Lecture 21 – Cycle performance limits

Purdue ME 200, Thermodynamics I

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Outline

Cycle performance limits

Example

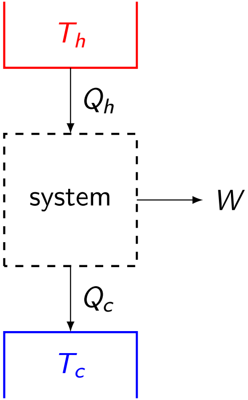
Reminder: Clausius inequality

- consider any cycle where a system
  - absorbs Q_{in} from a reservoir whose temperature is T_{in}
  - emits Q_{out} to a reservoir whose temperature is T_{out}
- the 2nd law implies the Clausius inequality,

$$\frac{Q_{\text{in}}}{T_{\text{in}}} \leq \frac{Q_{\text{out}}}{T_{\text{out}}}$$

- it holds with equality if and only if the cycle is reversible

Power cycle



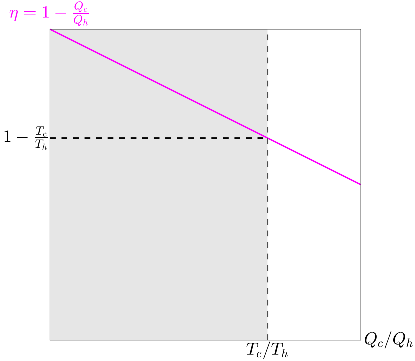
efficiency: $\eta = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$

Power cycle efficiency limit

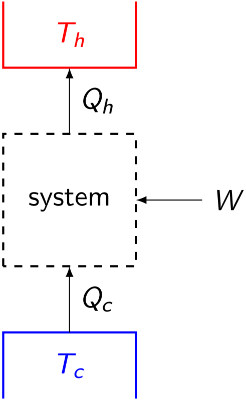
- Clausius inequality:

$$\frac{Q_h}{T_h} \leq \frac{Q_c}{T_c} \quad \text{or equivalently} \quad \frac{Q_c}{Q_h} \geq \frac{T_c}{T_h}$$

- so $\eta \leq 1 - T_c/T_h$



Refrigeration cycle



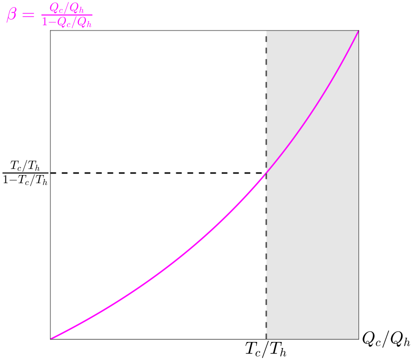
$$\text{COP: } \beta = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{Q_c/Q_h}{1 - Q_c/Q_h}$$

Refrigeration cycle COP limit

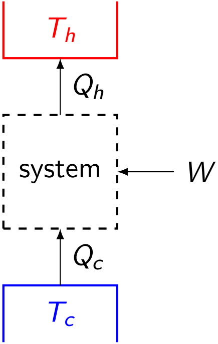
- Clausius inequality:

$$\frac{Q_c}{T_c} \leq \frac{Q_h}{T_h} \quad \text{or equivalently} \quad \frac{Q_c}{Q_h} \leq \frac{T_c}{T_h}$$

- so $\beta \leq (T_c/T_h)/(1 - T_c/T_h)$



Heat pump cycle



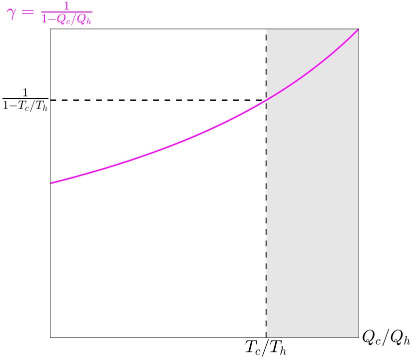
$$\text{COP: } \gamma = \frac{Q_h}{W} = \frac{Q_h}{Q_h - Q_c} = \frac{1}{1 - Q_c/Q_h}$$

Heat pump cycle COP limit

- Clausius inequality:

$$\frac{Q_c}{T_c} \le \frac{Q_h}{T_h} \quad \text{or equivalently} \quad \frac{Q_c}{Q_h} \le \frac{T_c}{T_h}$$

- so $\gamma \le 1/(1 - T_c/T_h)$



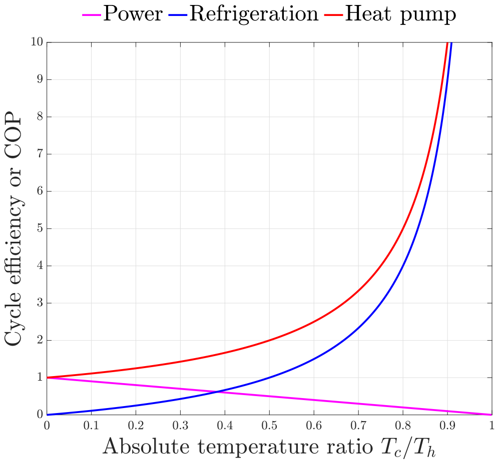
Summary

cycle	metric	limit
power	efficiency η	$\leq 1 - T_c/T_h$
refrigeration	COP β	$\leq (T_c/T_h)/(1 - T_c/T_h)$
heat pump	COP γ	$\leq 1/(1 - T_c/T_h)$

- limits hold with equality ($=$) if and only if cycles are reversible (so all reversible cycles have the same efficiencies/COPs)
- for real (irreversible) cycles, limits are strict inequalities ($<$)
- always use absolute temperatures (K, $^\circ$R) in these limits

Scaling of performance limits with T_c/T_h

- all limits depend only on absolute temperature ratio T_c/T_h
- $T_c > 0$, $T_h > 0$ and $T_h > T_c$, so $0 < T_c/T_h < 1$



Outline

Problem statement

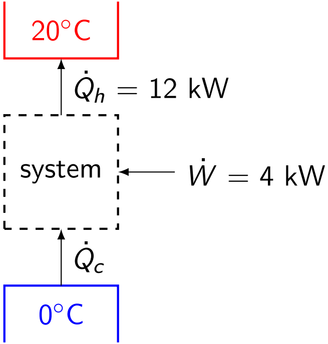
An air-source heat pump delivers 12 kW via heat transfer to 20 $^\circ$C air inside a house and receives 4 kW of electric power. Find

(a) the rate of heat transfer from the 0 $^\circ$C outdoor air,

(b) the heat pump COP, and

(c) the maximum possible heat pump COP.

System diagram and assumptions



- **assumptions:**
  - ◇ closed stationary system
  - ◇ heat pump operates in steady cycles
  - ◇ indoor and outdoor air temperatures are constant

Solution to parts (a) and (b)

(a) from 1st law, $\dot{Q}_c = \dot{Q}_h - \dot{W} = 12 \text{ kW} - 4 \text{ kW} = 8 \text{ kW}$

(b) heat pump COP is

$$\gamma = \frac{\dot{Q}_h}{\dot{W}} = \frac{12\text{kW}}{4\text{kW}} = 3$$

Solution to part (c)

- maximum possible COP is

$$\gamma^\star = \frac{1}{1 - T_c/T_h} = \frac{1}{1 - (0^\circ\text{C})/(20^\circ\text{C})} = 1$$

- nope! always use absolute temperatures in performance limits

$$\gamma^\star = \frac{1}{1 - (273\text{K})/(293\text{K})} = 14.65$$

$\Longrightarrow$ real heat pump only hits 20.5% of maximum possible COP

Bonus: Verifying the Clausius inequality

- in rate form, the Clausius inequality for heat pumps is

$$\frac{\dot{Q}_c}{T_c} \leq \frac{\dot{Q}_h}{T_h}$$

- in this problem,

$$\frac{\dot{Q}_c}{T_c} = \frac{8\text{kW}}{273\text{K}} = 0.029\text{kW/K}$$

and

$$\frac{\dot{Q}_h}{T_h} = \frac{12\text{kW}}{293\text{K}} = 0.041\text{kW/K}$$

- so the Clausius inequality holds

Double bonus: Reversible ground-source heat pump COP

- suppose that
  - instead of the outdoor air at 0 °C
  - the input heat transfer comes from the 13 °C ground
- then the maximum possible COP is

$$\gamma^{\star} = \frac{1}{1 - T_c/T_h} = \frac{1}{1 - (286\text{K})/(293\text{K})} = 41.86$$

- so a reversible ground-source heat pump's COP is

$$\frac{41.86}{14.65} = 285\%$$

higher than a reversible air-source heat pump