Lecture 18 – Cycles

Purdue ME 200, Thermodynamics I

Kevin J. Kircher, kircher@purdue.edu

Outline

Vocabulary reminders

- the **state** of a system determines all its properties
- a **process** is a transition between two equilibrium states
- a **cycle** is a sequence of processes that ends in the initial state

$\Longrightarrow$ at the end of a cycle, all properties return to their initial values

1st law for cycles

- for any closed system undergoing any process, $\Delta E = Q - W$
  - E is the system's total energy
  - Q is the net heat transfer into the system
  - W is the net work done by the system
- since E is a property, $\Delta E = 0$ over any cycle
- so for cycles, 1st law reduces to $Q = W$

Sign conventions, revisited

- so far, we've defined
  - heat transfer into the system as positive
  - work done by the system as positive
- we scrap that convention for cycles (sorry)
- new convention:

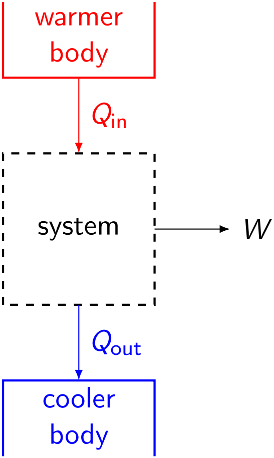
  arrows point in the direction of positive energy flow

- with this convention, 1st law for cycles becomes

$$\sum \text{energy inflows} = \sum \text{energy outflows}$$

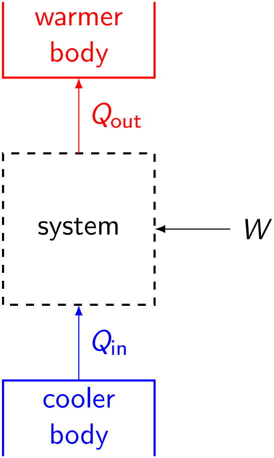
- to distinguish inflows from outflows, check the picture

Power cycles



- 1st law: $Q_{in} = W + Q_{out}$

Refrigeration and heat pump cycles



- 1st law: $Q_{in} + W = Q_{out}$

Efficiencies and coefficients of performance (COPs)

- we quantify cycle performance through benefit-to-cost ratios:

$$\text{performance metric} = \frac{\text{beneficial thing}}{\text{costly thing}}$$

- the metric, benefit and cost definitions vary between cycles

cycle	metric	benefit	cost
power	efficiency η	W	Q_{in}
refrigeration	COP β	Q_{in}	W
heat pump	COP γ	Q_{out}	W

Efficiencies and COPs (continued)

- for power cycles,

$$\eta = \frac{W}{Q_{\text{in}}} = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}}$$

- for refrigeration cycles,

$$\beta = \frac{Q_{\text{in}}}{W} = \frac{Q_{\text{in}}}{Q_{\text{out}} - Q_{\text{in}}}$$

- for heat pump cycles,

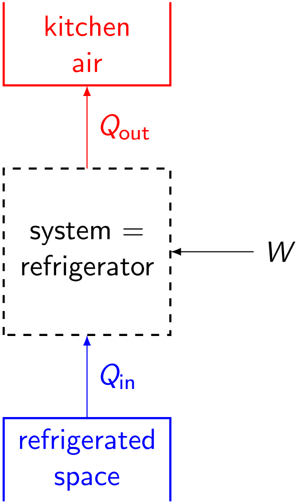
$$\gamma = \frac{Q_{\text{out}}}{W} = \frac{Q_{\text{out}}}{Q_{\text{out}} - Q_{\text{in}}}$$

Outline

Problem statement

A refrigerator steadily draws 200 W of electric power and rejects energy by heat transfer to the kitchen air at a rate of 800 W. Determine the rate of heat transfer from the refrigerated space and the refrigerator's COP.

System diagram



Given and find

- **given:**
  - $\dot{W} = 200$ W
  - $\dot{Q}_{\text{out}} = 800$ W
- **find:**
  - $\dot{Q}_{\text{in}}$
  - β

Assumptions and basic equations

- **assume:**
  - ◇ steady, cyclic operation
- **basic equations:**
  - ◇ 1st law for refrigeration cycles: $Q_{in} + W = Q_{out}$
  - ◇ refrigerator COP: $\beta = Q_{in}/W$

Solution

- call the time to complete one cycle Δt
- then the rate of heat transfer from the refrigerated space is

$$\dot{Q}_{\text{in}} = \frac{Q_{\text{in}}}{\Delta t} = \frac{Q_{\text{out}} - W}{\Delta t}$$

- but $Q_{\text{out}} = \dot{Q}_{\text{out}} \Delta t$ and $W = \dot{W} \Delta t$, so

$$\dot{Q}_{\text{in}} = \frac{\dot{Q}_{\text{out}} \Delta t - \dot{W} \Delta t}{\Delta t} = \dot{Q}_{\text{out}} - \dot{W}$$
$$= 800\text{W} - 200\text{W} = 600\text{W}$$

Solution (continued)

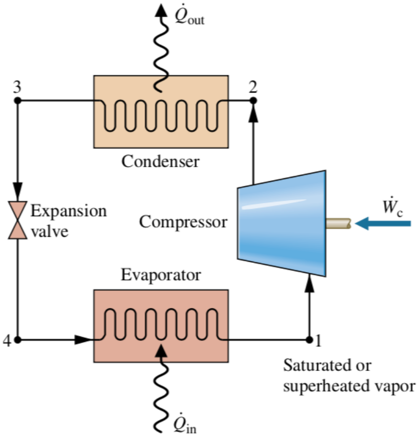
- similarly, the refrigerator's COP is

$$\beta = \frac{Q_{\text{in}}}{W} = \frac{\dot{Q}_{\text{in}}\Delta t}{\dot{W}\Delta t} = \frac{\dot{Q}_{\text{in}}}{\dot{W}}$$
$$= \frac{600\text{W}}{200\text{W}} = 3$$

Outline

Problem statement

Viewing the steady vapor-compression cycle pictured below as (a) a refrigerator and (b) a heat pump, express the COPs in terms of the specific enthalpies h_1, h_2 and h_3.



Given and find

- **given:**
  - h_1, h_2, h_3
- **find:**
  - $\beta = \dot{Q}_{\text{in}}/\dot{W}$
  - $\gamma = \dot{Q}_{\text{out}}/\dot{W}$

Assumptions and basic equations

- **assume:**
  - ◇ steady cyclic operation
  - ◇ no changes in KE or PE
  - ◇ no heat transfer across the compressor boundary
- **basic equations:** 1st law in rate form for the
  - ◇ compressor: $\dot{m}h_1 + \dot{W} = \dot{m}h_2$
  - ◇ condenser: $\dot{m}h_2 = \dot{Q}_{\text{out}} + \dot{m}h_3$
  - ◇ expansion valve: $\dot{m}h_3 = \dot{m}h_4$
  - ◇ evaporator: $\dot{m}h_4 + \dot{Q}_{\text{in}} = \dot{m}h_1$

⋆ sign convention: energy flow is + in direction of $\rightarrow$

Solution to part (a)

- from the compressor energy balance,

$$\dot{W} = \dot{m}(h_2 - h_1)$$

- from the evaporator energy balance,

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4)$$

- from the expansion valve energy balance, $h_4 = h_3$, so

$$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}} = \frac{\dot{m}(h_1 - h_4)}{\dot{m}(h_2 - h_1)} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{h_1 - h_3}{h_2 - h_1}$$

Solution to part (b)

- from the compressor energy balance,

$$\dot{W} = \dot{m}(h_2 - h_1)$$

- from the condenser energy balance,

$$\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3)$$

- so

$$\gamma = \frac{\dot{Q}_{\text{out}}}{\dot{W}} = \frac{\dot{m}(h_2 - h_3)}{\dot{m}(h_2 - h_1)} = \frac{h_2 - h_3}{h_2 - h_1}$$