Lecture 33 – The Rankine cycle

Purdue ME 200, Thermodynamics I

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Outline

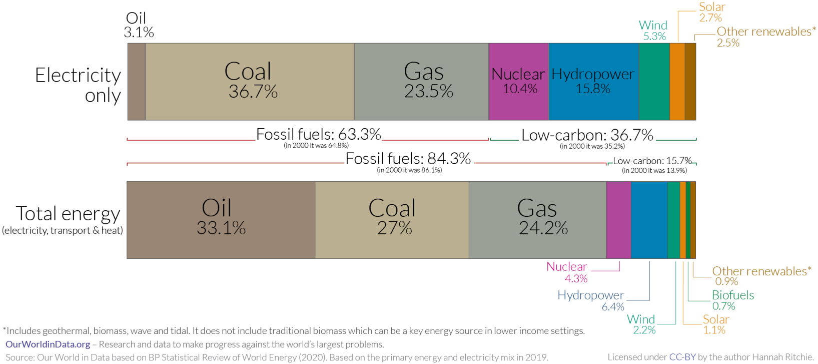
Electricity generation

The Rankine cycle in general

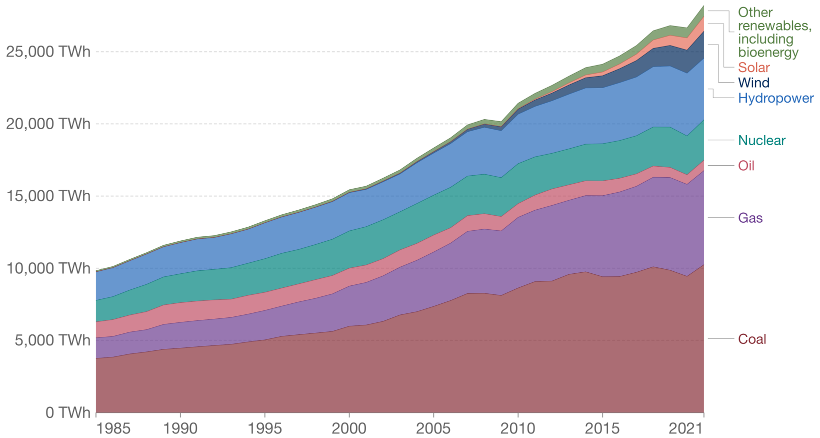
The ideal Rankine cycle

Example

World energy and electricity sources



World electricity generation by source



Source: Our World in Data based on BP Statistical Review of World Energy (2022); Our World in Data based on Ember's Global Electricity Review (2022); Our World in Data based on Ember's European Electricity Review (2022)
Note: 'Other renewables' includes biomass and waste, geothermal, wave and tidal.
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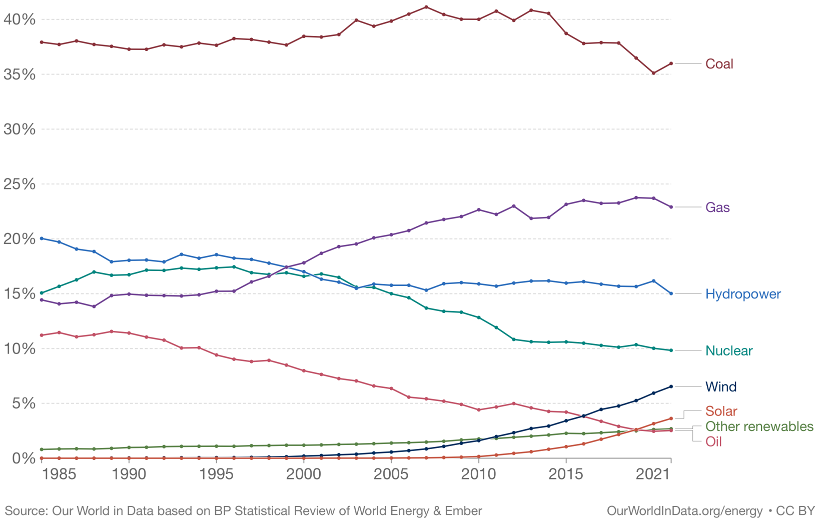
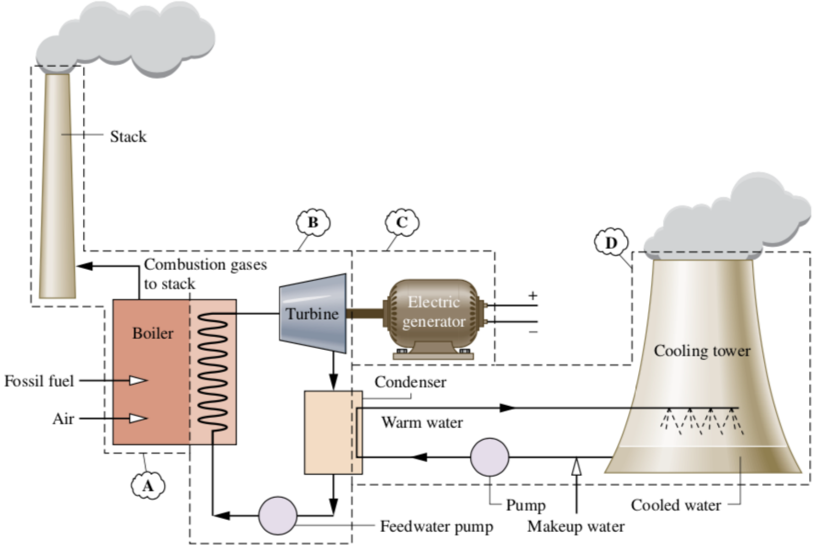
World electricity generation shares by source



The Rankine cycle is widely used

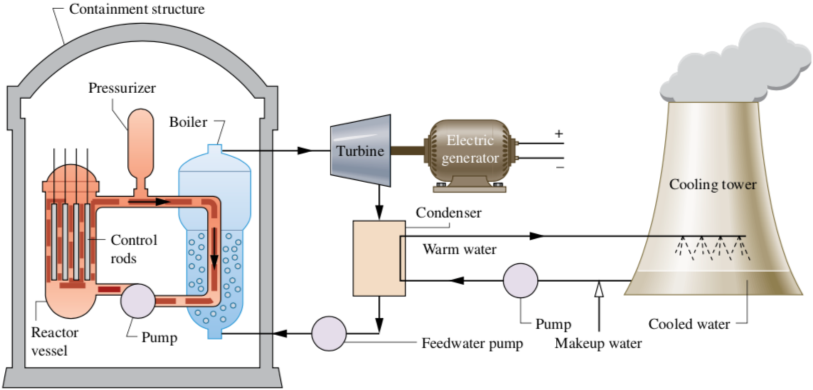
- many types of power plant use the Rankine cycle
  - ◇ coal (36% of global electricity generation)
  - ◇ nuclear (10%)
  - ◇ oil (2.5%)
  - ◇ geothermal (<1%)
  - ◇ solar thermal (<1%)
  - ◇ biomass (<1%)
- collectively, they generate ∼half of global electricity

A typical coal power plant



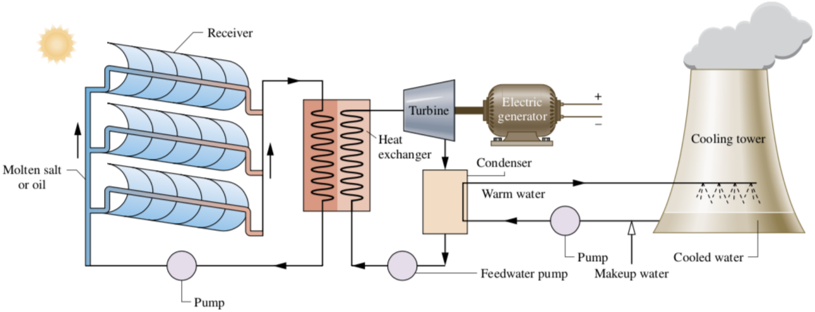
Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

A typical nuclear power plant



Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

A typical solar thermal power plant

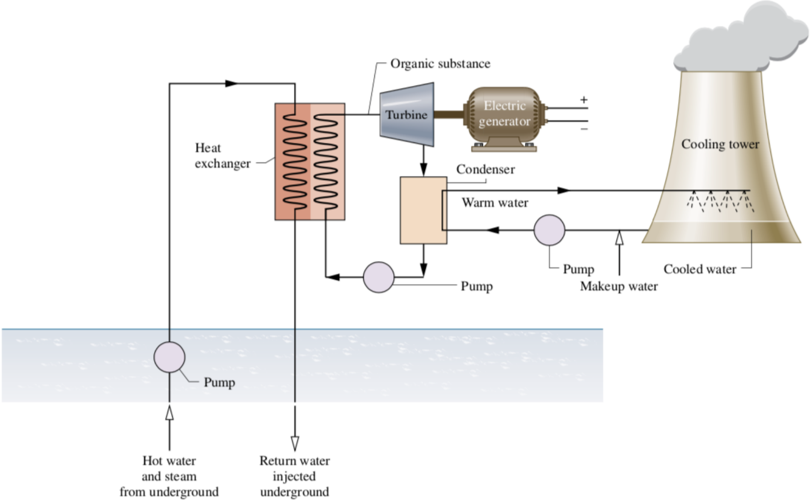


Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

A parabolic trough solar thermal collector

A solar thermal power tower

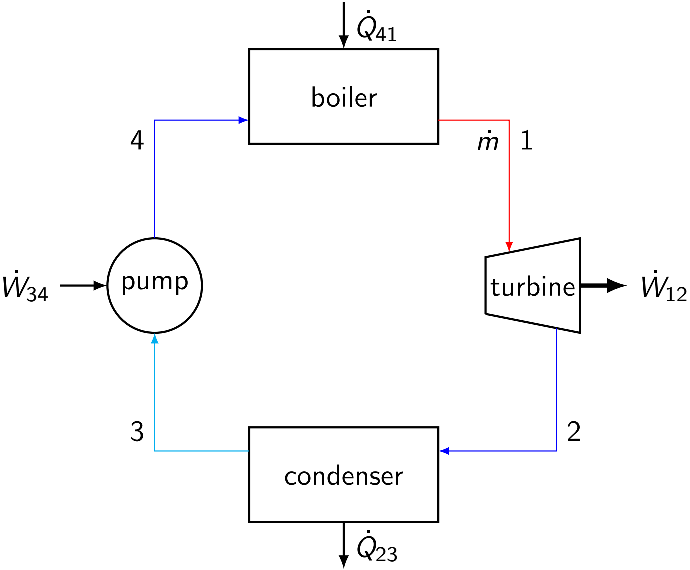
A typical geothermal power plant



Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

Outline

Rankine cycle schematic



Assumptions and energy balances

- standing assumptions:
  - ◇ steady state
  - ◇ no stray heat transfer
  - ◇ no kinetic or potential energy effects
  - ◇ flows are positive in the directions the arrows point
- under these assumptions, the component energy balances are
  - ◇ turbine: $\dot{W}_{12} = \dot{m}(h_1 - h_2)$
  - ◇ condenser: $\dot{Q}_{23} = \dot{m}(h_2 - h_3)$
  - ◇ pump: $\dot{W}_{34} = \dot{m}(h_4 - h_3)$
  - ◇ boiler: $\dot{Q}_{41} = \dot{m}(h_4 - h_1)$

Performance metrics

- 1st law on full cycle: $\dot{W}_{34} + \dot{Q}_{41} = \dot{W}_{12} + \dot{Q}_{23}$
- so the Rankine cycle efficiency is

$$\eta = \frac{\text{net work output}}{\text{heat input}} = \frac{\dot{W}_{12} - \dot{W}_{34}}{\dot{Q}_{41}}$$
$$= \frac{\dot{Q}_{41} - \dot{Q}_{23}}{\dot{Q}_{41}} = 1 - \frac{\dot{Q}_{23}}{\dot{Q}_{41}}$$
$$\implies \eta = 1 - \frac{h_2 - h_3}{h_4 - h_1}$$

- the **heat rate** is $1/\eta$ (often in units of BTU/kWh)
- the **back work ratio** is

$$\frac{\text{pump work input}}{\text{turbine work output}} = \frac{\dot{W}_{34}}{\dot{W}_{12}} = \frac{h_4 - h_3}{h_1 - h_2}$$

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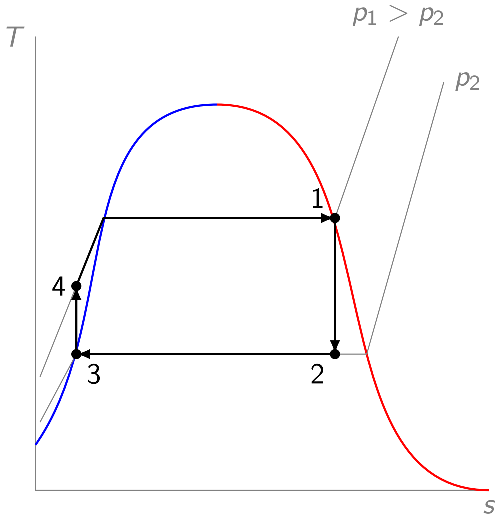
Additional assumptions

- the ideal Rankine cycle has no irreversibilities anywhere
  - no pressure drops due to friction in the condenser or boiler
  - no friction or heat transfer within the turbine or pump
- under these assumptions,
  - processes $4 \to 1$ and $2 \to 3$ are isobaric
  - processes $1 \to 2$ and $3 \to 4$ are isentropic

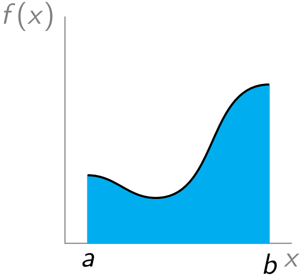
T-s diagram of the ideal Rankine cycle

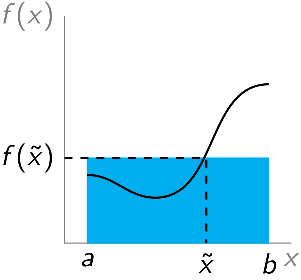


Reminder: the Mean Value Theorem for definite integrals

- suppose a function f is continuous on an interval $[a, b]$
- then there exists a point $\tilde{x}$ between a and b such that

$$\int_a^b f(x)\mathrm{d}x = f(\tilde{x})(b - a)$$





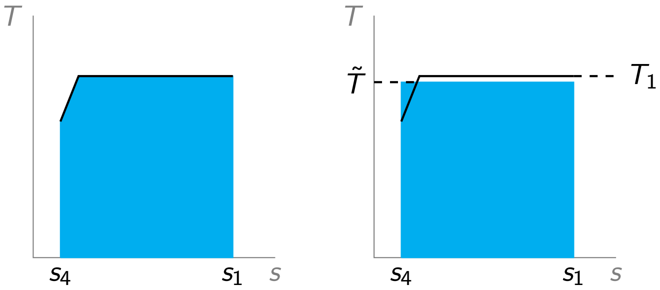
Heat transfer in the ideal Rankine cycle

- if the condenser is internally reversible, then

$$\frac{\dot{Q}_{23}}{\dot{m}} = -\int_{s_2}^{s_3} T\mathrm{d}s = \int_{s_3}^{s_2} T\mathrm{d}s = T_2(s_3 - s_2)$$

- if the boiler is too, then there exists a $\tilde{T} \approx T_1$ such that

$$\frac{\dot{Q}_{41}}{\dot{m}} = \int_{s_4}^{s_1} T\mathrm{d}s = \tilde{T}(s_4 - s_1) \approx T_1(s_4 - s_1)$$



Efficiency of the ideal Rankine cycle

- because the ideal pump and turbine are isentropic,

$$s_1 = s_2 \text{ and } s_3 = s_4$$

- so the ideal Rankine cycle efficiency is

$$\eta = 1 - \frac{\dot{Q}_{23}}{\dot{Q}_{41}} = 1 - \frac{T_2(s_3 - s_2)}{\tilde{T}(s_4 - s_1)} = 1 - \frac{T_2(s_4 - s_1)}{\tilde{T}(s_4 - s_1)}$$

$$= 1 - \frac{T_2}{\tilde{T}}$$

$$\Longrightarrow \eta \approx 1 - \frac{T_2}{T_1}$$

$\Longrightarrow$ $\eta \uparrow$ when $T_2 \downarrow$ in condenser or $\tilde{T} \approx T_1 \uparrow$ in boiler

- temperature increases with pressure, so we want $p_2 \downarrow$ and $p_1 \uparrow$

Outline

Problem statement

A nuclear power cycle runs between pressurized water at 345 °C and the outdoor air at 20 °C.

(a) Find the maximum theoretical efficiency.

(b) Estimate the efficiency of an ideal Rankine cycle with $T_1 =$ 275 °C and $T_2 = 30$ °C.

(c) Repeat part (b) with $T_1 = 315$ °C and $T_2 = 25$ °C.

Solution

(a) $T_h = 345\ ^\circ\text{C},\ T_c = 20\ ^\circ\text{C} \implies$ maximum efficiency is

$$1 - \frac{T_c}{T_h} = 1 - \frac{293\text{K}}{618\text{K}} = 52.6\%$$

(b) $T_1 = 275\ ^\circ\text{C},\ T_2 = 30\ ^\circ\text{C} \implies$ ideal Rankine efficiency

$$\approx 1 - \frac{T_2}{T_1} = 1 - \frac{303\text{K}}{548\text{K}} = 44.7\%$$

(c) $T_1 = 315\ ^\circ\text{C},\ T_2 = 25\ ^\circ\text{C} \implies$ ideal Rankine efficiency

$$\approx 1 - \frac{T_2}{T_1} = 1 - \frac{298\text{K}}{588\text{K}} = 49.3\%$$