Lecture 28 – Entropy balance for open systems

Purdue ME 200, Thermodynamics I

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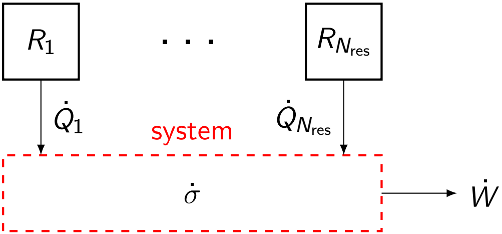
Outline

Entropy balance for open systems

Example #1

Example #2

Reminder: rate form of entropy balance for closed systems



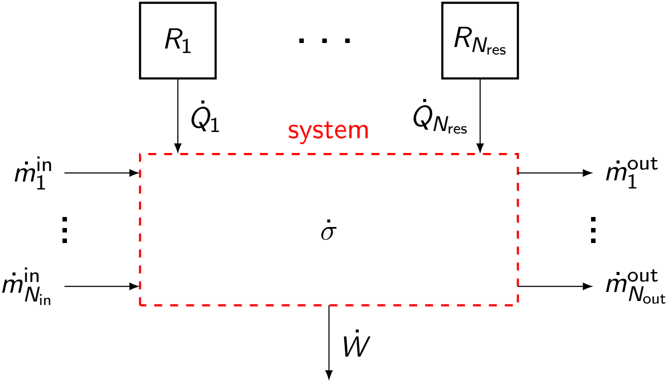
$$\frac{\mathrm{d}S}{\mathrm{d}t} = \sum_{j=1}^{N_{res}} \frac{\dot{Q}_j}{T_j} + \dot{\sigma}$$

- N_{res} is the number of reservoirs the system interacts with
- T_j is the temperature at the boundary with reservoir j
- $\dot{Q}_j$ is the instantaneous rate of heat transfer from reservoir j
- $\dot{\sigma}$ is the instantaneous rate of internal entropy generation

Mass carries entropy

- entropy is a property; all matter has it
- suppose that
  - mass m_{in} enters a system
  - the specific entropy of the entering matter is s_{in}
- then the system entropy increases by $m_{\text{in}} s_{\text{in}}$
- in other words, entropy transfer accompanies mass transfer

Entropy balance for open systems



$$\frac{\mathrm{d}S}{\mathrm{d}t} = \sum_{j=1}^{N_{res}} \frac{\dot{Q}_j}{T_j} + \sum_{i=1}^{N_{in}} \dot{m}_i^{in} s_i^{in} - \sum_{k=1}^{N_{out}} \dot{m}_k^{out} s_k^{out} + \dot{\sigma}$$

Interpreting terms in the entropy balance

- N_{in} is the number of mass inflows
- s_i^{in} is the specific entropy of the mass inflow $\dot{m}_i^{\text{in}}$
- N_{out} is the number of mass outflows
- s_k^{out} is the specific entropy of the mass outflow $\dot{m}_k^{\text{out}}$

Steady state

- in steady state, $(\mathrm{d}S/\mathrm{d}t = 0)$, the entropy balance simplifies to

$$0 = \sum_{j=1}^{N_{\text{res}}} \frac{\dot{Q}_j}{T_j} + \sum_{i=1}^{N_{\text{in}}} \dot{m}_i^{\text{in}} s_i^{\text{in}} - \sum_{k=1}^{N_{\text{out}}} \dot{m}_k^{\text{out}} s_k^{\text{out}} + \dot{\sigma}$$

Steady state, single inflow, single outflow

- in steady state with $N_{in} = N_{out} = 1$, CoM implies that

$$\dot{m}_1^{in} = \dot{m}_1^{out} \quad \text{(so just call them both } \dot{m}\text{)}$$

- in this case, the entropy balance simplifies to

$$0 = \sum_{j=1}^{N_{res}} \frac{\dot{Q}_j}{T_j} + \dot{m}(s_{in} - s_{out}) + \dot{\sigma}$$

Steady state, single inflow, single outflow, no heat transfer

- in steady state with $N_{in} = N_{out} = 1$ and $N_{res} = 0$,

$$0 = \dot{m}(s_{in} - s_{out}) + \dot{\sigma}$$

- equivalently,

$$s_{out} - s_{in} = \frac{\dot{\sigma}}{\dot{m}}$$

- so in steady state with no heat transfer,
  - ◇ if mass flows through an internally irreversible system ($\dot{\sigma} > 0$)
  - ◇ then it exits with more entropy than it entered with ($s_{out} > s_{in}$)

Delta form of entropy balance for time-varying systems

- if $\mathrm{d}S/\mathrm{d}t \neq 0$, we want an entropy balance in delta form
- to get it, integrate rate form over time:

$$\underbrace{\int_0^t \frac{\mathrm{d}S}{\mathrm{d}t}\mathrm{d}\tau}_{\Delta S} = \sum_{j=1}^{N_{\text{res}}} \int_0^t \frac{\dot{Q}_j}{T_j}\mathrm{d}\tau + \sum_{i=1}^{N_{\text{in}}} \int_0^t \dot{m}_i^{\text{in}} s_i^{\text{in}} \mathrm{d}\tau$$

$$- \sum_{k=1}^{N_{\text{out}}} \int_0^t \dot{m}_k^{\text{out}} s_k^{\text{out}} \mathrm{d}\tau + \underbrace{\int_0^t \dot{\sigma}\mathrm{d}\tau}_{\sigma}$$

Delta form of entropy balance (continued)

- if all the T_j, s_i^{in} and s_k^{out} are time-invariant, then

$$\int_0^t \frac{\dot{Q}_j}{T_j} \mathrm{d}\tau = \frac{1}{T_j} \int_0^t \dot{Q}_j \mathrm{d}\tau = \frac{Q_j}{T_j}$$
$$\int_0^t \dot{m}_i^{\text{in}} s_i^{\text{in}} \mathrm{d}\tau = s_i^{\text{in}} \int_0^t \dot{m}_i^{\text{in}} \mathrm{d}\tau = s_i^{\text{in}} m_i^{\text{in}}$$
$$\int_0^t \dot{m}_k^{\text{out}} s_k^{\text{out}} \mathrm{d}\tau = s_k^{\text{out}} \int_0^t \dot{m}_k^{\text{out}} \mathrm{d}\tau = m_k^{\text{out}} s_k^{\text{out}}$$

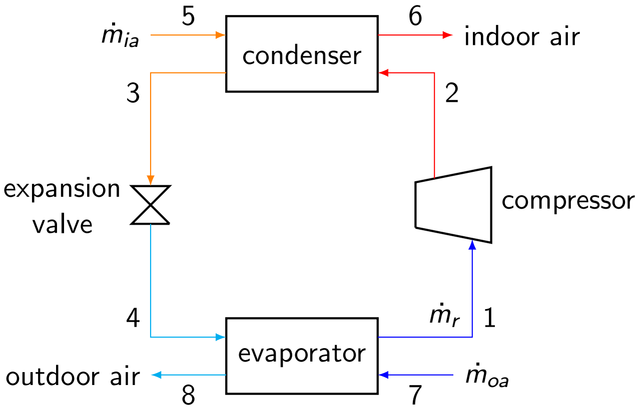
- under these assumptions, delta form of entropy balance is

$$\Delta S = \sum_{j=1}^{N_{\text{res}}} \frac{Q_j}{T_j} + \sum_{i=1}^{N_{\text{in}}} m_i^{\text{in}} s_i^{\text{in}} - \sum_{k=1}^{N_{\text{out}}} m_k^{\text{out}} s_k^{\text{out}} + \sigma$$

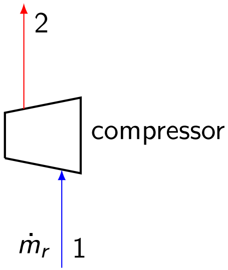
Outline

Problem statement

Write down an entropy balance in rate form for each component of the vapor-compression heat pump pictured below.



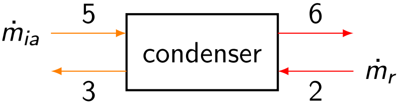
Compressor



- assumptions:
  - steady state, steady flow
  - no external heat transfer
  - one inflow, one outflow
- entropy balance:

$$0 = \dot{m}_r(s_1 - s_2) + \dot{\sigma}_{\text{comp}}$$

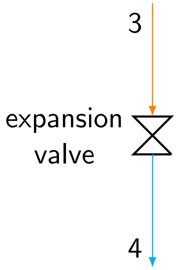
Condenser



- assumptions:
  - ◇ steady state, steady flow
  - ◇ no external heat transfer
  - ◇ two inflows, two outflows
- entropy balance:

$$0 = \dot{m}_r(s_2 - s_3) + \dot{m}_{ia}(s_5 - s_6) + \dot{\sigma}_{\text{cond}}$$

Expansion valve



- assumptions:
  - steady state, steady flow
  - no external heat transfer
  - one inflow, one outflow
- entropy balance:

$$0 = \dot{m}_r(s_3 - s_4) + \dot{\sigma}_{\text{exp}}$$

Evaporator

- assumptions:
  - steady state, steady flow
  - no external heat transfer
  - two inflows, two outflows
- entropy balance:

$$0 = \dot{m}_r(s_4 - s_1) + \dot{m}_{oa}(s_7 - s_8) + \dot{\sigma}_{\text{evap}}$$

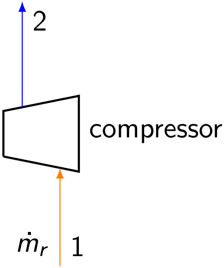
Outline

Problem statement

Given the following input data, rank the components of the vapor-compression heat pump in the previous example in order of internal entropy generation.

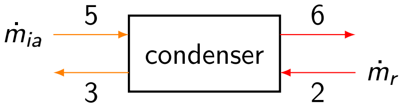
quantity	units	value
$\dot{m}_r$	kg/s	0.07
$\dot{m}_{ia}$	kg/s	0.5
$\dot{m}_{oa}$	kg/s	0.5
s_1	kJ/kg/K	0.9572
s_2	kJ/kg/K	0.9823
s_3	kJ/kg/K	0.2936
s_4	kJ/kg/K	0.3078
$s_6 - s_5$	kJ/kg/K	0.0980
$s_8 - s_7$	kJ/kg/K	-0.0765

Compressor



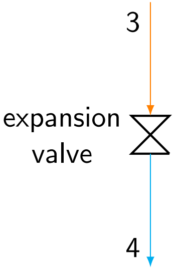
$$
\begin{aligned}
\dot{\sigma}_{\text{comp}} &= \dot{m}_r(s_2 - s_1) \\
&= (0.07\text{kg/s})(0.9823 - 0.9572)\text{kJ/kg/K} \\
&= 0.001757\text{kW/K}
\end{aligned}
$$

Condenser



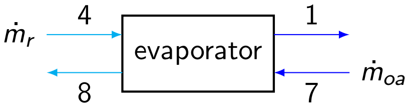
$$
\begin{aligned}
\dot{\sigma}_{\text{cond}} &= \dot{m}_r(s_3 - s_2) + \dot{m}_{ia}(s_6 - s_5) \\
&= (0.07\text{kg/s})(0.2936 - 0.9823)\text{kJ/kg/K} \\
&\quad + (0.5\text{kg/s})(0.098\text{kJ/kg/K}) \\
&= 0.000791\text{kW/K}
\end{aligned}
$$

Expansion valve



$$
\begin{aligned}
\dot{\sigma}_{\text{exp}} &= \dot{m}_r(s_4 - s_3) \\
&= (0.07\text{kg/s})(0.3078 - 0.2936)\text{kJ/kg/K} \\
&= 0.000994\text{kW/K}
\end{aligned}
$$

Evaporator



$$\begin{aligned}
\dot{\sigma}_{\text{evap}} &= \dot{m}_r(s_1 - s_4) + \dot{m}_{oa}(s_8 - s_7) \\
&= (0.07\text{kg/s})(0.9572 - 0.3078)\text{kJ/kg/K} \\
&\quad + (0.5\text{kg/s})(-0.0765\text{kJ/kg/K}) \\
&= 0.04163\text{kW/K}
\end{aligned}$$

Component ranking

component	$\dot{\sigma}$ (W/K)	source(s) of irreversibilities
evaporator	41.63	internal heat transfer
compressor	1.757	fluid friction, internal heat transfer
expansion valve	0.994	fluid friction
condenser	0.791	internal heat transfer