Lecture 24 – Entropy

Purdue ME 200, Thermodynamics I

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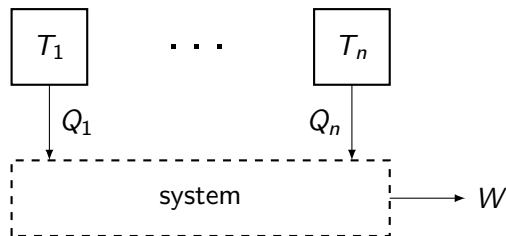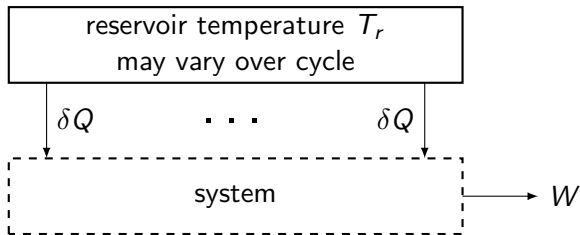# Outline

Reminder: the Clausius inequality

- if system interacts over a cycle with reservoirs at $T_1, \ldots, T_n$
- and exchanges heat Q_i with the reservoir at T_i, then

$$\sum_{i=1}^{n} \frac{Q_i}{T_i} \leq 0$$

- equality holds if and only if the cycle is reversible

Generalization to continuous reservoir distributions

- if system interacts with continuous distribution of reservoirs
- then $n \to \infty$, each $Q_i \to \delta Q$, and Clausius inequality becomes

$$\oint \frac{\delta Q}{T_r} \leq 0$$

- $\oint$ denotes integral over cycle, T_r is reservoir temperature
- equality holds if and only if cycle is reversible

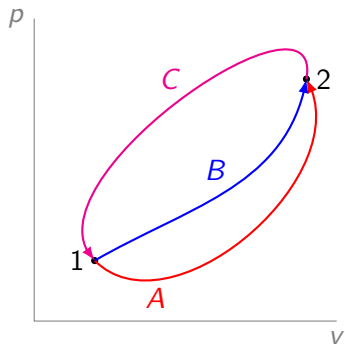# Continuous Clausius (in)equality for reversible cycles

- for any reversible cycle,
  - ◇ the Clausius inequality holds with equality
  - ◇ all heat transfer must be reversible
  - ◇ so system T equals reservoir T_r whenever heat transfers
- $\implies$ for reversible cycles and continuous reservoir distributions,

$$\oint_{\text{rev}} \frac{\delta Q}{T} = 0$$

- $\oint_{\text{rev}}$ denotes integral over reversible cycle

(Reversible) path-independence of $\int \delta Q / T$

- consider reversible paths A and B from state 1 to state 2
- and another reversible path C from 2 back to 1
- combined path (A, C) is a reversible cycle, as is (B, C)

(Reversible) path-independence of $\int \delta Q / T$ (continued)

- by the Clausius inequality,

$$\oint_{\text{rev}} \frac{\delta Q}{T} = \int_A \frac{\delta Q}{T} + \int_C \frac{\delta Q}{T} = 0$$

$$\text{and} \oint_{\text{rev}} \frac{\delta Q}{T} = \int_B \frac{\delta Q}{T} + \int_C \frac{\delta Q}{T} = 0$$

- rearranging and combining the right-hand equations,

$$-\int_C \frac{\delta Q}{T} = \boxed{\int_A \frac{\delta Q}{T} = \int_B \frac{\delta Q}{T}}$$

$\implies$ $\int_P \delta Q / T$ has the same value for any reversible P from 1 to 2

- integral depends only on end-points, not path between them

Path-independence, properties and entropy

- any path-independent quantity defines a property
- for example, $\int_P (\delta Q - \delta W)$ is independent of the path P
- this defines the property **internal energy** U (kJ) satisfying

$$dU = \delta Q - \delta W$$

- we showed $\int_P \delta Q / T$ is independent of the (reversible) path P
- so we define a new property, called **entropy** S (kJ/K)
- over reversible processes, entropy satisfies

$$dS = \frac{\delta Q}{T}$$

Outline

Reminder: 2nd law for closed systems

- there is an <u>extensive property</u> called **entropy**, S (kJ/K)
- heat transfer δQ into system at system temperature T
- and entropy generation $\delta\sigma$ (always ≥ 0) within system
- change system entropy by

$$dS = \frac{\delta Q}{T} + \delta\sigma$$

- for any <u>isolated</u> system,
  - $\diamond$ $dS = 0$ in reversible processes
  - $\diamond$ $dS > 0$ in irreversible processes

Calculating entropy changes

- the change in entropy over any process is

$$\Delta S = \int \mathrm{d}S = \int \frac{\delta Q}{T} + \int \delta\sigma$$

$$\implies \Delta S = \int \frac{\delta Q}{T} + \sigma$$

- T is system temperature when and where δQ happens
- $\sigma = \int \delta\sigma \geq 0$ is entropy generated within system
- for internally reversible systems, $\sigma = 0$ and

$$\Delta S = \int \frac{\delta Q}{T}$$

Entropy changes for isothermal processes

- if system temperature is constant throughout process, then

$$\Delta S = \frac{1}{T} \int \delta Q + \sigma$$

$$\implies \Delta S = \frac{Q}{T} + \sigma$$

- if system is also internally reversible ($\sigma = 0$), then

$$\Delta S = \frac{Q}{T}$$
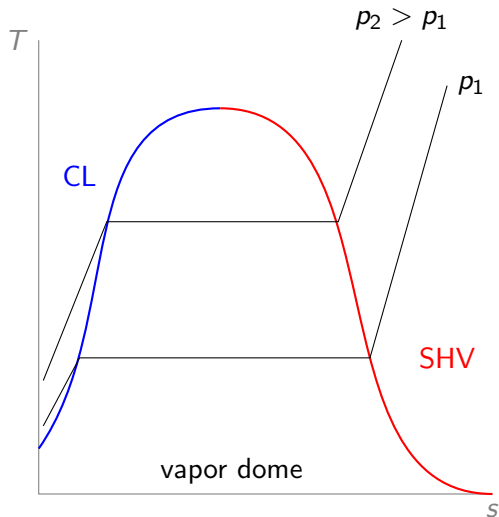
Outline

Using property tables

- specific entropy $s = S/m$ (kJ/kg/K) is an intensive property
- s is tabulated alongside v, u, h, ...
- superheated vapor tables have s for various substances
- so do compressed liquid tables
- if compressed liquid data is unavailable, use
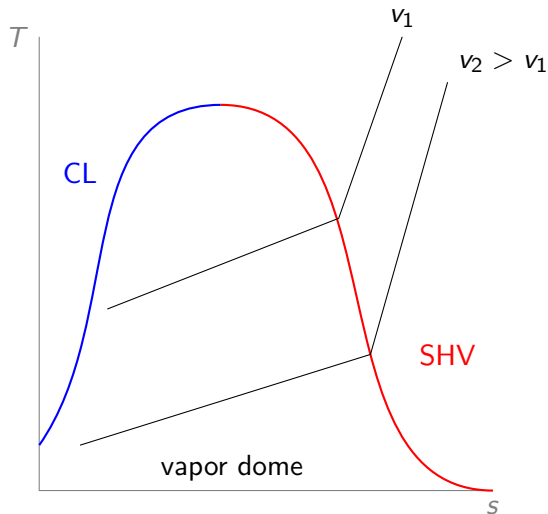
$$s(T, p) \approx s_\ell(T)$$

- for saturated liquid-vapor mixtures, use quality x as usual:

$$s = s_\ell + x(s_v - s_\ell)$$

Drawing temperature-entropy diagrams

Drawing temperature-entropy diagrams (continued)

Outline

Problem statement

A closed rigid tank contains 5 kg of R134a, initially at 20 °C and 140 kPa, that is cooled and stirred until it reaches 100 kPa.

(a) Calculate the entropy change over the process.

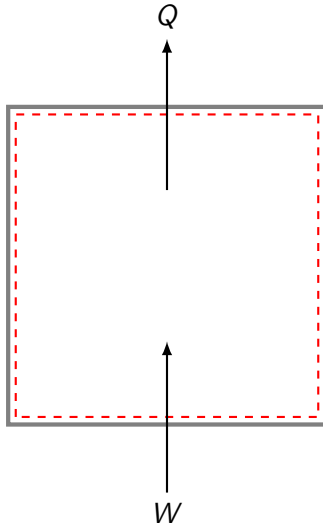(b) Sketch the process on a T-s diagram.

Given and find

- **given:**
    - $m = 5$ kg
    - $T_1 = 20\ ^\circ$C, $p_1 = 140$ kPa
    - $p_2 = 100$ kPa
- **find:**
    - (a) $\Delta S = S_2 - S_1$
    - (b) process sketch on T-s diagram

System diagram

Assumptions and basic equations

- **assume:**
  - $\diamond$ closed ($\Delta m = 0$)
  - $\diamond$ rigid ($\Delta V = 0$)
- **basic equations:**
  - $\diamond$ $\Delta S = m \Delta s$
  - $\diamond$ for SLVM, $s = s_\ell + x(s_v - s_\ell)$

Solution to part (a)

- to find $\Delta S = m\Delta s$, we'll find s in states 1 and 2
- superheated vapor table at $p_1 = 1.4$ bar and $T_1 = 20\,^\circ$C:

$$s_1 = 1.0624 \text{kJ/kg/K}, \ v_1 = 0.165 \text{m}^3/\text{kg}$$

- since m and V are constant, $v_2 = v_1 = 0.165$ m^3/kg
- at $p_2 = 1$ bar, v_2 is between v_ℓ and v_v, so state 2 is SLVM
- in state 2, quality is

$$x = \frac{v_2 - v_\ell}{v_v - v_\ell} = \frac{(0.165 - 0.000726)\text{m}^3/\text{kg}}{(0.193 - 0.000726)\text{m}^3/\text{kg}} = 0.854$$

Solution to part (a) (continued)

- specific entropy in state 2 is

$$s_2 = s_\ell + x(s_v - s_\ell)$$
$$= 0.07195 \text{kJ/kg/K} + 0.854(0.9519 - 0.07195)\text{kJ/kg/K}$$
$$= 0.8234 \text{kJ/kg/K}$$

- so entropy change over process is

$$\Delta S = m(s_2 - s_1)$$
$$= (5\text{kg})(0.8234 - 1.0624)\text{kJ/kg/K}$$
$$= -1.195 \text{kJ/K}$$

- since $\Delta S < 0$, system entropy decreases

Solution to part (b)