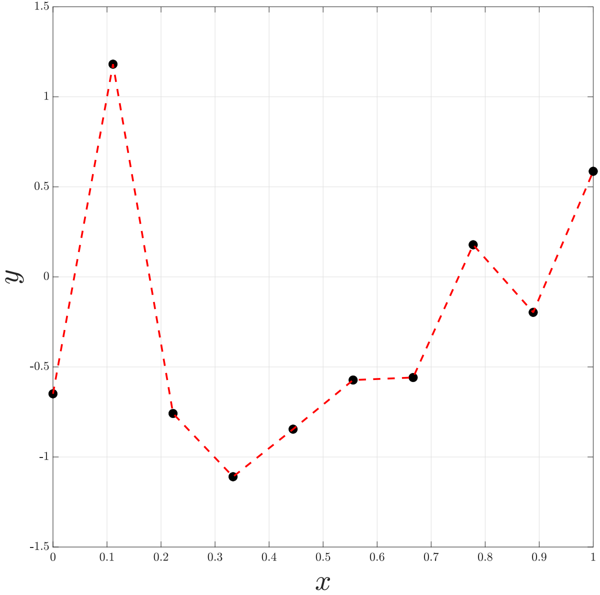
The interpolation problem

- suppose we have n data points (x_1, y_1), ..., (x_n, y_n)
- we'd like to know y corresponding to some x, where
  - $x_1 \leq x \leq x_n$
  - but $x \neq x_1$, ..., $x \neq x_n$
- in **interpolation**, we
  - find a function f such that $y_1 = f(x_1)$, ..., $y_n = f(x_n)$
  - approximate the unknown y by $f(x)$
- interpolation methods usually fit polynomials to the data

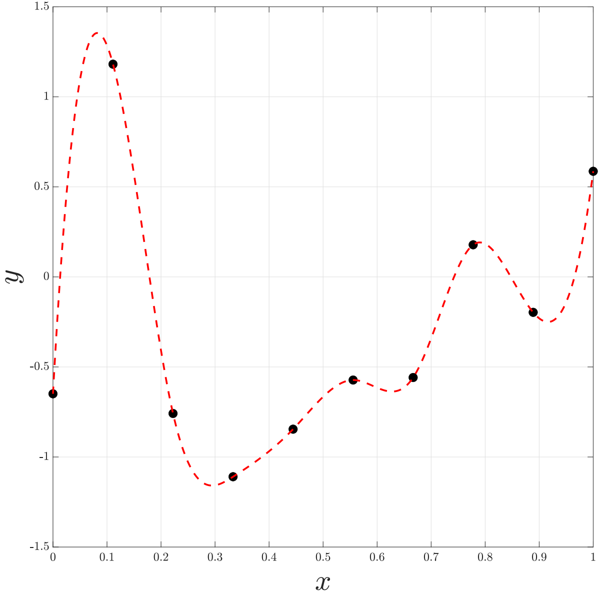
Constant interpolation



Linear interpolation



Cubic interpolation



Linear interpolation

- linear interpolation uses polynomials of degree 1 (lines)
  - simple enough to do by hand
  - accurate enough for most purposes
- for x between x_i and x_{i+1}, approximate y by

$$y = y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i}(x - x_i)$$

- this is the point-slope formula, $y - y_i = m(x - x_i)$, with

$$m = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$

Linear interpolation (continued)