Lecture 35 – Vapor-compression refrigeration

Purdue ME 200, Thermodynamics I

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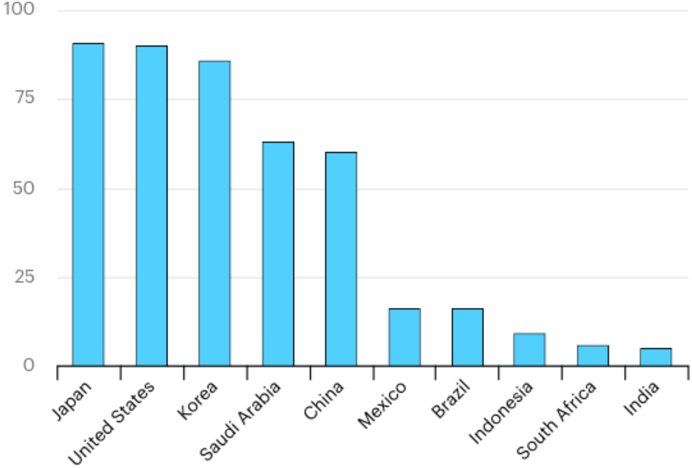
Outline

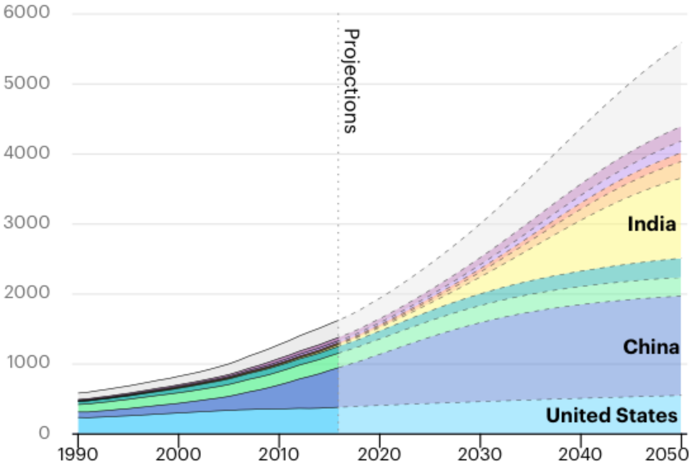
Many people in hot climates lack air conditioning



y-axis: percent of population that has air conditioning

International Energy Agency, *The Future of Cooling* (2018)

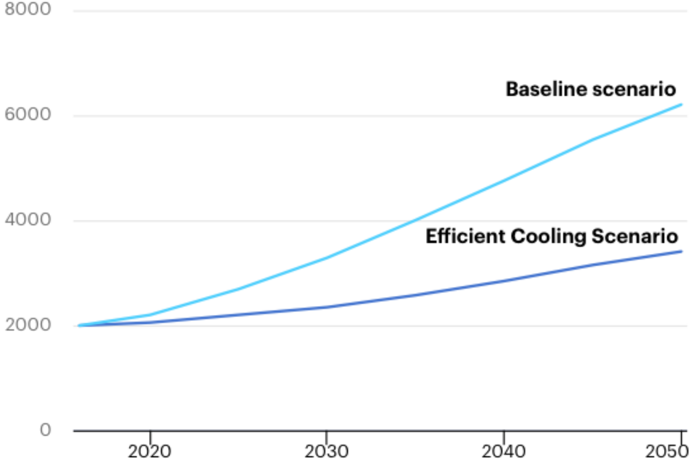
But many people will soon get air conditioning



y-axis: millions of air conditioners

International Energy Agency, *The Future of Cooling* (2018)

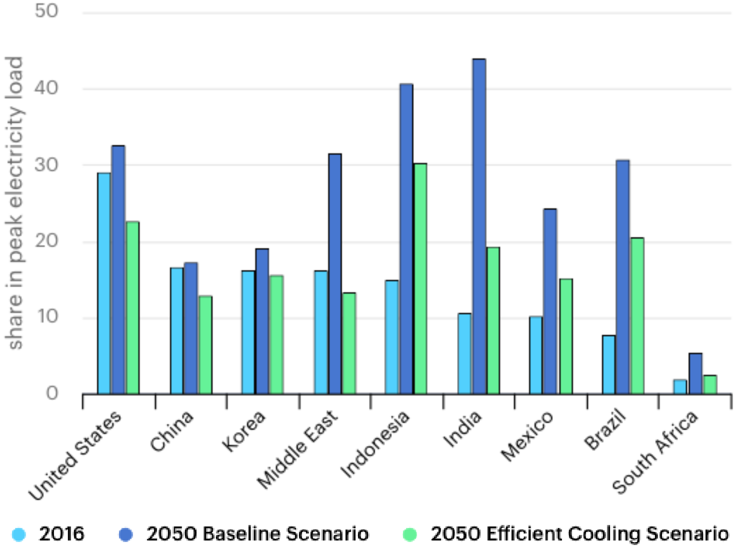
Air conditioning uses a lot of electricity



y-axis: annual TWh of electricity (2019 global total: 23000 TWh)

International Energy Agency, *The Future of Cooling* (2018)

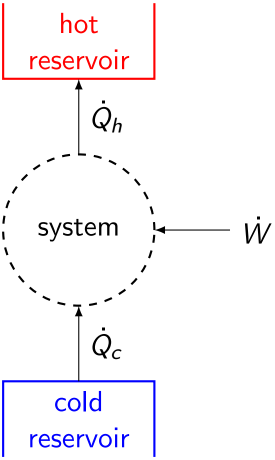
Air conditioning drives electricity demand peaks



International Energy Agency, *The Future of Cooling* (2018)

Outline

Reminder: refrigeration cycles



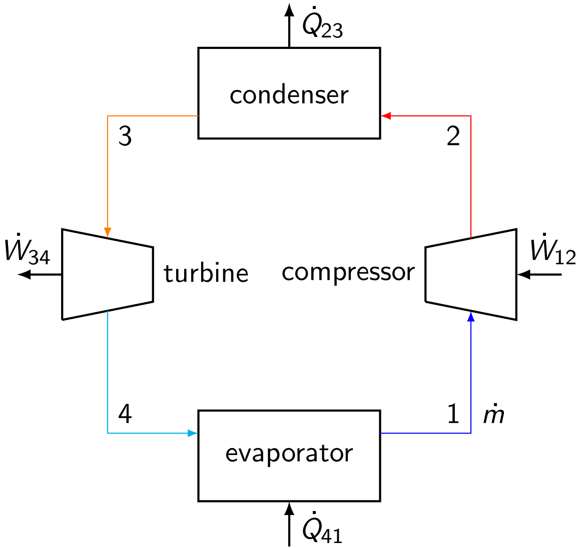
- cooling capacity: $\dot{Q}_c$
- coefficient of performance:

$$\beta = \frac{\text{heat transfer input}}{\text{net work input}} = \frac{\dot{Q}_c}{\dot{W}} = \frac{\dot{Q}_c}{\dot{Q}_h - \dot{Q}_c} = \frac{\dot{Q}_c/\dot{Q}_h}{1 - \dot{Q}_c/\dot{Q}_h}$$

- Carnot performance limit:

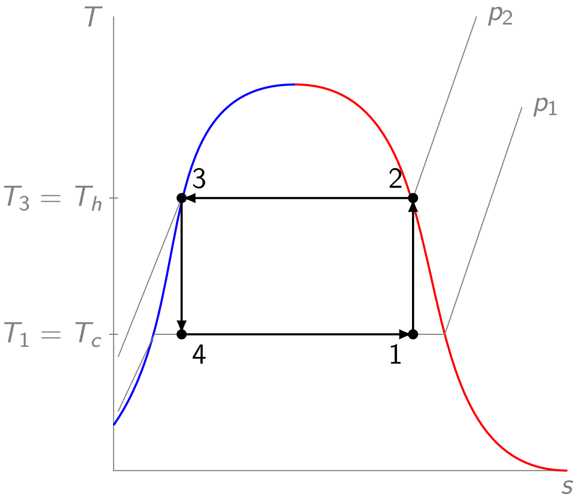
$$\beta \leq \frac{T_c/T_h}{1 - T_c/T_h}$$

Carnot refrigeration cycle schematic



sign convention: energy flows are positive in the arrow directions

T-s diagram of the Carnot refrigeration cycle



Verifying the Carnot refrigeration cycle COP

- 1st law on the full system: $\dot{Q}_{41} + \dot{W}_{12} = \dot{Q}_{23} + \dot{W}_{34}$
- so the Carnot COP is

$$\beta^\star = \frac{\dot{Q}_{41}}{\dot{W}_{12} - \dot{W}_{34}} = \frac{\dot{Q}_{41}}{\dot{Q}_{23} - \dot{Q}_{41}} = \frac{\dot{Q}_{41}/\dot{Q}_{23}}{1 - \dot{Q}_{41}/\dot{Q}_{23}}$$

- from the T-s diagram, $\dot{Q}_{23}/\dot{m} = \int_{s_2}^{s_3} T\mathrm{d}s = T_3(s_3 - s_2)$
- similarly, $\dot{Q}_{41}/\dot{m} = \int_{s_4}^{s_1} T\mathrm{d}s = T_1(s_4 - s_1)$
- but $s_3 = s_4$ and $s_2 = s_1$, so $\dot{Q}_{41}/\dot{Q}_{23} = T_1/T_3$ and

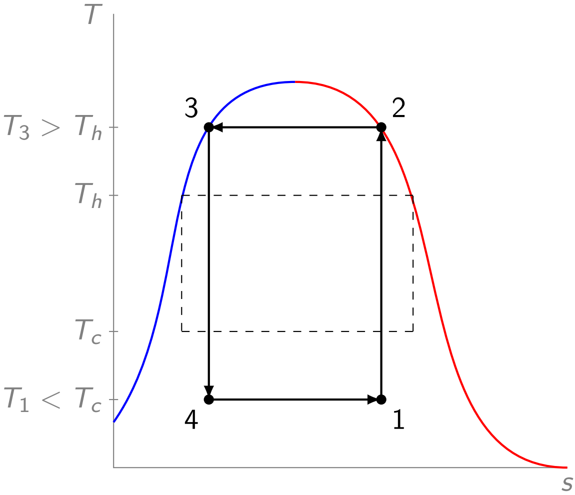
$$\beta^\star = \frac{T_1/T_3}{1 - T_1/T_3} = \frac{T_c/T_h}{1 - T_c/T_h}$$

Outline

Heat transfer through finite temperature differences

- in the Carnot cycle, $T_1 = T_c$ and $T_3 = T_h$
- so $\Delta T = 0$ for heat transfer in the evaporator and condenser
- in reality, the evaporator and condenser ΔT must be finite

T-s diagram with finite temperature differences



COP with finite temperature differences

- all but one step of the Carnot COP derivation still hold
- exception: the last step used $T_1 = T_c$ and $T_3 = T_h$
- with finite temperature differences ($T_1 \neq T_c$ and $T_3 \neq T_h$),

$$\beta = \frac{T_1/T_3}{1 - T_1/T_3}$$

- but $T_1 < T_c$ and $T_3 > T_h$, so

$$\frac{T_1}{T_3} < \frac{T_c}{T_h} \quad \text{and} \quad 1 - \frac{T_1}{T_3} > 1 - \frac{T_c}{T_h}$$

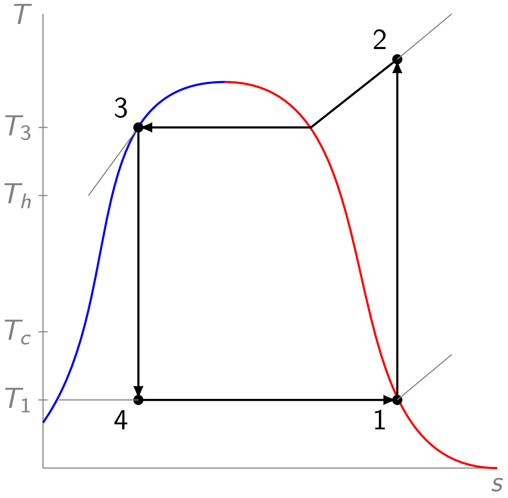
- therefore, the COP is below the Carnot limit:

$$\beta = \frac{T_1/T_3}{1 - T_1/T_3} < \frac{T_c/T_h}{1 - T_c/T_h}$$

Requiring 'dry' compression

- in the Carnot cycle, compression happens in the vapor dome
- but liquid droplets can damage real compressors
- in practice, state 1 is usually saturated or superheated vapor

T-s diagram with finite ΔT and dry compression



Replacing the turbine with an expansion valve

- the Carnot cycle extracts work in a turbine after the condenser
- this turbine operates in the vapor dome at fairly low quality
- as with compressors, liquid droplets can damage real turbines
- also, the amount of work extracted is not large
- in practice, an expansion valve usually replaces the turbine

Outline

Energy and air conditioning

The Carnot refrigeration cycle

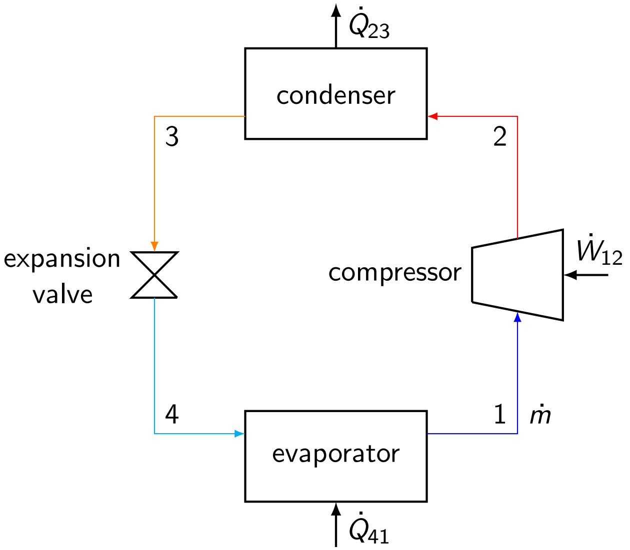
Departures from the Carnot refrigeration cycle

The ideal vapor-compression refrigeration cycle

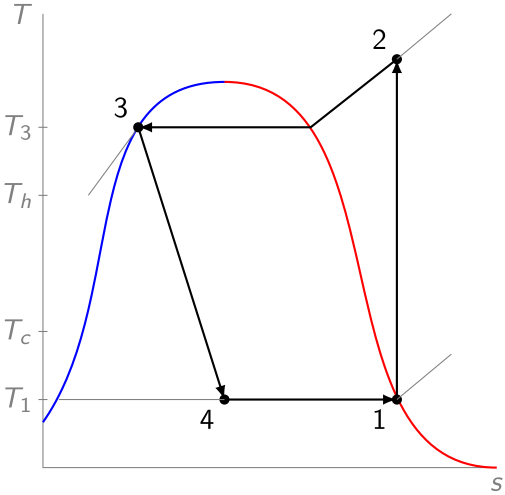
The ideal vapor-compression refrigeration cycle

- the ideal vapor-compression cycle is like the Carnot refrigerator
- but it uses the 3 modifications in the previous section
- it's still ideal in that
  - ◇ compression is isentropic
    (the compressor is adiabatic and internally reversible)
  - ◇ expansion is isenthalpic
    (no stray heat transfer in the expansion valve)
  - ◇ the condenser, evaporator and connecting pipes are isobaric
    (no pressure drops due to fluid friction)

Ideal vapor-compression refrigeration cycle schematic



T-s diagram of the ideal vapor-compression cycle



1st and 2nd laws in the ideal vapor-compression refrigerator

- from the steady-state 1st law in rate form,
  - ◇ whole system: $\dot{W}_{12} + \dot{Q}_{41} = \dot{Q}_{23}$
  - ◇ compressor: $\dot{W}_{12} = \dot{m}(h_2 - h_1)$
  - ◇ condenser: $\dot{Q}_{23} = \dot{m}(h_2 - h_3)$
  - ◇ expansion valve: $h_4 = h_3$
  - ◇ evaporator: $\dot{Q}_{41} = \dot{m}(h_1 - h_4)$
- from the steady-state 2nd law in rate form,
  - ◇ whole system: $\dot{\sigma} = \dot{Q}_{23}/T_{23}^b - \dot{Q}_{41}/T_{41}^b$
  - ◇ compressor: $\dot{\sigma}_{12} = \dot{m}(s_2 - s_1)$
  - ◇ condenser: $\dot{\sigma}_{23} = \dot{Q}_{23}/T_{23}^b + \dot{m}(s_3 - s_2)$
  - ◇ expansion valve: $\dot{\sigma}_{34} = \dot{m}(s_4 - s_3)$
  - ◇ evaporator: $\dot{\sigma}_{41} = \dot{m}(s_1 - s_4) - \dot{Q}_{41}/T_{41}^b$

The ideal vapor-compression refrigerator COP

- compressor energy balance: $\dot{W}_{12} = \dot{m}(h_2 - h_1)$
- evaporator energy balance: $\dot{Q}_{41} = \dot{m}(h_1 - h_4)$
- so the ideal vapor-compression refrigerator COP is

$$\beta = \frac{\dot{Q}_{41}}{\dot{W}_{12}} = \frac{\dot{m}(h_1 - h_4)}{\dot{m}(h_2 - h_1)} = \frac{h_1 - h_4}{h_2 - h_1}$$