Lecture 39 – The Brayton cycle

Purdue ME 200, Thermodynamics I

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Outline

Gas power cycles

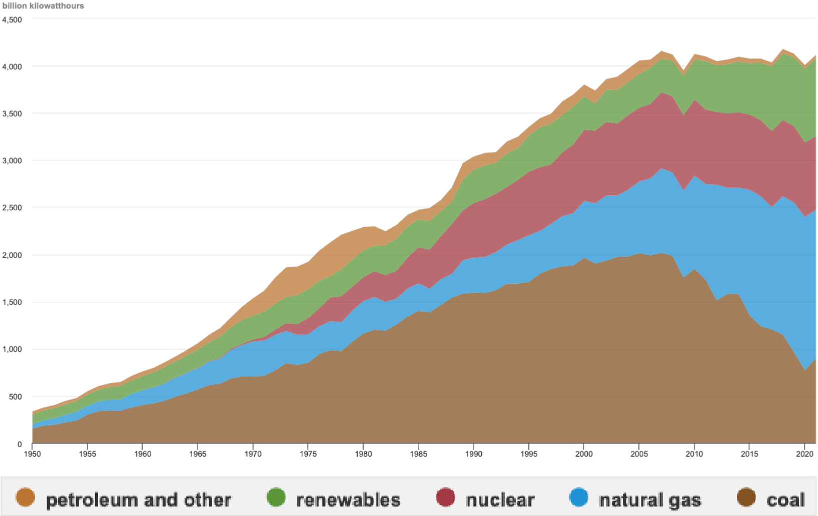
The ideal air-standard Brayton cycle

Example

Why study gas power cycles?

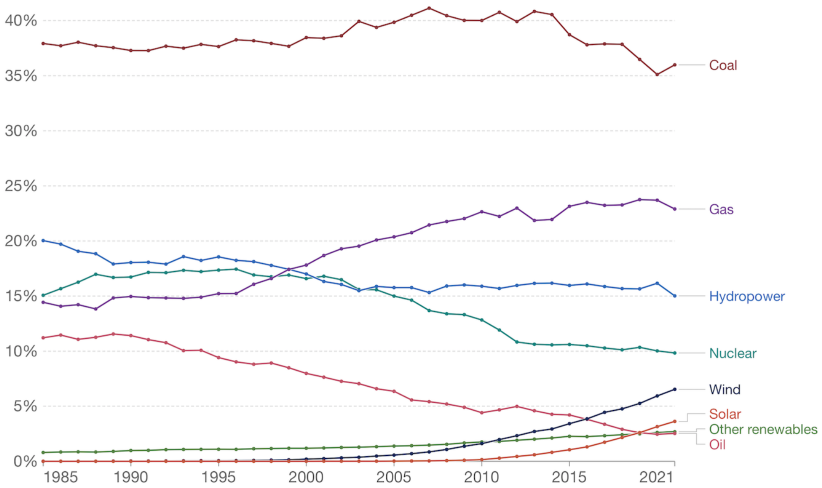
- compared to vapor power cycles, gas power cycles can
  - ◇ generate more power for a given machine weight
  - ◇ adjust their power output much faster
- so gas power cycles run most airplanes and many ships
- they also generate a lot of electricity

Gas power cycles generate 38% of US electricity



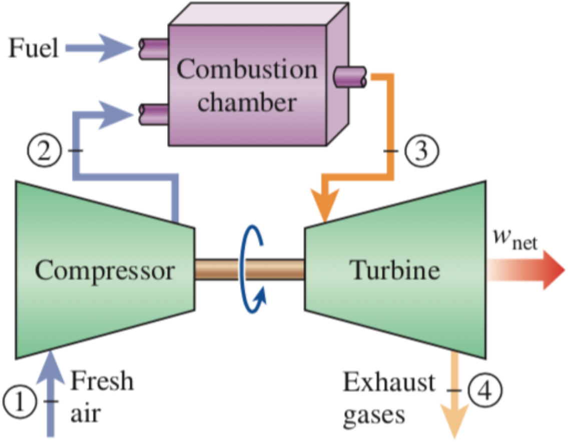
US Energy Information Administration, *Electricity explained* (2022)

Gas power cycles generate 23% of world electricity



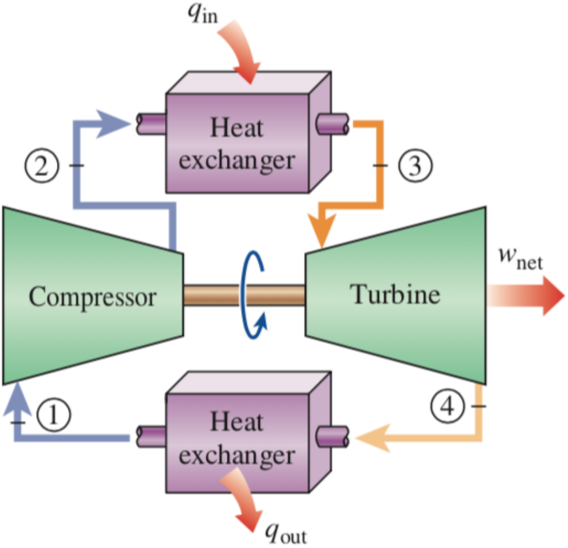
Our World in Data, *Electricity Mix* (2022)

Most gas power plants are open systems



Cengel and Boles, *Thermodynamics: An Engineering Approach* (2019)

But they're easier to model as closed systems



Cengel and Boles, *Thermodynamics: An Engineering Approach* (2019)

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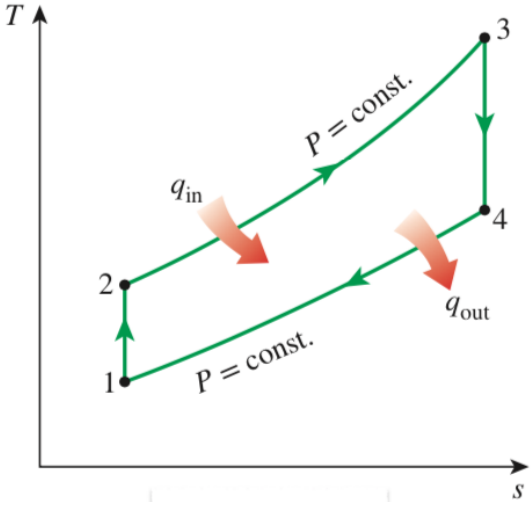
Simplifying assumptions

- no KE or PE effects
- no stray heat transfer
- no pressure drops due to friction
- steady mass flow, steady cyclic operation
- air-standard assumptions:
  - ◇ all processes are internally reversible
  - ◇ the working fluid is air, behaving as an ideal gas
  - ◇ combustion is heat transfer from an external reservoir
  - ◇ gas exhaust/air intake is heat transfer to an external reservoir

The ideal air-standard Brayton cycle has four processes

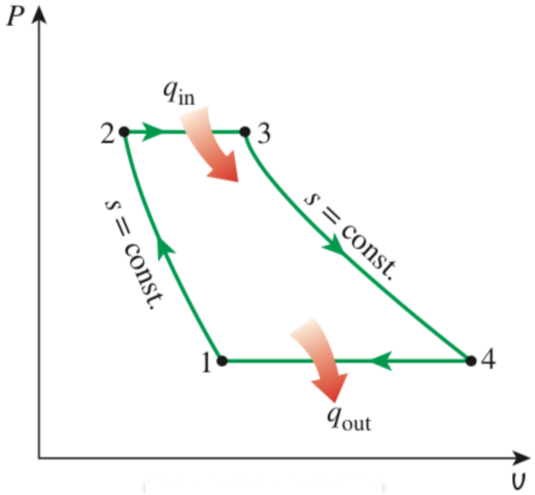
- constant-entropy compression in a compressor (1$\rightarrow$2)
- constant-pressure input heat transfer (2$\rightarrow$3)
- constant-entropy expansion in a turbine (3$\rightarrow$4)
- constant-pressure output heat transfer (4$\rightarrow$1)

Ideal air-standard Brayton cycle T-s diagram



Cengel and Boles, *Thermodynamics: An Engineering Approach* (2019)

Ideal air-standard Brayton cycle *p*-*v* diagram



Cengel and Boles, *Thermodynamics: An Engineering Approach* (2019)

Ideal air-standard Brayton cycle energy balances

- adiabatic compression: $\dot{W}_{12} = \dot{m}(h_2 - h_1)$
- input heat transfer: $\dot{Q}_{23} = \dot{m}(h_3 - h_2)$
- adiabatic expansion: $\dot{W}_{34} = \dot{m}(h_3 - h_4)$
- output heat transfer: $\dot{Q}_{41} = \dot{m}(h_4 - h_1)$
- full cycle: $\dot{W}_{12} + \dot{Q}_{23} = \dot{W}_{34} + \dot{Q}_{41}$

Ideal air-standard Brayton cycle back-work ratio

- the Brayton cycle's **back work ratio** is

$$\text{BWR} = \frac{\text{compressor work}}{\text{turbine work}} = \frac{\dot{W}_{12}}{\dot{W}_{34}} = \frac{h_2 - h_1}{h_3 - h_4}$$

- Brayton cycles have much higher BWRs than Rankine cycles
  - ◇ 40–80% for most Brayton cycles
  - ◇ 1–3% for most Rankine cycles
- most jet engine turbines generate just enough work to
  - ◇ run the compressor
  - ◇ power on-board electronics, HVAC equipment, etc.

  (exhaust gases push the plane)

Ideal air-standard Brayton cycle efficiency

- the Brayton cycle efficiency is

$$\eta = \frac{\text{net work output}}{\text{heat transfer input}} = \frac{\dot{W}_{34} - \dot{W}_{12}}{\dot{Q}_{23}}$$

$$= \frac{\dot{Q}_{23} - \dot{Q}_{41}}{\dot{Q}_{23}} = 1 - \frac{\dot{Q}_{41}}{\dot{Q}_{23}}$$

$$\implies \eta = 1 - \frac{h_4 - h_1}{h_3 - h_2}$$

Ideal **cold** air-standard Brayton cycle efficiency

- if specific heats are constant, then

$$h_3 - h_2 = c_p(T_3 - T_2),\ h_4 - h_1 = c_p(T_4 - T_1)$$

- so the Brayton cycle efficiency becomes

$$\begin{aligned}\eta &= 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \\ &= 1 - \frac{T_1}{T_2}\left(\frac{T_4/T_1 - 1}{T_3/T_2 - 1}\right)\end{aligned}$$

- but $T_4/T_1 = T_3/T_2$ (see next slide), so

$$\eta = 1 - \frac{T_1}{T_2}$$

Cold air-standard Brayton cycle efficiency (continued)

- 1→2 and 3→4 are isentropic, $p_3 = p_2$, and $p_4 = p_1$, so

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{(k-1)/k} = \left(\frac{p_1}{p_2}\right)^{(k-1)/k} = \frac{T_1}{T_2}$$

- rearranging gives $T_4/T_1 = T_3/T_2$

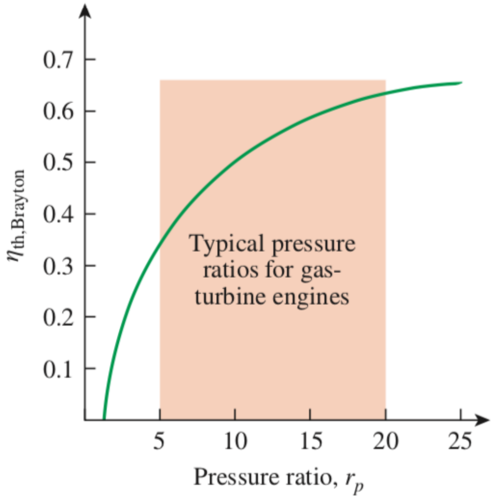
Efficiency and the pressure ratio

- define the **pressure ratio** $r_p = p_2/p_1$
- then the cold air-standard Brayton cycle efficiency is

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{T_2/T_1} = 1 - \frac{1}{(p_2/p_1)^{(k-1)/k}}$$

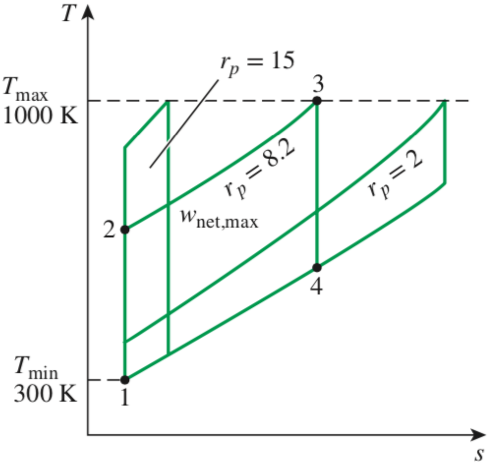
$$\implies \eta = 1 - \frac{1}{r_p^{(k-1)/k}}$$

Efficiency increases with the pressure ratio



Cengel and Boles, *Thermodynamics: An Engineering Approach* (2019)

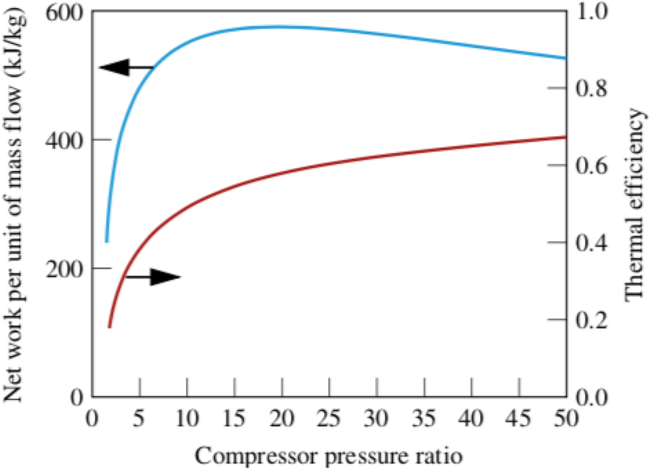
But there's a 'sweet spot' pressure ratio for work output



(int. rev. cycle $\implies$ enclosed area = net heat transfer = net work)

Cengel and Boles, *Thermodynamics: An Engineering Approach* (2019)

Balancing efficiency and work output



Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

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Problem statement

An ideal air-standard Brayton cycle with a pressure ratio of 10 has an inlet pressure and temperature of 100 kPa and 300 K.

(a) Find the efficiency using cold air-standard analysis.

(b) Find the efficiency and back-work ratio using the specific enthalpy data below.

state	specific enthalpy (kJ/kg)
1	300.19
2	579.9
3	1515.4
4	808.5

Solution to part (a)

- in cold air-standard analysis, $k = c_p(T_1)/c_v(T_1) = 1.4$
- the pressure ratio $r_p = 10$ is given, so

$$\eta = 1 - \frac{1}{r_p^{(k-1)/k}} = 1 - \frac{1}{10^{0.4/1.4}} = 0.482$$

Solution to part (b)

- given the specific enthalpy data, the efficiency is

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{(808.5 - 300.19)\text{kJ/kg}}{(1515.4 - 579.9)\text{kJ/kg}} = 0.457$$

- so assuming constant specific heats introduces error

$$0.482 - 0.457 = 0.025$$

- this is 6% error, relative to the true efficiency 0.457

Solution to part (b) (continued)

- given the specific enthalpy data, the BWR is

$$\frac{h_2 - h_1}{h_3 - h_4} = \frac{(579.9 - 300.19)\text{kJ/kg}}{(1515.4 - 808.5)\text{kJ/kg}} = 0.396$$

- so 39.6% of the turbine work goes to the compressor