Lecture 31 – Isentropic efficiency

Purdue ME 200, Thermodynamics I

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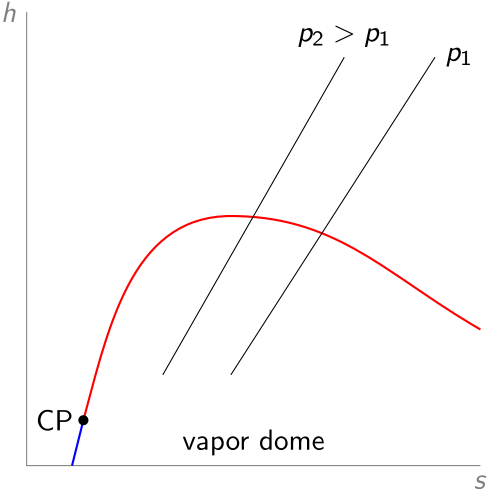
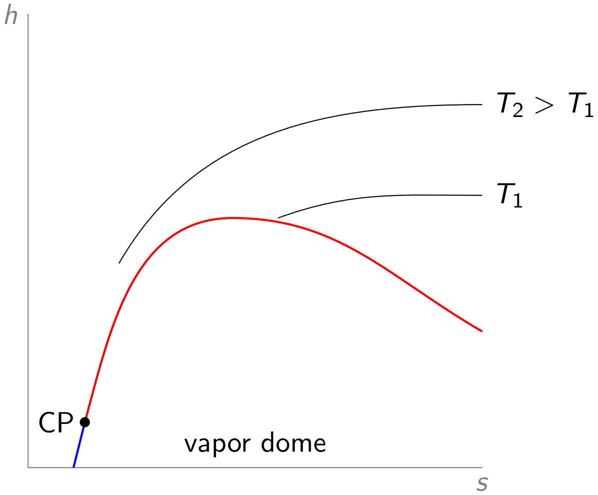
Outline

Enthalpy-entropy diagram (with isobars)



Enthalpy-entropy diagram (with isotherms)



Enthalpy-entropy diagrams for ideal gases

- the ideal-gas region is the upper right of these *h*-*s* diagrams
- isotherms are flat in the ideal-gas region, where $h = h(T)$
- $h(T)$ increases with T, so higher isotherm = higher T
- isobars slope up and right $\nearrow$
- pressure is highest in the top left corner $\nwarrow$

Outline

Enthalpy-entropy (Mollier) diagrams

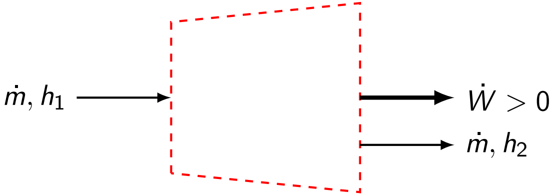
Device isentropic efficiencies

Example

Isentropic efficiency

- isentropic processes are idealizations: the best devices can do
- isentropic efficiencies
  - ◇ quantify how close real devices get to their isentropic limits
  - ◇ are defined differently for different devices
- we define device isentropic efficiencies through optimization:
  - ◇ given the initial state and the final pressure
  - ◇ find the final state that maximizes the useful output (or minimizes the costly input)
  - ◇ subject to the constraint that $s_2 \geq s_1$ (assuming $\dot{Q} = 0$)

Turbine reminders



- turbines change the state of a working fluid to produce power

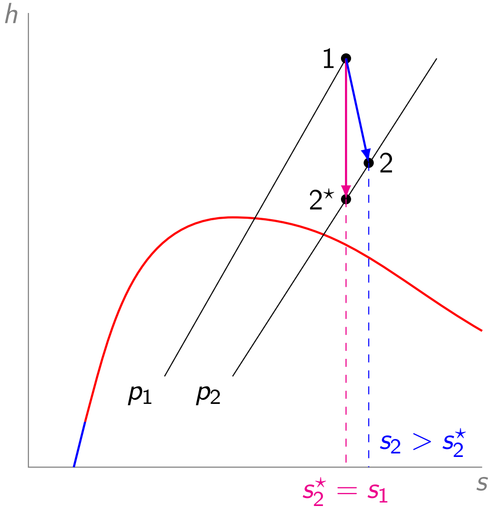
1st law for turbines

- typical assumptions:
  - ◇ steady state
  - ◇ one-dimensional flow
  - ◇ no change in PE (except for some hydro turbines)
  - ◇ no change in KE (except for hydro and wind turbines)
  - ◇ negligible heat transfer ($\dot{Q} = 0$)
- under these assumptions,

$$\dot{W} = \dot{m}(h_1 - h_2)$$

- given $\dot{m}$, h_1 and p_2, find the h_2 that maximizes $\dot{W}$

Turbine isentropic efficiency



- isentropic work:

$$\dot{W}^\star = \dot{m}(h_1 - h_2^\star)$$

- real work:

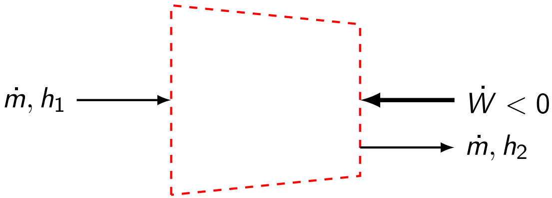
$$\dot{W} = \dot{m}(h_1 - h_2)$$

- isentropic efficiency:

$$\eta_t = \frac{\dot{W}}{\dot{W}^\star} = \frac{h_1 - h_2}{h_1 - h_2^\star}$$

- $\eta_t \approx$ 70–90%

Compressor and pump reminders



- compressors and pumps are ‘backwards turbines’
- they consume power to change the state of a working fluid
- the working fluid is a gas in compressors, a liquid in pumps

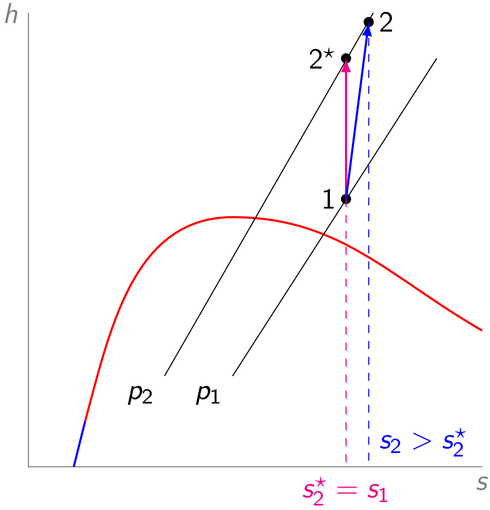
1st law for compressors and (most) pumps

- typical assumptions:
  - ◇ steady state
  - ◇ one-dimensional flow
  - ◇ no change in PE
  - ◇ no change in KE
  - ⋆ in some settings, pumps will have KE and/or PE effects
  - ◇ negligible heat transfer ($\dot{Q} = 0$)
- under these assumptions,

$$-\dot{W} = \dot{m}(h_2 - h_1)$$

- given $\dot{m}$, h_1 and p_2, find the h_2 that minimizes $-\dot{W}$

Compressor and pump isentropic efficiencies



- isentropic work:

$$-\dot{W}^\star = \dot{m}(h_2^\star - h_1)$$

- real work:

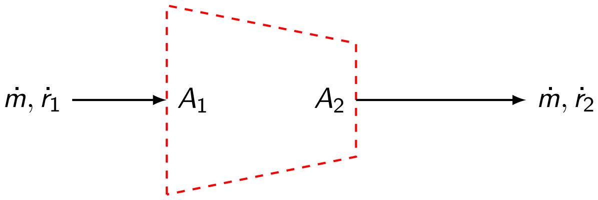
$$-\dot{W} = \dot{m}(h_2 - h_1)$$

- isentropic efficiency:

$$\eta_c = \frac{-\dot{W}^\star}{-\dot{W}} = \frac{h_2^\star - h_1}{h_2 - h_1}$$

- $\eta_c \approx$ 75–85%

Nozzle reminders



- nozzles increase flow velocity: $\dot{r}_2 > \dot{r}_1$
- and decrease pressure: $p_2 < p_1$

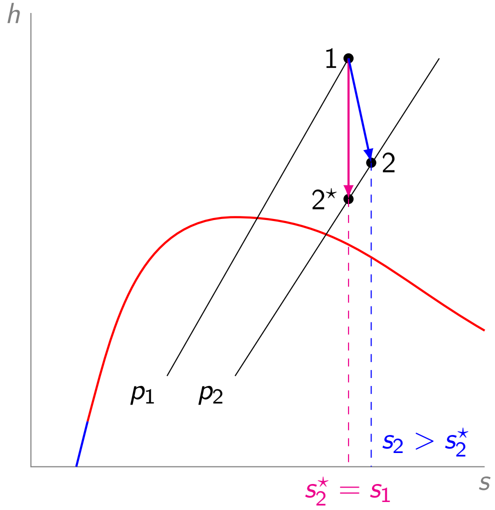
1st law for nozzles

- typical assumptions:
  - steady state
  - one-dimensional flow
  - no change in PE
  - no boundary/shaft/electrical/etc. work
  - negligible heat transfer ($\dot{Q} = 0$)
- under these assumptions,

$$\dot{r}_2^2 = \dot{r}_1^2 - 2(h_2 - h_1)$$

- given $\dot{r}_1^2$, h_1 and p_2, find the h_2 that maximizes $\dot{r}_2^2$

Nozzle isentropic efficiency



- isentropic $\dot{r}_2^2$:

$$(\dot{r}_2^\star)^2 = \dot{r}_1^2 - 2(h_2^\star - h_1)$$

- real $\dot{r}_2^2$:

$$\dot{r}_2^2 = \dot{r}_1^2 - 2(h_2 - h_1)$$

- isentropic efficiency:

$$\eta_n = \frac{\dot{r}_2^2}{(\dot{r}_2^\star)^2} = \frac{\dot{r}_1^2 - 2(h_2 - h_1)}{\dot{r}_1^2 - 2(h_2^\star - h_1)}$$

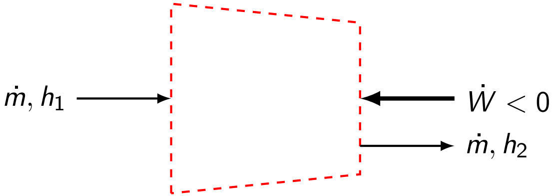
- $\eta_n \approx$ 90–98%

Outline

Problem statement

A steady air flow of 0.2 kg/s enters an adiabatic compressor at 100 kPa and 12 $^\circ$C and exits at 800 kPa. If the compressor's isentropic efficiency is 80%, find (a) the exit air temperature and (b) the rate of work input for the real compressor and for an ideal isentropic compressor operating under the same conditions.

System diagram



Given and find

- **given:**
  - $\dot{m} = 0.2$ kg/s
  - $p_1 = 100$ kPa
  - $T_1 = 12\ ^\circ\text{C} = 285$ K
  - $p_2 = 800$ kPa
  - $\eta_c = 0.8$
- **find:**
  (a) T_2
  (b) $\dot{W}$ and $\dot{W}^\star$

Assumptions and basic equations

- **assume:**
  - ◇ ideal gas
  - ◇ steady state
  - ◇ one-dimensional flow
  - ◇ no change in PE
  - ◇ no change in KE
  - ◇ negligible heat transfer ($\dot{Q} = 0$)
- **basic equations:**
  - ◇ 1st law for compressors: $\dot{W} = \dot{m}(h_2 - h_1)$
  - ◇ compressor isentropic efficiency:

$$\eta_c = \frac{-\dot{W}^\star}{-\dot{W}} = \frac{h_2^\star - h_1}{h_2 - h_1}$$

  - ◇ relative pressure equation for air in an isentropic process:

$$\frac{p_2}{p_1} = \frac{p_r(T_2^\star)}{p_r(T_1)}$$

Solution to part (a)

- we know p_1, T_1, p_2 and η_c and want to find T_2
- this requires fixing another intensive property in state 2
- we can find h_2 from the isentropic efficiency equation given $h_2^\star$
- and we can find $h_2^\star$ from the relative pressure equation

Solution to part (a) (continued)

- relative pressure equation:

$$\frac{p_2}{p_1} = \frac{p_r(T_2^\star)}{p_r(T_1)} \implies p_r(T_2^\star) = \frac{p_r(T_1)p_2}{p_1}$$

- ideal gas table: $p_r(T_1) = 1.1584$, so

$$p_r(T_2^\star) = \frac{1.1584 \cdot 800\text{kPa}}{100\text{kPa}} = 9.2672$$

- ideal gas table:
  - ◇ $p_r(510\text{K}) = 9.031$ and $h(510\text{K}) = 513.6$ kJ/kg
  - ◇ $p_r(520\text{K}) = 9.684$ and $h(520\text{K}) = 523.9$ kJ/kg
- interpolating gives $h_2^\star = 517.05$ kJ/kg

Solution to part (a) (continued)

- isentropic efficiency equation:

$$\eta_c = \frac{h_2^\star - h_1}{h_2 - h_1} \implies h_2 = h_1 + \frac{h_2^\star - h_1}{\eta_c}$$

- interpolating ideal gas table gives $h_1 = 285.14$ kJ/kg, so

$$\begin{aligned} h_2 &= 285.14\text{kJ/kg} + \frac{(517.05 - 285.14)\text{kJ/kg}}{0.8} \\ &= 575.03\text{kJ/kg} \end{aligned}$$

- ideal gas table: $h(570\text{K}) = 575.8$ kJ/kg, so $T_2 = 569.5$ K

Solution to part (b)

- ideal gas table at $T_1 = 285$ K: $h_1 = 285.1$ kJ/kg
- 1st law for compressors:

$$\dot{W} = \dot{m}(h_2 - h_1) = (0.2\text{kg/s})(575.03 - 285.1)\text{kJ/kg} = 58.0\text{kW}$$

- isentropic efficiency definition:

$$\eta_c = \frac{-\dot{W}^\star}{-\dot{W}} \implies \dot{W}^\star = \eta_c \dot{W} = (0.8)(58.0\text{kW}) = 46.4\text{kW}$$