Homework 3: Buildings

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Directions:

- Students may work individually or in groups, but each student must upload their own solutions to Gradescope by **11:59 PM ET on Thursday, February 8**.
- Use any outside resources you want, but **cite your sources**. (If you really want to learn the material, I recommend seriously attempting the problems yourself before looking for outside help.)
- The TA will grade each problem quickly on a three-tier scale:
  - Zero points for a solution that's mostly unreadable or missing.
  - One point for a serious attempt that's not easy to read or is substantially incorrect.
  - Two points for a solution that's clearly readable and nearly or completely correct.

Problems:

1. Consider a 1R1C thermal circuit with continuous-time dynamics

$$C\frac{\mathrm{d}T(t)}{\mathrm{d}t} = \frac{\theta(t) - T(t)}{R} + q_c(t) + q_e(t).$$

   Show that with uniform time step Δt and piecewise constant θ, q_c, and q_e, the equivalent discrete-time dynamics are $T(k+1) = aT(k) + (1-a)R\left(q_c(k) + w(k)\right)$, where $a = e^{-\Delta t/(RC)}$ and $w(k) = q_e(k) + \theta(k)/R$.

2. Suppose in the 1R1C model from #1, $T(t) = \hat{T}$ and $q_e(t) = \hat{q}_e$ for constants $\hat{T}$ and $\hat{q}_e$ and all t.
   (a) Find a relationship between the cumulative heat demand $H = \int \max\left\{0, q_c(t)\right\} \mathrm{d}t$ (in kWh) and the number of heating degree-hours $D = \int \max\left\{0, \theta_0 - \theta(t)\right\}$ (in °Ch), where the balance temperature θ_0 is defined such that $q_c(t) = 0$ if $\theta(t) = \theta_0$.
   (b) For a given building, what factors might increase or decrease the balance temperature θ_0? Explain.

3. Download the Matlab files in the Github repository buildings and open the script `simulate2R2C`.
   (a) Fill in the missing code in the cell `2R2C system matrices`, which should generate the continuous-time and discrete-time dynamics and input matrices.
   (b) Fill in the missing code in the function `perfectTrackingControl`, which should implement near-perfect setpoint tracking control (but saturate the HVAC thermal power at equipment capacity limits if necessary).
   (c) Fill in the missing code in the function `thermostaticControl`, which should implement thermostatic control. Report the cumulative heat demand (in kWh) under each control policy.