Lecture 32 – Internally reversible flow

Purdue ME 200, Thermodynamics I

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Outline

Heat transfer and work

Polytropic reversible flow work

Example

Standing assumptions throughout this lecture

- open system
- steady state, steady flow
- single mass input, single output
- internal reversibility ($\dot{\sigma} = 0$)
- boundary temperature T_b is system temperature T
- under these assumptions, energy and entropy balances are

$$0 = \dot{Q} - \dot{W} + \dot{m}\left[h_1 - h_2 + \frac{\dot{r}_1^2 - \dot{r}_2^2}{2} + g(z_1 - z_2)\right]$$

$$0 = \frac{\dot{Q}}{T} + \dot{m}(s_1 - s_2)$$

⋆ $\dot{W}$ is the net rate of boundary/shaft/electrical/spring work

Heat transfer

- if temperature is spatially uniform, entropy balance is

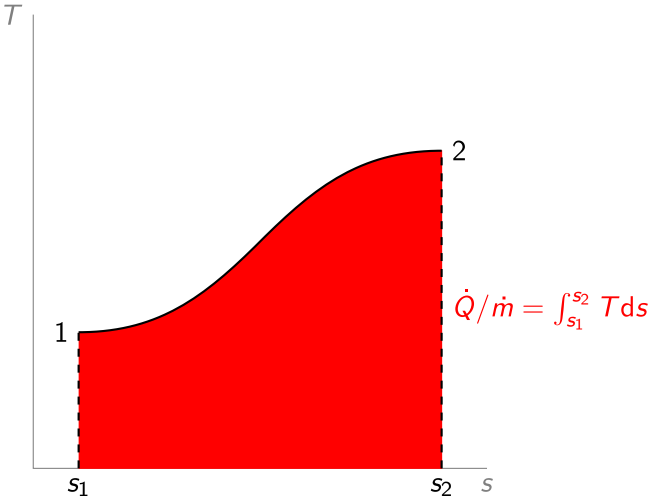
$$\frac{\dot{Q}}{\dot{m}} = T(s_2 - s_1)$$

- if temperature varies spatially, then

$$\frac{\dot{Q}}{\dot{m}} = \int_{s_1}^{s_2} T \mathrm{d}s$$

- the left-hand sides of these equations are
  - the specific heat transfer input (units of $\dot{Q}/\dot{m}$ are kJ/kg)
  - to an internally reversible device in a steady flow process

Visualizing heat transfer



Reversible flow work in general

- 1st law:

$$\frac{\dot{W}}{\dot{m}} = \frac{\dot{Q}}{\dot{m}} + h_1 - h_2 + \frac{\dot{r}_1^2 - \dot{r}_2^2}{2} + g(z_1 - z_2)$$
$$= \int_{s_1}^{s_2} T\mathrm{d}s + h_1 - h_2 + \frac{\dot{r}_1^2 - \dot{r}_2^2}{2} + g(z_1 - z_2)$$

- 2nd $T\mathrm{d}s$ equation: $T\mathrm{d}s = \mathrm{d}h$ - $v\mathrm{d}p$, so

$$\int_{s_1}^{s_2} T\mathrm{d}s = h_2 - h_1 - \int_{p_1}^{p_2} v\mathrm{d}p$$
$$\implies \frac{\dot{W}}{\dot{m}} = -\int_{p_1}^{p_2} v\mathrm{d}p + \frac{\dot{r}_1^2 - \dot{r}_2^2}{2} + g(z_1 - z_2)$$

- the left-hand side of this equation is
  - ◇ the specific work output (units of $\dot{W}/\dot{m}$ are kJ/kg)
  - ◇ of an internally reversible device in a steady-flow process

Reversible flow work in special cases

- if $\dot{W} = 0$ (no boundary/shaft/electrical/spring work), then

$$\int_{p_1}^{p_2} v \mathrm{d}p + \frac{\dot{r}_2^2 - \dot{r}_1^2}{2} + g(z_2 - z_1) = 0$$

  (this is a form of the Bernoulli equation from fluid mechanics)

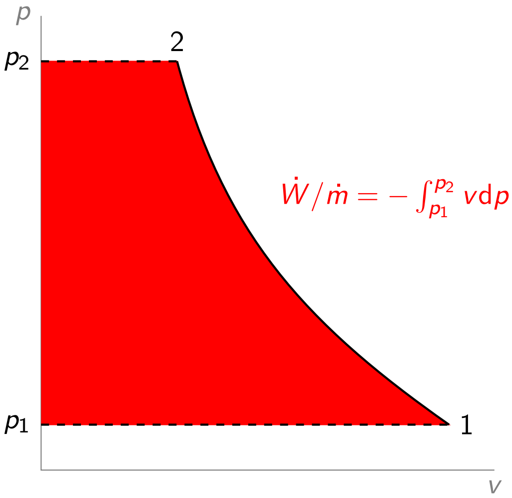
- if there are no KE or PE effects, then

$$\frac{\dot{W}}{\dot{m}} = -\int_{p_1}^{p_2} v \mathrm{d}p$$

- if v is also ~constant (e.g. ~incompressible substance), then

$$\frac{\dot{W}}{\dot{m}} \approx -v(p_2 - p_1)$$

Visualizing reversible flow work with no KE or PE effects



Outline

Polytropic reversible flow work ($n = 1$)

- in a polytropic ($pv^n = c$) process with no KE or PE effects,

$$\frac{\dot{W}}{\dot{m}} = -\int_{p_1}^{p_2} v \mathrm{d}p = -\int_{p_1}^{p_2} \left(\frac{c}{p}\right)^{1/n} \mathrm{d}p = -c^{1/n} \int_{p_1}^{p_2} p^{-1/n} \mathrm{d}p$$

- if $n = 1$, then $c = p_1 v_1 = p_2 v_2$ and

$$-c^{1/n} \int_{p_1}^{p_2} p^{-1/n} \mathrm{d}p = -c \int_{p_1}^{p_2} \frac{1}{p} \mathrm{d}p = -p_1 v_1 \ln\left(\frac{p_2}{p_1}\right)$$

Polytropic reversible flow work ($n \neq 1$)

- if $n \neq 1$, then $v = (c/p)^{1/n}$ and

$$
\begin{aligned}
-c^{1/n}\int_{p_1}^{p_2} p^{-1/n}\mathrm{d}p &= \frac{-c^{1/n}}{1-1/n}\left(p_2^{1-1/n} - p_1^{1-1/n}\right) \\
&= \frac{-1}{1-1/n}\left[p_2\left(\frac{c}{p_2}\right)^{1/n} - p_1\left(\frac{c}{p_1}\right)^{1/n}\right] \\
&= \frac{-\left(p_2 v_2 - p_1 v_1\right)}{1-1/n} \\
&= \frac{-n\left(p_2 v_2 - p_1 v_1\right)}{n-1}
\end{aligned}
$$

Polytropic reversible flow work (summary)

- in a polytropic ($pv^n = c$) reversible flow process,

$$\frac{\dot{W}}{\dot{m}} = \begin{cases} -p_1 v_1 \ln\left(p_2/p_1\right) & \text{if } n = 1 \\ -n\left(p_2 v_2 - p_1 v_1\right)/(n-1) & \text{if } n \neq 1 \end{cases}$$

Ideal gas polytropic reversible flow work ($n = 1$)

- for an ideal gas, polytropic with $n = 1$ means isothermal:

$$pv = c \implies T = \frac{c}{R}$$

- in this case, polytropic work is

$$\frac{\dot{W}}{\dot{m}} = -RT \ln\left(\frac{p_2}{p_1}\right)$$

Ideal gas polytropic reversible flow work ($n \neq 1$)

- for an ideal gas, polytropic work with $n \neq 1$ is

$$\frac{\dot{W}}{\dot{m}} = -\frac{n(p_2 v_2 - p_1 v_1)}{n-1} = -\frac{nR(T_2 - T_1)}{n-1}$$

- reminder: for an ideal gas in a polytropic process,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n}$$

- so polytropic work can also be written as

$$\frac{\dot{W}}{\dot{m}} = -\frac{nRT_1}{n-1}\left(\frac{T_2}{T_1} - 1\right) = -\frac{nRT_1}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right]$$

Ideal gas polytropic reversible flow work (summary)

- in an ideal gas polytropic ($pv^n = c$) reversible flow process,

$$\frac{\dot{W}}{\dot{m}} = \begin{cases} -RT\ln\left(p_2/p_1\right) & \text{if } n = 1 \\ -nR\left(T_2 - T_1\right)/(n-1) & \text{if } n \neq 1 \end{cases}$$

- sometimes in the $n \neq 1$ case, it may be more convenient to use

$$\frac{\dot{W}}{\dot{m}} = -\frac{nRT_1}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right]$$

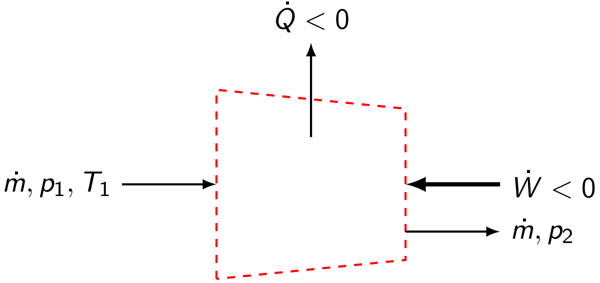
Outline

Problem statement

Air steadily flows into an internally reversible compressor at 1 bar and 20 °C and exits at 5 bar. If the process is polytropic with $n = 1.3$, find the specific (a) work and (b) heat transfer.

Moran et al., *Fundamentals of Engineering Thermodynamics* (2018), Example 6.15

System diagram



Given, find and assumptions

- **given:**
  - ◇ $p_1 = 1$ bar
  - ◇ $T_1 = 20\ ^\circ\mathrm{C}$
  - ◇ $p_2 = 5$ bar
  - ◇ $n = 1.3$
- **find:**
  (a) $\dot{W}/\dot{m}$
  (b) $\dot{Q}/\dot{m}$
- **assume:**
  - ◇ ideal gas
  - ◇ steady state, steady flow
  - ◇ no KE or PE effects
  - ◇ internal reversibility

Basic equations

- **basic equations:**
  - steady rate form of 1st law for compressors:

$$0 = \dot{Q} - \dot{W} + \dot{m}(h_1 - h_2)$$

  - polytropic work in internally reversible ideal gas flow:

$$\frac{\dot{W}}{\dot{m}} = -\frac{nRT_1}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right]$$

  - ideal gas in a polytropic process:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n}$$

Solution to part (a)

- from the polytropic work equation,

$$\begin{aligned}\frac{\dot{W}}{\dot{m}} &= -\frac{nRT_1}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right]\\ &= -\frac{1.3(0.287\text{kJ/kg/K})(293\text{K})}{0.3}\left[\left(\frac{5\text{bar}}{1\text{bar}}\right)^{0.3/1.3} - 1\right]\\ &= -163.9\text{kJ/kg}\end{aligned}$$

Solution to part (b)

- from the 1st law,

$$0 = \dot{Q} - \dot{W} + \dot{m}(h_1 - h_2) \implies \frac{\dot{Q}}{\dot{m}} = \frac{\dot{W}}{\dot{m}} + h_2 - h_1$$

- ideal gas table: $h_1 = 293.1$ kJ/kg at $T_1 = 293$ K
- ideal gas in a polytropic process:

$$\begin{aligned} T_2 &= T_1 \left(\frac{p_2}{p_1}\right)^{(n-1)/n} \\ &= (293\text{K}) \left(\frac{5\text{bar}}{1\text{bar}}\right)^{0.3/1.3} \\ &= 424.8\text{K} \end{aligned}$$

- ideal gas table: $h_2 = 426.5$ kJ/kg at $T_2 = 424.8$ K

Solution to part (b) (continued)

- plugging in numbers, the heat transfer per unit mass is

$$\begin{aligned}\frac{\dot{Q}}{\dot{m}} &= \frac{\dot{W}}{\dot{m}} + h_2 - h_1 \\ &= (-163.9 + 426.5 - 293.1)\text{kJ/kg} \\ &= -30.5\text{kJ/kg}\end{aligned}$$