Thermal storage and water heaters

Purdue ME 597, Distributed Energy Resources

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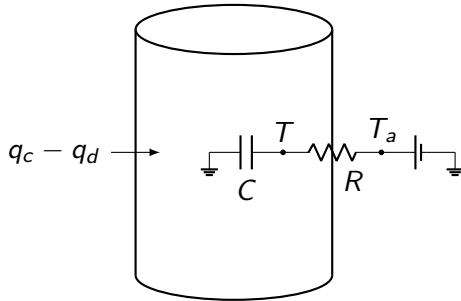# Outline

Lumped sensible thermal storage

- C (kWh/$^\circ$C) is thermal capacitance of storage medium
- R ($^\circ$C/kW) is thermal resistance of tank wall
- q_c (kW) is charging thermal power
- q_d (kW) is discharging thermal power
- assumption: temperature is spatially uniform inside tank

- suppose tank temperature satisfies $T \in [\underline{T}, \overline{T}]$
- choose an arbitrary reference temperature ('datum') T_{LS}
- define stored energy x (kWh) as zero when $T = T_{LS}$:

$$x = C(T - T_{LS}) \in [C(\underline{T} - T_{LS}), C(\overline{T} - T_{LS})]$$

- from conservation of energy,

$$\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= \frac{T_a - T}{R} + q_c - q_d \\
&= \frac{T_a - (T_{LS} + x/C)}{R} + q_c - q_d \\
&= -\frac{x}{RC} + q_c + w \text{ where } w = \frac{T_a - T_{LS}}{R} - q_d
\end{aligned}$$
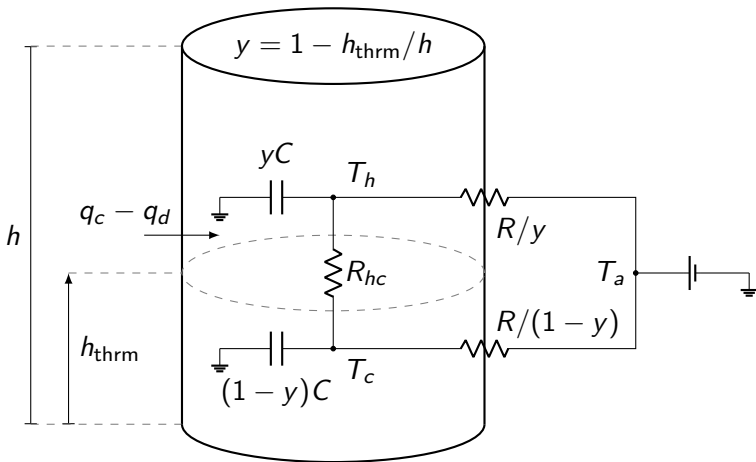
Lumped sensible thermal storage in a 'box of rocks'

Green Energy Times: Electric Thermal Storage

Steffes: Comfort Plus Forced Air

Stratified sensible thermal storage

- hot column sits on cold column, separated by thermocline
- T_h and T_c are constant; charging moves thermocline down

- define column energies relative to (arbitrary) T_{SS}:

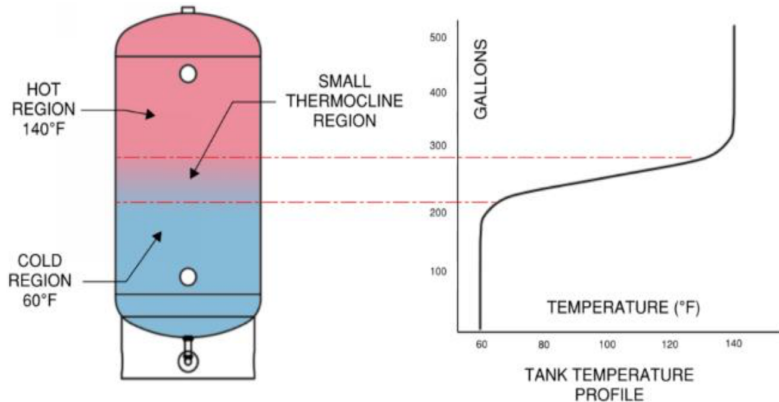$$x_h = yC(T_h - T_{SS}), \ x_c = (1 - y)C(T_c - T_{SS})$$

- then total energy is

$$x = x_h + x_c = \cdots = C[y(T_h - T_c) + T_c - T_{SS}]$$
$$\in [C(T_c - T_{SS}), C(T_h - T_{SS})]$$

- from conservation of energy,

$$\frac{dx}{dt} = \frac{(T_a - T_h)y}{R} + \frac{(T_a - T_c)(1 - y)}{R} + q_c - q_d$$
$$= \cdots = -\frac{x}{RC} + q_c + w \text{ where } w = \frac{T_a - T_{SS}}{R} - q_d$$

Latent thermal storage

- tank contains M kg of material that freezes and melts
- latent heat of fusion L (kWh/kg), melting point T_m (°C)
- liquid mass $m_\ell \in [0, M]$ (kg) increases as tank charges
- define energy relative to (arbitrary) T_L:

$$x = C(T_m - T_L) + Lm_\ell \in [C(T_m - T_L), C(T_m - T_L) + LM]$$
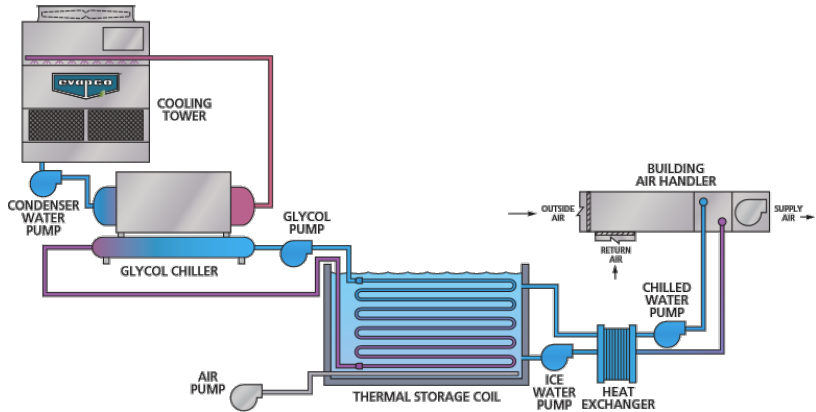
- from conservation of energy,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = q_c + w \text{ where } w = \frac{T_a - T_m}{R} - q_d$$

Evapco: Extra-Pak Ice Coils

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Summary

- all three storage types follow 'heat battery' model:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{x}{\tau} + q_c + w, \ \underline{x} \le x \le \overline{x}$$

- in discrete time with $a = e^{-\Delta t/\tau}$,

$$x(k+1) = \begin{cases} ax(k) + (1-a)\tau(q_c(k) + w(k)) & \text{if } \tau < \infty \\ x(k) + \Delta t(q_c(k) + w(k)) & \text{if } \tau = \infty \end{cases}$$

	Lumped sensible	Stratified sensible	Latent
x	$C(T - T_{\text{LS}})$	$C[y(T_h - T_c) + T_c - T_{\text{SS}}]$	$C(T_m - T_{\text{L}}) + Lm_\ell$
$\overline{x}$	$C(\overline{T} - T_{\text{LS}})$	$C(T_h - T_{\text{SS}})$	$C(T_m - T_{\text{L}}) + LM$
$\underline{x}$	$C(\underline{T} - T_{\text{LS}})$	$C(T_c - T_{\text{SS}})$	$C(T_m - T_{\text{L}})$
τ	RC	RC	∞
w	$(T_a - T_{\text{LS}})/R - q_d$	$(T_a - T_{\text{SS}})/R - q_d$	$(T_a - T_m)/R - q_d$

One model can capture switches between storage types

- example: suppose tank goes stratified $\rightarrow$ lumped as it heats
- goal: choose T_{LS}, T_{SS}, $\underline{T}$ so model switches smoothly
- to get $w_{LS} = w_{SS}$, need $T_{LS} = T_{SS}$
  (might as well choose $T_{LS} = T_{SS} = T_a$ so $w_{LS} = w_{SS} = 0$)
- to get $\underline{x}_{LS} = \overline{x}_{SS}$, need $\underline{T} = T_h$
- with these choices, one model captures full operating range:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{x}{\tau} + q_c - q_d$$
$$C(T_c - T_a) \leq x \leq C(\overline{T} - T_a)$$

- stratified $\leftrightarrow$ lumped switch happens when $x = C(T_h - T_a)$

- discharging typically takes little energy (just a pump or fan)
- charging typically uses a heat pump or resistance heat
- heat pump COPs depend on tank and ambient temperatures
- for resistance, COP ≈ 1

- thermal capacitance is $C = \rho c V$
- thermal resistance is $R = 1/(UA)$
- typically, $U \approx 0.0005$ to 0.001 kW/($^\circ$C m^2)
- for liquid water at 50 $^\circ$C,
  - $\rho = 988$ kg/m^3
  - $c = 4.17$ kJ/($^\circ$C kg) $= 0.00116$ kWh/($^\circ$C kg)
  - $\rho c = 1.15$ kWh/($^\circ$C m^3)
  - $L = 334$ kJ/kg $= 0.09$ kWh/kg
- for a cylinder with radius r and height h,
  - $V = \pi r^2 h$
  - $A = 2\pi r h + 2\pi r^2 = 2V(1/r + 1/h)$

How much energy does a domestic water heater store?

- typical $V \approx 0.2$ to 0.3 m^3, so $C \approx 0.25$ to 0.35 kWh/$^\circ$C
- stratified sensible thermal storage capacity:

$$\overline{x} - \underline{x} = C(T_h - T_c)$$

- typically, $T_c \approx 15\ ^\circ$C
- below $\sim$49 $^\circ$C, legionella bacteria can grow
- above $\sim$60 $^\circ$C, water can scald
- with $T_h \approx 51\ ^\circ$C, energy capacity ≈ 9 to 13 kWh

Outline

- typical home draws $\sim(16n + 4)$ gal of hot water per day
  - $\diamond$ $\sim16n$ gal for one shower or bath apiece from n occupants
  - $\diamond$ ~4 gal for everything else (like washing dishes and clothes)
- typical shower lasts ~8 min, draws volumetric flow rate

$$\dot{V} \approx 2 \text{ gal/min} = 0.45 \text{ m}^3/\text{h}$$

Florida Solar Energy Center (2015): Estimating daily domestic hot water use in
North American homes

Typical hot water thermal power draws

- thermal power associated with $\dot{V}$ is $\rho c \dot{V}(T - T_\text{in})$

- inlet water temperature on day d of the year ($d = 1$ on Jan 1):

$$T_\text{in} \approx 15\ ^\circ\text{C} + (6\ ^\circ\text{C})\sin(d - 130)$$
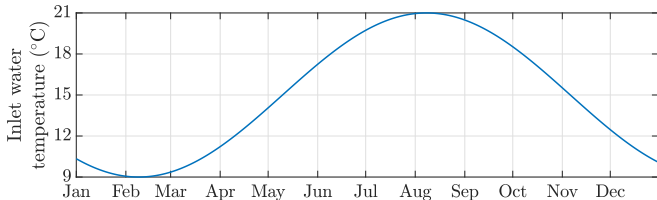
- with $T \approx 51\ ^\circ\text{C}$, $T_\text{in} \approx 15\ ^\circ\text{C}$, typical shower draws

$$\sim \left(1.15\ \text{kWh}/(^\circ\text{C m}^3)\right)(0.45\ \text{m}^3/\text{h})(36^\circ\text{C}) \approx 19\ \text{kW}$$

Hendron (2010): Building America house simulation protocols.

Most heat withdrawals do not depend on tank temperature

- typical shower or bath has constant mixed water flow $\dot{m}^\star$
- typical bather wants constant mixed water temperature $T^\star$
- so

$$\overbrace{\frac{\dot{m}T + (\dot{m}^\star - \dot{m})T_{\text{in}}}{\dot{m}^\star}}^{\text{mixed water temperature}} = T^\star$$

$$\iff \dot{m}(T - T_{\text{in}}) + \dot{m}^\star T_{\text{in}} = \dot{m}^\star T^\star$$

$$\iff \underbrace{\dot{m}c(T - T_{\text{in}})}_{q_d} = \underbrace{\dot{m}^\star c(T^\star - T_{\text{in}})}_{\text{independent of } T}$$

$\implies$ heat withdrawal q_d does not depend on tank temperature T
   (if T decreases, bather can mix in more hot water, less cold)

- showers and baths dominate hot water use
- so generate *n* showers per day, each at ~19 kW for ~10 min
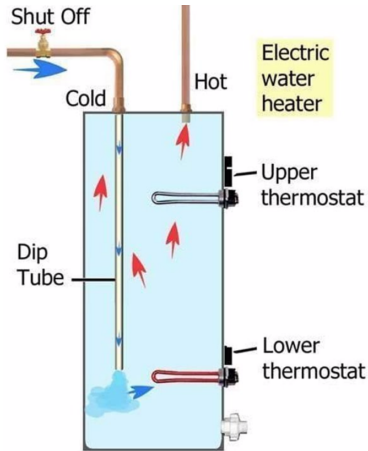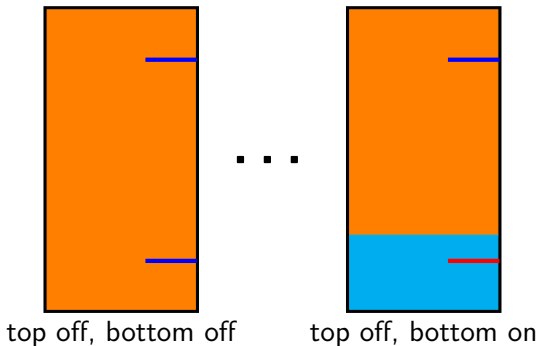- and place them at random plausible times

Outline

- typically designed to maintain stratification
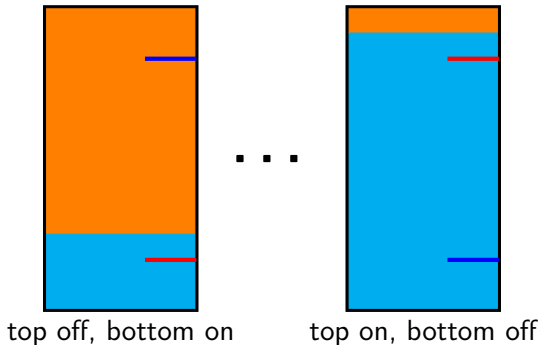- hot water drawn from top, cold added to bottom

Resistance-only water heater controls

- water heater controls vary by manufacturer
- one approach:
    - ◇ one resistor near bottom, one near top
    - ◇ each resistor has a water temperature sensor
    - ◇ run bottom resistor if its temperature drops below setpoint
    - ◇ switch to top resistor if top temperature drops below setpoint
    - ◇ when top temperature returns to setpoint, switch to bottom
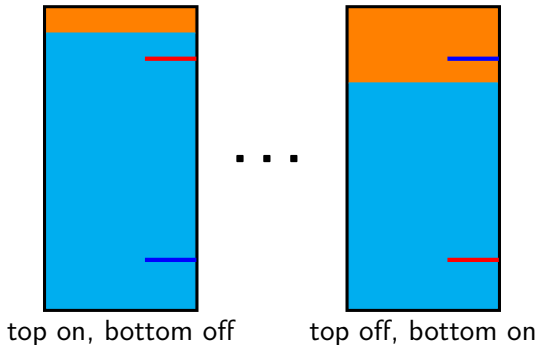   - ◇ when bottom temperature returns to setpoint, turn bottom off

top off, bottom off   top off, bottom on

top off, bottom on   top on, bottom off

top on, bottom off   top off, bottom on

top on, bottom on   top off, bottom off

- anytime tank is not fully charged, exactly one resistor runs
$\implies$ model:
    - $\diamond$ try to place $x(k+1) = ax(k) + (1-a)\tau(q_c(k) + w(k))$ at $\overline{x}$
    - $\diamond$ but saturate $q_c(k)$ at capacity limits $[0, \overline{p}_r]$

$$q_c(k) = \max\left\{0, \min\left\{\overline{p}_r, \frac{\overline{x} - ax(k)}{(1-a)\tau} - w(k)\right\}\right\}$$
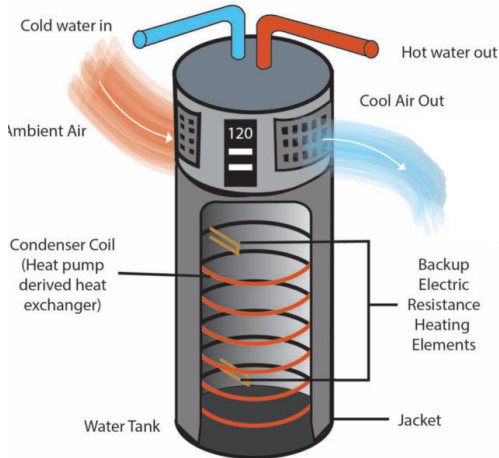
$$p(k) = q_c(k)$$

Outline

- most HPWHs are hybrids with resistance backup
- heat pump COP depends on temperatures of water, air

TRC Solutions: Heat pump water heater technology

- anytime tank is not fully charged, heat pump runs
$\implies$ try to place $x(k+1)$ at $\overline{x}$
- but saturate $q_c(k)$ at capacity limits $[0, \eta(k)\overline{p}]$

$$q_c(k) = \max\left\{0, \min\left\{\eta(k)\overline{p}, \frac{\overline{x} - ax(k)}{(1-a)\tau} - w(k)\right\}\right\}$$

$$p(k) = q_c(k)/\eta(k)$$

Hybrid water heater controls

- anytime tank is not fully charged, heat pump runs
- if charge drops below a threshold x_r, resistor also runs
$\implies$ try to place $x(k+1)$ at $\overline{x}$, but
  - ◇ if $x(k) \geq x_r$, same as heat-pump-only case
  - ◇ if $x(k) < x_r$, saturate $q_c(k)$ at combined limits $[0, \eta(k)\overline{p} + \overline{p}_r]$

$$q_c(k) = \max\left\{0, \min\left\{\eta(k)\overline{p} + \overline{p}_r, \frac{\overline{x} - ax(k)}{(1-a)\tau} - w(k)\right\}\right\}$$

$$p(k) = \max\left\{\frac{q_c(k)}{\eta(k)}, q_c(k) + (1-\eta(k))\overline{p}\right\}$$
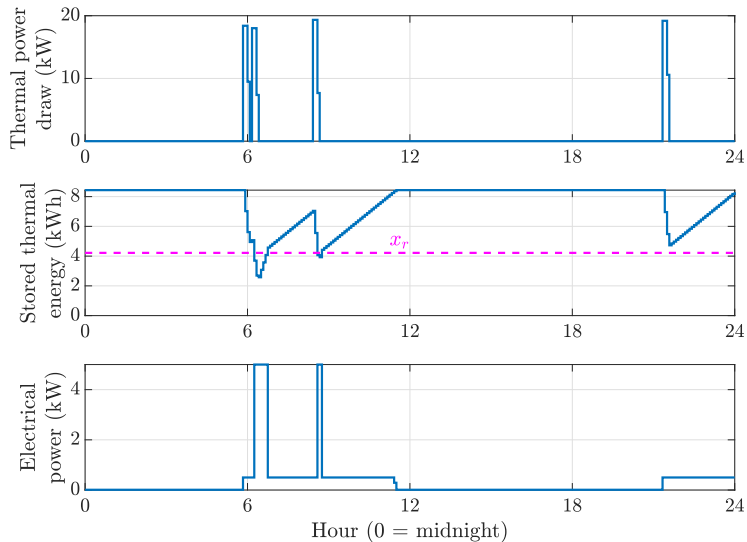
4 occupants, 50 gal (0.19 m^3) hybrid HPWH