Lecture 27 – Entropy balance for closed systems

Purdue ME 200, Thermodynamics I

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Outline

Visualizing internally reversible heat transfer

Rate form of entropy balance

What temperatures to use in entropy balances?

Example

Reminder: internally reversible processes

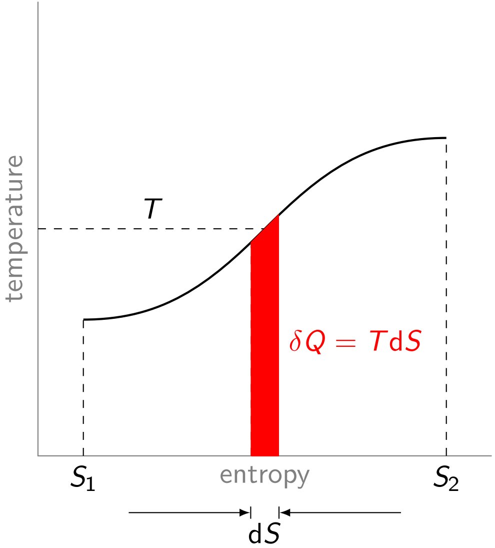
- for a closed system in an internally reversible process ($\delta\sigma = 0$),

$$\mathrm{d}S = \frac{\delta Q}{T} \quad \text{or equivalently} \quad \delta Q = T\mathrm{d}S$$

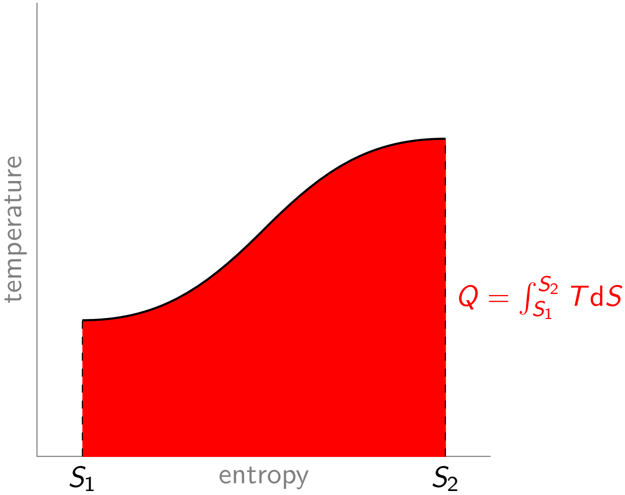
- so the total heat transfer is

$$Q = \int \delta Q = \int T\mathrm{d}S$$

Visualizing differential heat transfer



Visualizing total heat transfer



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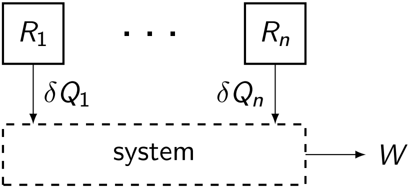
Delta form of entropy balance with multiple reservoirs

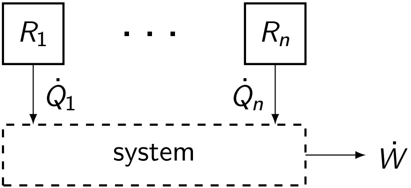


- suppose system exchanges heat with reservoirs $R_1, \ldots, R_n$
- then the change in system entropy over a process is

$$\Delta S = \sum_{j=1}^{n} \int \frac{\delta Q_j}{T_j} + \sigma$$

- T_j is system temperature at boundary with reservoir j
- δQ_j is infinitesimal heat transfer from reservoir j to system
- $\sigma \geq 0$ is entropy generated within system

Rate form of entropy balance with multiple reservoirs



- in rate form, the instantaneous change in system entropy is

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \sum_{j=1}^{n} \frac{\dot{Q}_j}{T_j} + \dot{\sigma}$$

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Example

What temperatures to use in entropy balances?

- temperature is a continuous function of spatial coordinates

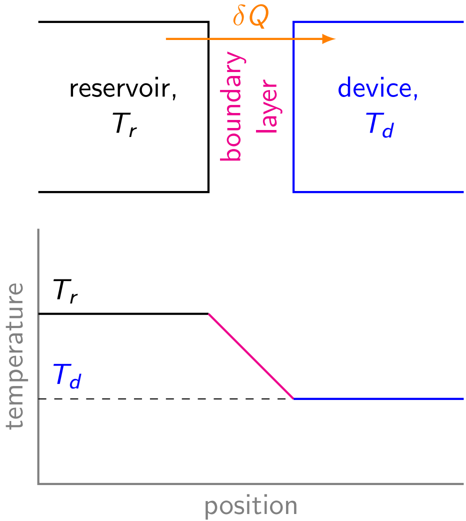
$\Longrightarrow$ temperature changes smoothly between reservoir and system

- this temperature change happens in a **boundary layer**
- should the system or surroundings contain the boundary layer?
- the answer determines the T_j values used in entropy balances

Standing assumption

- reservoirs are internally reversible
- in other words, $\delta\sigma_r = 0$ within reservoirs

Consider a reservoir and an internally reversible device

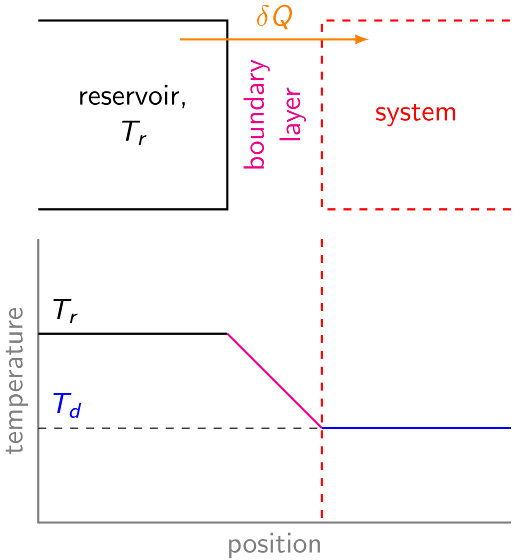


$$\mathrm{d}S_r = -\frac{\delta Q}{T_r}$$

$$\mathrm{d}S_b = \frac{\delta Q}{T_r} - \frac{\delta Q}{T_d} + \delta\sigma_b$$

$$\mathrm{d}S_d = \frac{\delta Q}{T_d}$$

Option 1: Include boundary layer in surroundings



Entropy changes for option 1

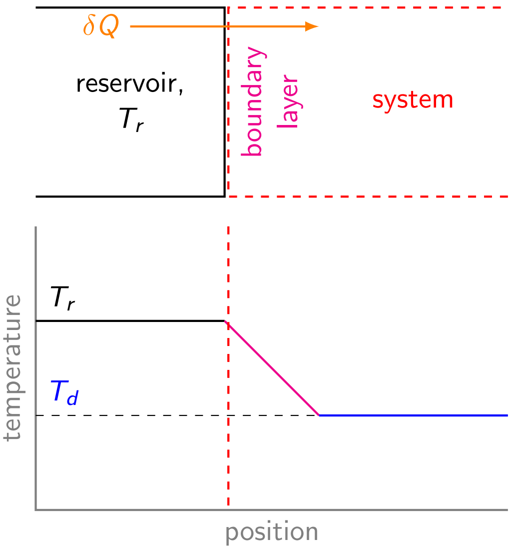
- system: $\mathrm{d}S = \mathrm{d}S_d = \delta Q / T_d$
- surroundings (boundary layer + reservoir):

$$\mathrm{d}\tilde{S} = \mathrm{d}S_b + \mathrm{d}S_r = \cancel{\frac{\delta Q}{T_r}} - \frac{\delta Q}{T_d} + \delta\sigma_b - \cancel{\frac{\delta Q}{T_r}} = -\frac{\delta Q}{T_d} + \delta\sigma_b$$

- universe (system + surroundings):

$$\mathrm{d}S + \mathrm{d}\tilde{S} = \cancel{\frac{\delta Q}{T_d}} - \cancel{\frac{\delta Q}{T_d}} + \delta\sigma_b = \boxed{\delta\sigma_b}$$

Option 2: Include boundary layer in system



Entropy changes for option 2

- system (device + boundary layer):

$$\mathrm{d}S = \mathrm{d}S_d + \mathrm{d}S_b = \cancel{\frac{\delta Q}{T_d}} + \frac{\delta Q}{T_r} - \cancel{\frac{\delta Q}{T_d}} + \delta\sigma_b = \frac{\delta Q}{T_r} + \delta\sigma_b$$

- surroundings (reservoir): $\mathrm{d}\tilde{S} = \mathrm{d}S_r = -\delta Q / T_r$
- universe (system + surroundings):

$$\mathrm{d}S + \mathrm{d}\tilde{S} = \cancel{\frac{\delta Q}{T_r}} + \delta\sigma_b - \cancel{\frac{\delta Q}{T_r}} = \boxed{\delta\sigma_b}$$

Comparing options 1 and 2

- in both options, the entropy change of the universe is $\delta\sigma_b$
- but the entropy change of the system differs between options
- so does the entropy change of the surroundings
- in option 2, temperature varies with position inside the system

$\Longrightarrow$ the system cannot be in equilibrium

- but it can be close if the boundary layer is very thin

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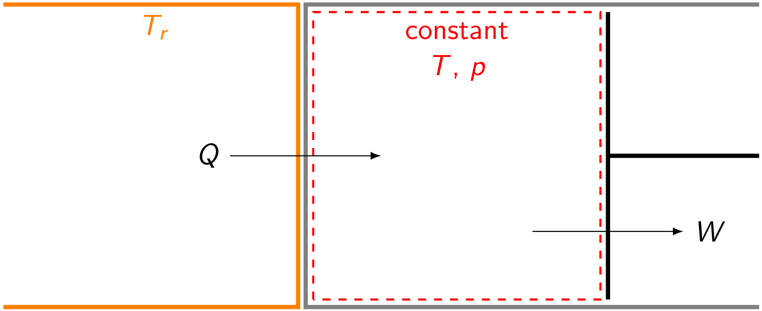
Problem statement

A closed stationary piston-cylinder device containing 1 kg of saturated liquid water at 100 °C comes into contact with a reservoir at 500 K. The water ends as saturated vapor. Including the boundary layer in the system, find the entropy changes of the (a) system, (b) surroundings, and (c) universe.

System diagram



Given and find

- **given:**
  - ◇ $m = 1$ kg
  - ◇ constant $T = 373$ K
  - ◇ $T_r = 500$ K
  - ◇ state 1 is saturated liquid
  - ◇ state 2 is saturated vapor
- **find:** entropy changes of the
  (a) system, ΔS
  (b) surroundings (the reservoir), ΔS_r
  (c) universe (system + surroundings), $\Delta S + \Delta S_r$

Assumptions and basic equations

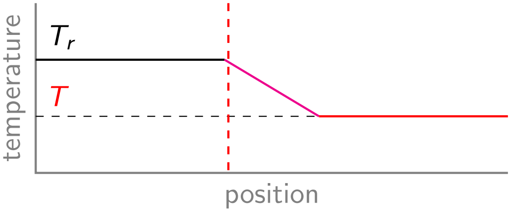
- **assume:**
  - ⋄ closed (constant mass)
  - ⋄ stationary (constant PE and KE)
  - ⋄ phase change (constant temperature and pressure)
- **basic equations:**
  - ⋄ 1st law for closed stationary systems: $\Delta U = Q - W$
  - ⋄ boundary work at constant pressure: $W = p\Delta V$
  - ⋄ 2nd law for closed systems: $\Delta S = \int \delta Q/T + \sigma$

Solution to part (a)

- from the saturation table, the system entropy change is

$$\begin{aligned}\Delta S &= m(s_g - s_\ell) = (1\text{kg})(7.3541 - 1.3072)\text{kJ/kg/K} \\ &= 6.0469\text{kJ/K}\end{aligned}$$

Solution to part (b)



- the boundary layer is included in the system
- so the temperature at the boundary is T_r (constant)
- and the entropy change of the surroundings (the reservoir) is

$$\Delta S_r = -\int \frac{\delta Q}{T_r} = -\frac{1}{T_r}\int \delta Q = -\frac{Q}{T_r}$$

- Q is heat transfer from reservoir to system

Solution to part (b) (continued)

- 1st law for system: $\Delta U = Q - W$
- constant pressure: $W = p\Delta V$
- so $Q = \Delta U + W = \Delta U + p\Delta V = \Delta H$
- and the entropy change of the surroundings is

$$\begin{aligned}\Delta S_r &= -\frac{Q}{T_r} = -\frac{\Delta H}{T_r} = -\frac{m(h_g - h_\ell)}{T_r} \\ &= -\frac{(1\text{kg})(2675.6 - 419.17)\text{kJ/kg}}{500\text{K}} \\ &= -4.5129\text{kJ/K}\end{aligned}$$

Solution to part (c)

- the entropy change of the universe (system + surroundings) is

$$\Delta S + \Delta S_r = (6.0469 - 4.5129)\text{kJ/K} = 1.5340\text{kJ/K}$$

- this is also the internal entropy generation σ