Lecture 19 – Time-varying systems

Purdue ME 200, Thermodynamics I

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Outline

Rate, steady-state and delta equation forms

Delta forms of CoM and 1st law

Example

Rate, steady-state and delta forms

- this class uses 3 laws of physics
  - ◇ conservation of mass
  - ◇ conservation of energy
  - ◇ increase of entropy

Rate, steady-state and delta forms

- this class uses 3 laws of physics
  - conservation of mass
  - conservation of energy
  - increase of entropy
- each law can be written in 3 forms
  - rate: $\frac{dE}{dt} = \dot{Q} - \dot{W}$
  - steady-state: $0 = \dot{Q} - \dot{W}$
  - delta: $\Delta E = Q - W$

Converting between forms

- the class equation sheet lists only rate forms
- to get steady-state, set time derivative to zero in rate
- to get delta, integrate rate over time:

$$\int_0^t \frac{\mathrm{d}E}{\mathrm{d}t}\mathrm{d}\tau = \int_0^t (\dot{Q} - \dot{W})\mathrm{d}\tau$$

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$$\iff \underbrace{E(t) - E(0)}_{\Delta E} = \underbrace{\int_0^t \dot{Q}\mathrm{d}\tau}_{Q} - \underbrace{\int_0^t \dot{W}\mathrm{d}\tau}_{W}$$

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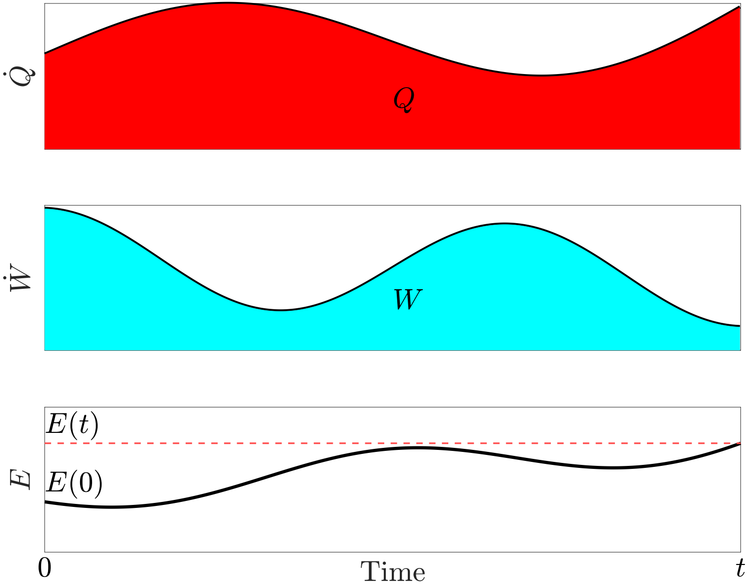
\end{aligned}
$$



$\dot{Q}$
Q
$\dot{W}$
W
E
$E(t)$
$E(0)$
0
Time
t

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Example

Delta form of conservation of mass

- integrating the rate form of CoM over time gives

$$\Delta m = \sum_{i=1}^{N_{\text{in}}} m_i^{\text{in}} - \sum_{j=1}^{N_{\text{out}}} m_j^{\text{out}}$$

where

$$m_i^{\text{in}} = \int_0^t \dot{m}_i^{\text{in}} \mathrm{d}\tau, \;\; m_j^{\text{out}} = \int_0^t \dot{m}_j^{\text{out}} \mathrm{d}\tau$$

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- if the system has 1 inlet and 1 outlet, then

$$\Delta m = m_{\text{in}} - m_{\text{out}}$$

Delta form of 1st law for open systems

- integrating the rate form of the 1st law gives

$$\Delta E = Q - W + \sum_{i=1}^{N_{\text{in}}} \int_0^t \dot{m}_i^{\text{in}} \left[\frac{(\dot{r}_i^{\text{in}})^2}{2} + g z_i^{\text{in}} + h_i^{\text{in}} \right] \mathrm{d}t$$
$$- \sum_{i=1}^{N_{\text{out}}} \int_0^t \dot{m}_j^{\text{out}} \left[\frac{(\dot{r}_j^{\text{out}})^2}{2} + g z_j^{\text{out}} + h_j^{\text{out}} \right] \mathrm{d}t$$

Delta form of 1st law for open systems (continued)

- if velocities, heights and enthalpies are time-invariant, then

$$\Delta E = Q - W + \sum_{i=1}^{N_{\text{in}}} m_i^{\text{in}} \left[\frac{(\dot{r}_i^{\text{in}})^2}{2} + g z_i^{\text{in}} + h_i^{\text{in}} \right]$$
$$- \sum_{j=1}^{N_{\text{out}}} m_j^{\text{out}} \left[\frac{(\dot{r}_j^{\text{out}})^2}{2} + g z_j^{\text{out}} + h_j^{\text{out}} \right]$$

Delta form of 1st law for open systems (continued)

- very often, KE and PE terms can be neglected, giving

$$\Delta U = Q - W + \sum_{i=1}^{N_{\text{in}}} m_i^{\text{in}} h_i^{\text{in}} - \sum_{j=1}^{N_{\text{out}}} m_j^{\text{out}} h_j^{\text{out}}$$

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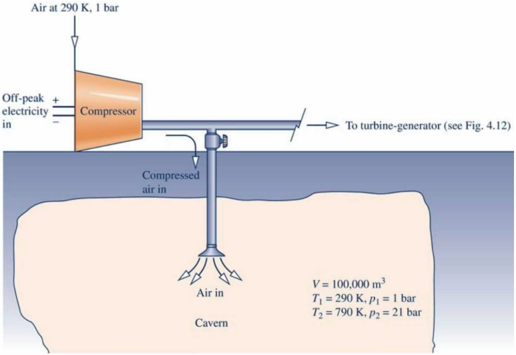
- if, in addition, the system has 1 inlet and 1 outlet, then

$$\Delta U = Q - W + m_{\text{in}} h_{\text{in}} - m_{\text{out}} h_{\text{out}}$$

Outline

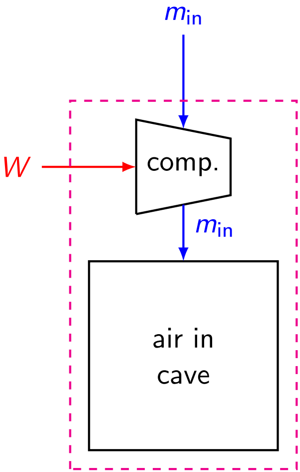
Problem statement

A 10^5 m^3 cave initially contains air at 290 K and 1 bar, the same conditions as the above-ground air. A compressor runs until the air in the cave reaches 790 K and 21 bar. Find (a) the initial and final mass of air in the cave, (b) the change in internal energy, and (c) the work done by the compressor.



Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)

System diagram



Given and find

- **given:**
    - ◇ $V = 10^5$ m^3
    - ◇ $T_1 = 290$ K, $p_1 = 1$ bar $= 100$ kPa
    - ◇ $T_2 = 790$ K, $p_2 = 21$ bar $= 2100$ kPa

Given and find

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  - $T_1 = 290$ K, $p_1 = 1$ bar $= 100$ kPa
  - $T_2 = 790$ K, $p_2 = 21$ bar $= 2100$ kPa
- **find:**
  (a) m_1 and m_2
  (b) ΔU
  (c) W

Assumptions and basic equations

- **assume:**
  - ideal gas
  - no air leaks
  - no KE or PE effects
  - no heat transfer across system boundary
  - spatially uniform intensive properties initially and finally

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  - ◇ no air leaks
  - ◇ no KE or PE effects
  - ◇ no heat transfer across system boundary
  - ◇ spatially uniform intensive properties initially and finally
- **basic equations:**
  - ◇ delta form of CoM with 1 inlet and 1 outlet:

$$\Delta m = m_{\text{in}} - m_{\text{out}}$$

  - ◇ delta form of 1st law with no KE or PE effects:

$$\Delta U = Q - W + m_{\text{in}} h_{\text{in}} - m_{\text{out}} h_{\text{out}}$$

Solution to part (a)

- from the ideal gas law,

$$m_1 = \frac{p_1 V}{RT_1} = \frac{(100\text{kPa})(10^5\text{m}^3)}{(0.287\text{kJ}/(\text{kg K}))(290\text{K})} = 1.20 \times 10^5\text{kg}$$

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- similarly,

$$m_2 = \frac{p_2 V}{R T_2} = \frac{(2100\text{kPa})(10^5\text{m}^3)}{(0.287\text{kJ}/(\text{kg K}))(790\text{K})} = 9.26 \times 10^5\text{kg}$$

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- assuming intensive properties are spatially uniform in the cave,

$$\Delta U = \Delta(mu) = m_2 u_2 - m_1 u_1$$

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- from ideal gas table for air,
  - ◇ $u_1 = 206.9$ kJ/kg at $T_1 = 290$ K
  - ◇ $u_2 = 584.2$ kJ/kg at $T_2 = 790$ K

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  - ◇ $u_2 = 584.2$ kJ/kg at $T_2 = 790$ K
- so

$$\begin{aligned}\Delta U &= m_2 u_2 - m_1 u_1 \\ &= (9.26 \times 10^5\text{kg})(584.2\text{kJ/kg}) - (1.20 \times 10^5\text{kg})(206.9\text{kJ/kg}) \\ &= 5.16 \times 10^8\text{kJ}\end{aligned}$$

Solution to part (c)

- delta form of 1st law for open systems with
  - ◇ 1 inlet, no outlet
  - ◇ no heat transfer
  - ◇ no changes in KE or PE:

$$W = \Delta U - m_{\text{in}} h_{\text{in}} = \Delta U - (m_2 - m_1) h_{\text{in}}$$

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- for incoming air from atmosphere, $h_{\text{in}} = 290.1$ kJ/kg, so

$$\begin{aligned} W &= \Delta U - (m_2 - m_1) h_{\text{in}} \\ &= (5.16 \times 10^8 \text{kJ}) - [(9.26 - 1.20) \times 10^5 \text{kg}](290.1 \text{kJ/kg}) \\ &= 2.82 \times 10^8 \text{kJ} \end{aligned}$$

Bonus: economics

- suppose input electricity to compressor costs 50 \$/MWh
- 1 kJ = 1 kWs, so compressor work is

$$W = \frac{(2.82 \times 10^8 \text{kWs})(1\text{h})}{3600\text{s}} = 78400\text{kWh} = 78.4\text{MWh}$$

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- so input electricity costs

$$(50\$/\text{MWh})(78.4\text{MWh}) = \$3920$$