Battery optimization examples

Purdue ME 597, Distributed Energy Resources

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Outline

Battery examples

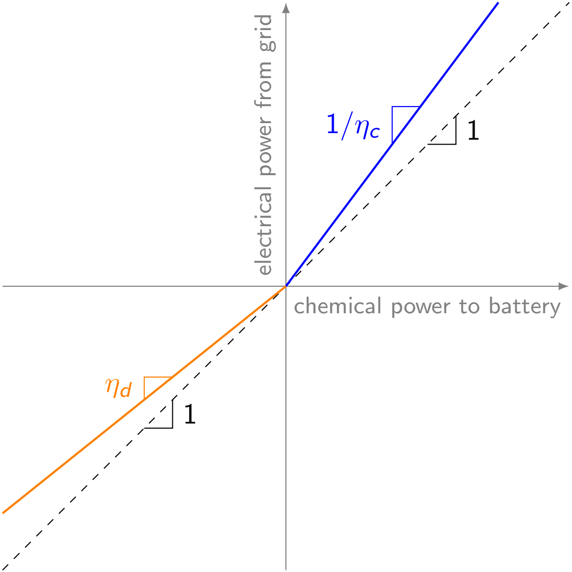
Dealing with nonmonotone objectives

Joint sizing and operation

Battery model reminders

$$
\begin{aligned}
x(k+1) &= ax(k) + (1-a)\tau p^{\text{chem}}(k),\;\; a = e^{-\Delta t/\tau} \\
p(k) &= \max\left\{ p^{\text{chem}}(k)/\eta_c, \eta_d p^{\text{chem}}(k) \right\} \\
\underline{x} &\leq x(k) \leq \overline{x} \\
-\overline{p}_d \leq p(k) \leq \overline{p}_c &\iff -\frac{\overline{p}_d}{\eta_d} \leq p^{\text{chem}}(k) \leq \eta_c \overline{p}_c
\end{aligned}
$$

Chemical and electrical power



Problem formulation

- choose
  - $x = (x(0), \ldots, x(K)) \in \mathbf{R}^{K+1}$
  - $p^{\text{chem}} = (p^{\text{chem}}(0), \ldots, p^{\text{chem}}(K-1)) \in \mathbf{R}^K$
  - $p = (p(0), \ldots, p(K-1)) \in \mathbf{R}^K$
- to minimize $\Delta t \pi^\top p$
- subject to $x(0) = x_0$ and for $k = 0, \ldots, K-1$,
  - $x(k+1) = ax(k) + (1-a)\tau p^{\text{chem}}(k)$
  - $p(k) = \max\left\{p^{\text{chem}}(k)/\eta_c, \eta_d p^{\text{chem}}(k)\right\}$
  - $\underline{x} \leq x(k+1) \leq \overline{x}$
  - $-\overline{p}_d/\eta_d \leq p^{\text{chem}}(k) \leq \eta_c \overline{p}_c$
- given Δt, $\pi \in \mathbf{R}^K$, x_0, a, τ, η_c, η_d, $\underline{x}$, $\overline{x}$, $\overline{p}_c$, $\overline{p}_d$

CVX code

```
cvx_begin
  variables x(K+1,1) pChem(K,1) p(K,1)
  minimize( dt*pie'*p )
  subject to
    x(1) == x0
    x(2:K+1) == a*x(1:K) + (1-a)*tau*pChem
    p == max(pChem/etac, etad*pChem)
    xMin <= x(2:K+1) <= xMax
    -pdMax/etad <= pChem <= etac*pcMax
cvx_end
```

```
Disciplined convex programming error:
  Invalid constraint: {real affine} == {convex}
```

CVX code

```
cvx_begin
  variables x(K+1,1) pChem(K,1)
  minimize( dt*pie'*max(pChem/etac, etad*pChem) )
  subject to
    x(1) == x0
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    xMin <= x(2:K+1) <= xMax
    -pdMax/etad <= pChem <= etac*pcMax
cvx_end
```
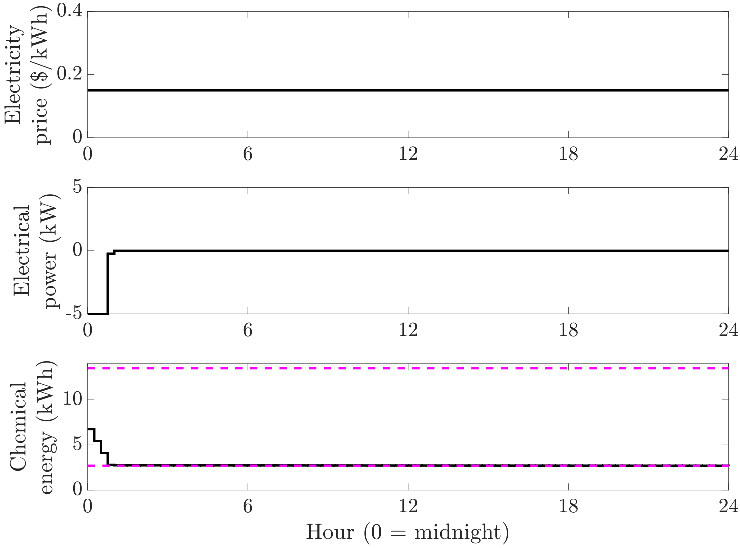

Constant energy price



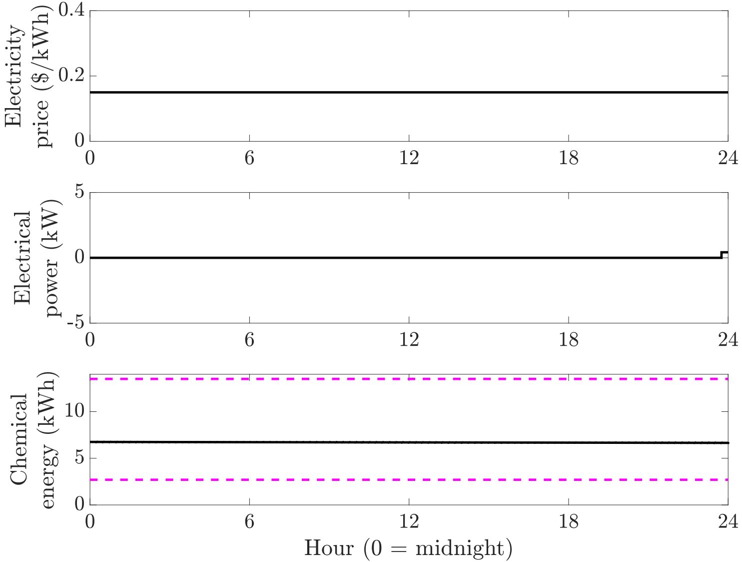
Problem revision options

1. add final constraint such as $x(K) = x(0)$ or $x(K) \geq x(0)$
2. add objective rewarding larger final state $x(K)$: minimize

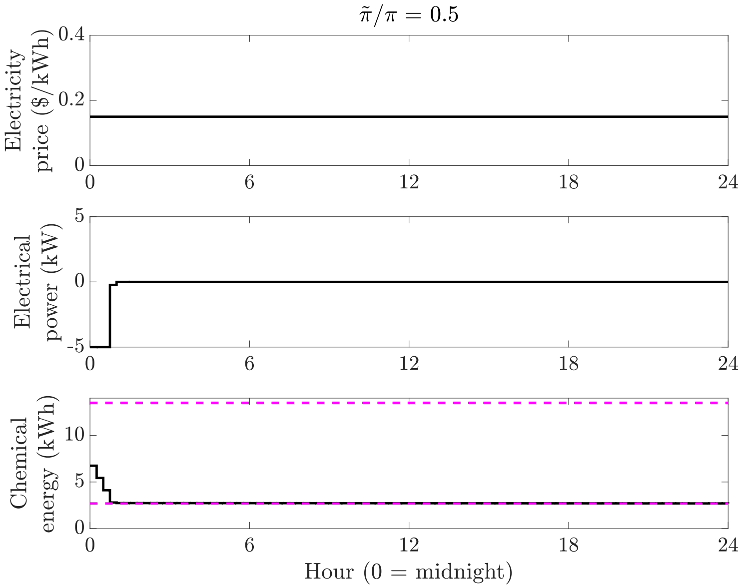
$$\Delta t(\pi(0)p(0) + \cdots + \pi(K-1)p(K-1)) - \tilde{\pi}x(K)$$

   for some tunable final price $\tilde{\pi}$ (\$/kWh)

Constant energy price, add constraint $x(K) \geq x(0)$



Constant energy price, objective $\Delta t \pi \mathbf{1}^\top p - \tilde{\pi} x(K)$



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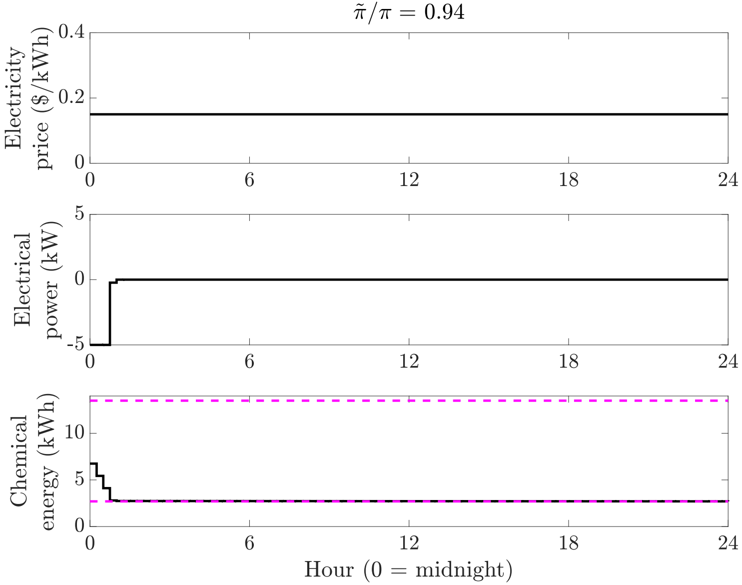

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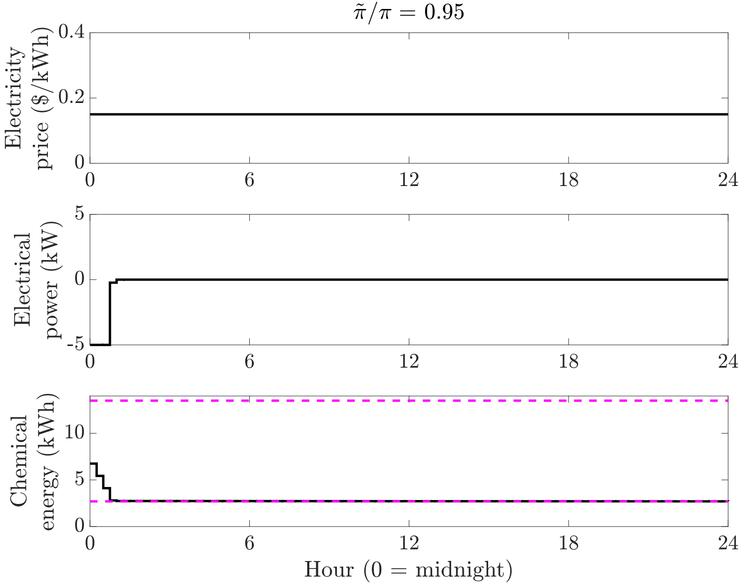
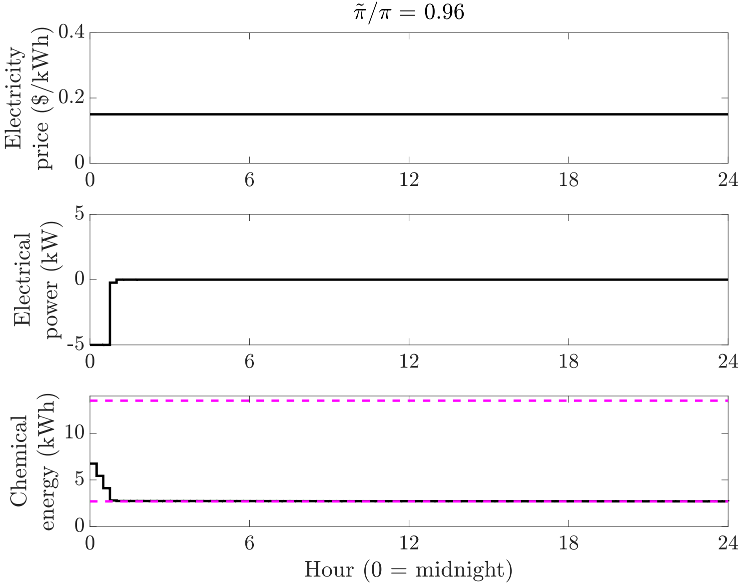
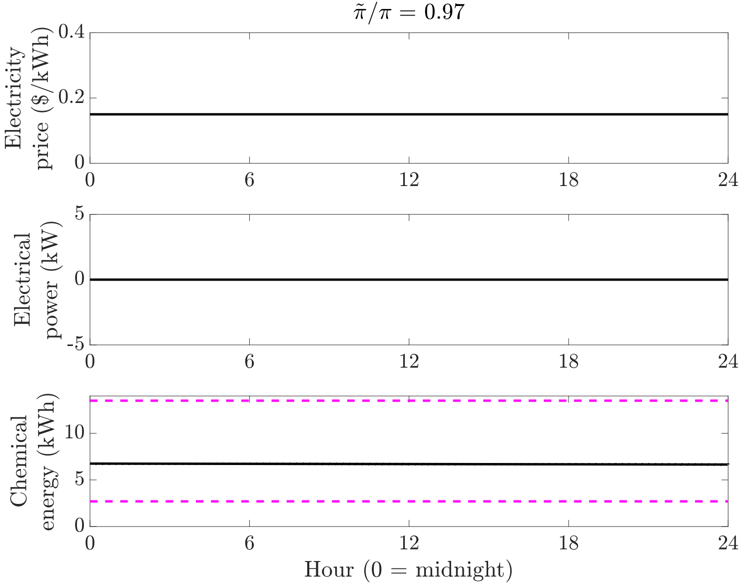


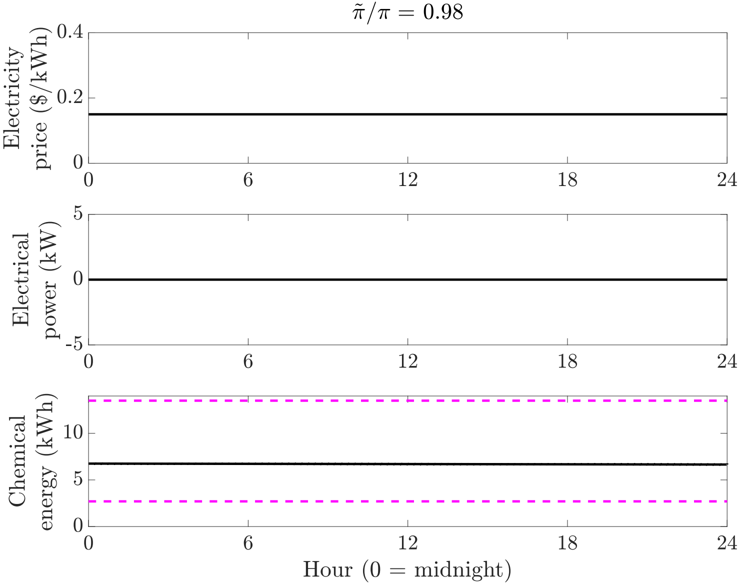
Constant energy price, objective $\Delta t \pi \mathbf{1}^\top p - \tilde{\pi} x(K)$



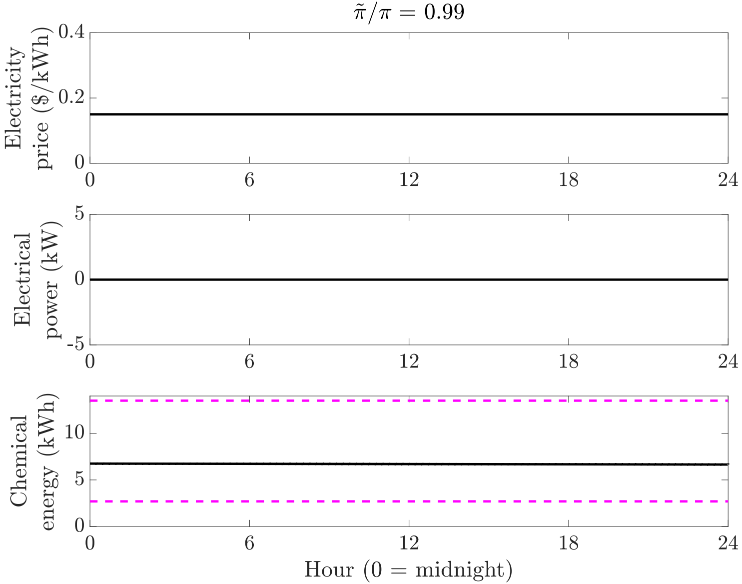
Constant energy price, objective $\Delta t \pi \mathbf{1}^\top p - \tilde{\pi} x(K)$



Constant energy price, objective $\Delta t \pi \mathbf{1}^\top p - \tilde{\pi} x(K)$



Constant energy price, objective $\Delta t \pi \mathbf{1}^\top p - \tilde{\pi} x(K)$



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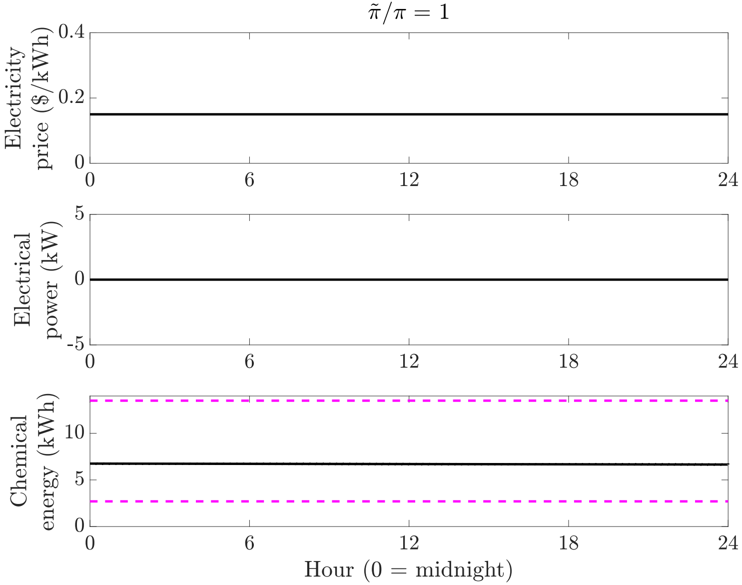
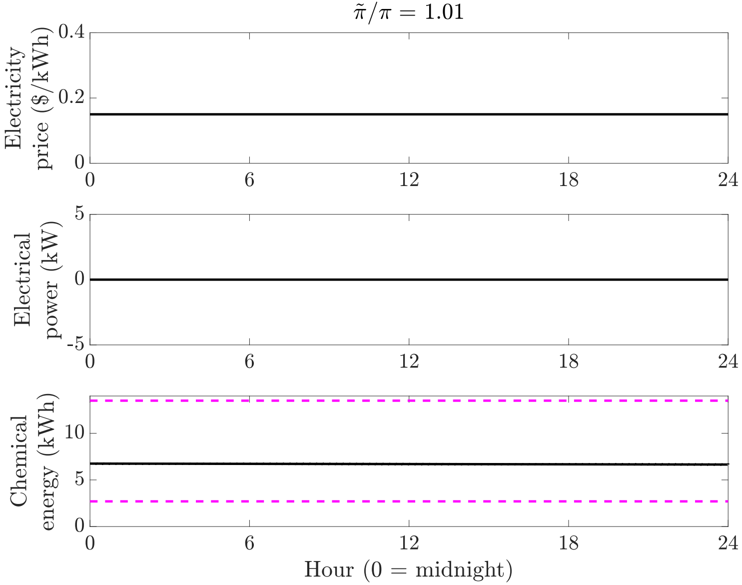


Constant energy price, objective $\Delta t \pi \mathbf{1}^\top p - \tilde{\pi} x(K)$



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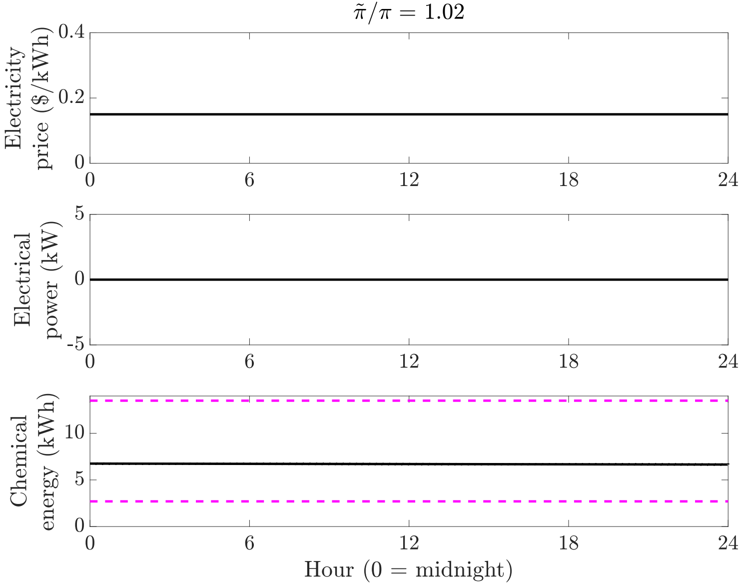
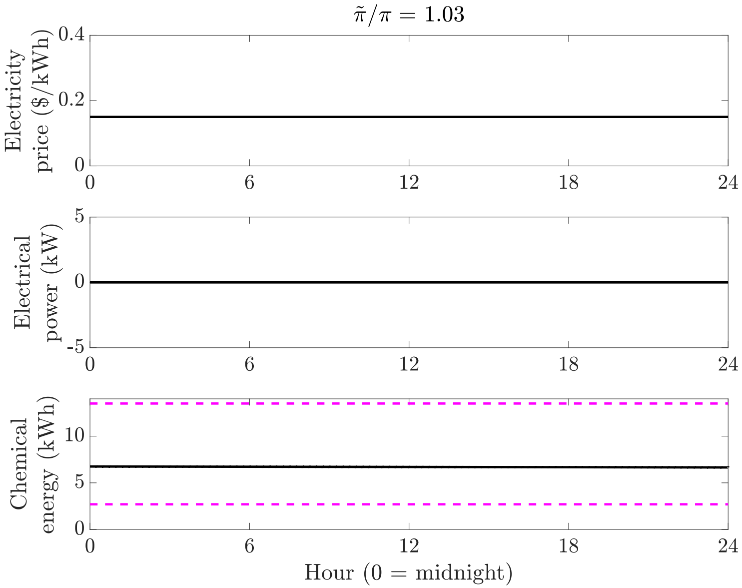
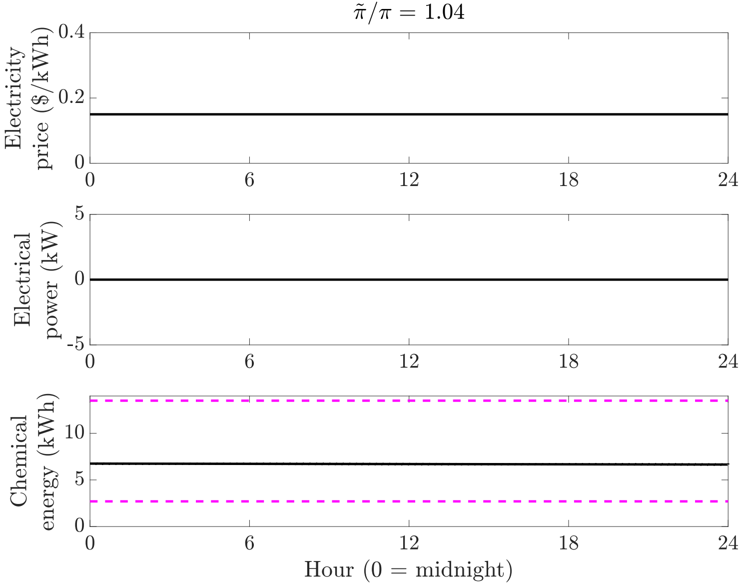


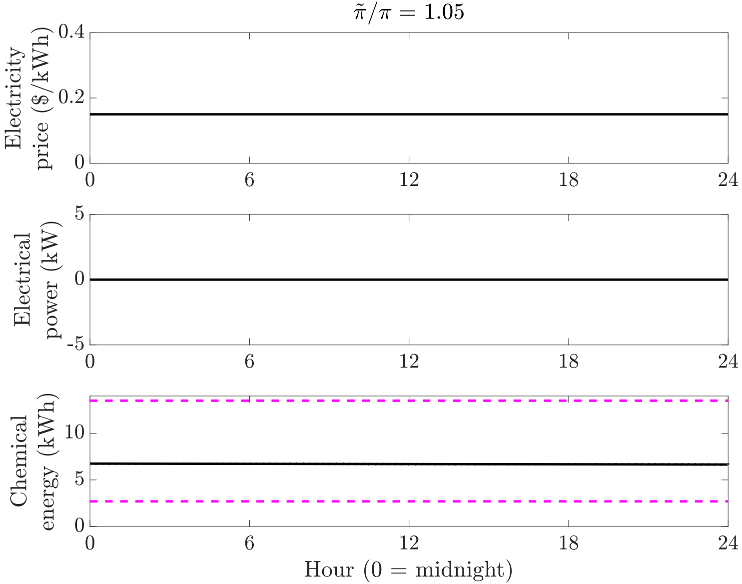
Constant energy price, objective $\Delta t \pi \mathbf{1}^\top p - \tilde{\pi} x(K)$



Constant energy price, objective $\Delta t \pi \mathbf{1}^\top p - \tilde{\pi} x(K)$



Constant energy price, objective $\Delta t \pi \mathbf{1}^\top p - \tilde{\pi} x(K)$



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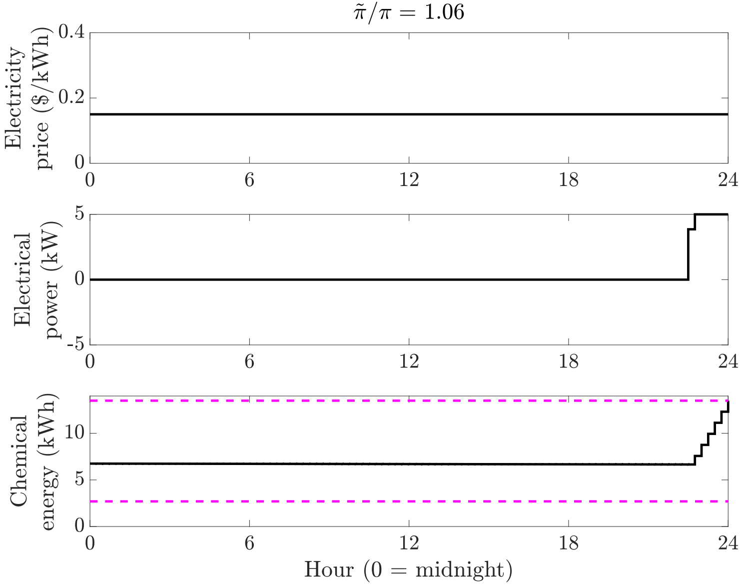

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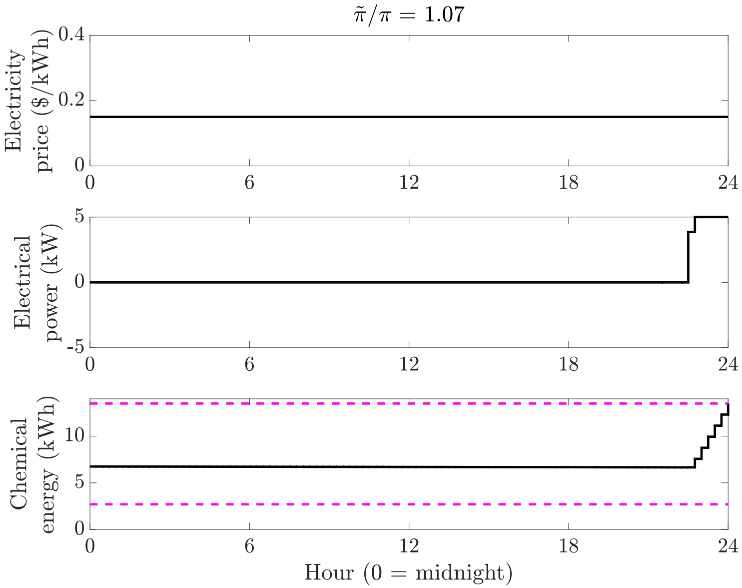

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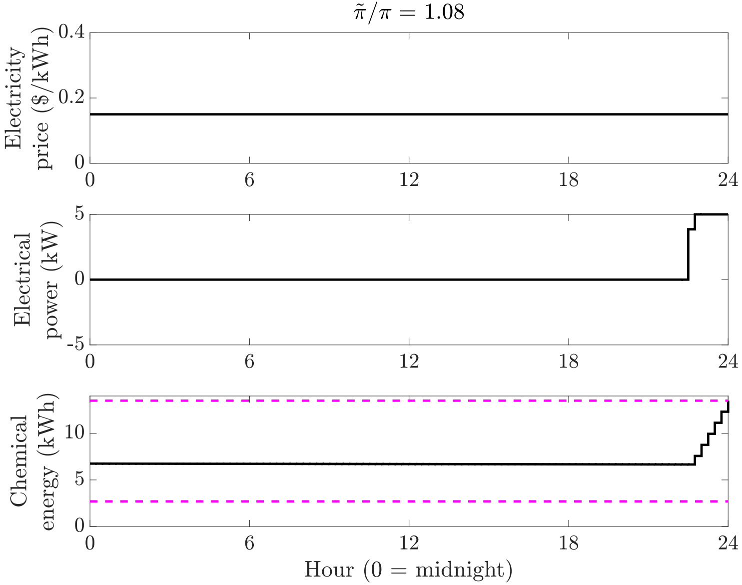
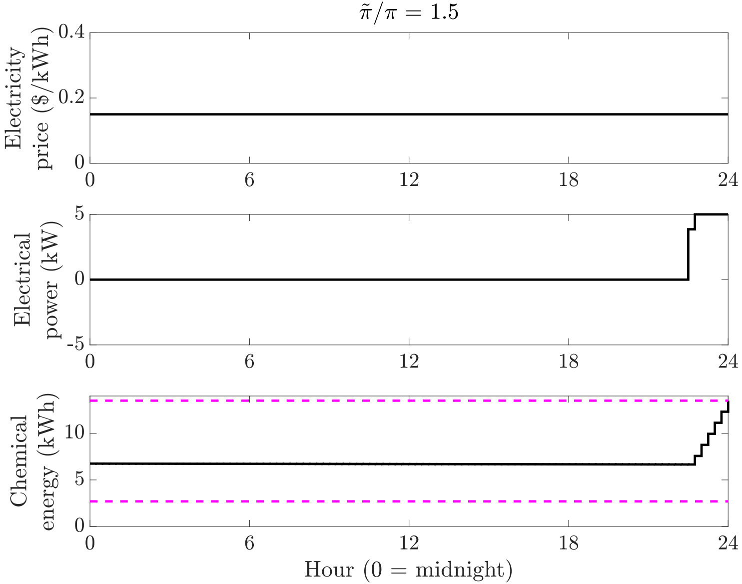
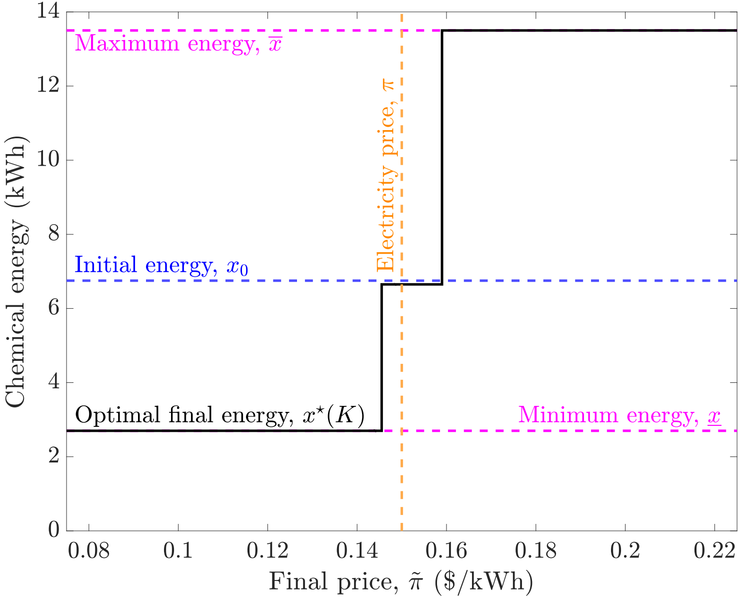


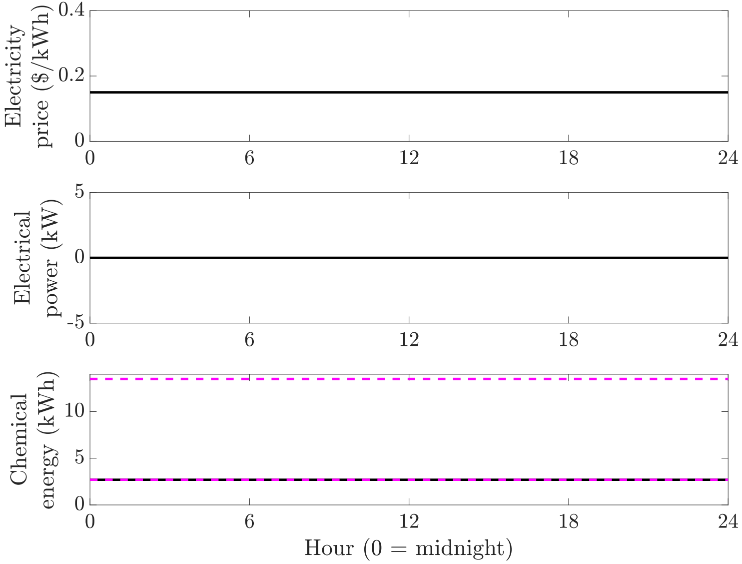
Constant energy price, objective $\Delta t \pi \mathbf{1}^\top p - \tilde{\pi} x(K)$



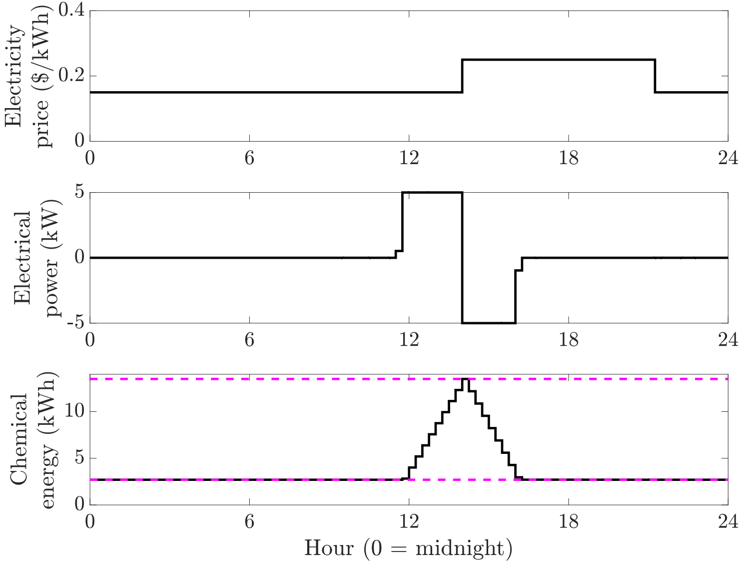
Optimal final energy depends nonsmoothly on final price



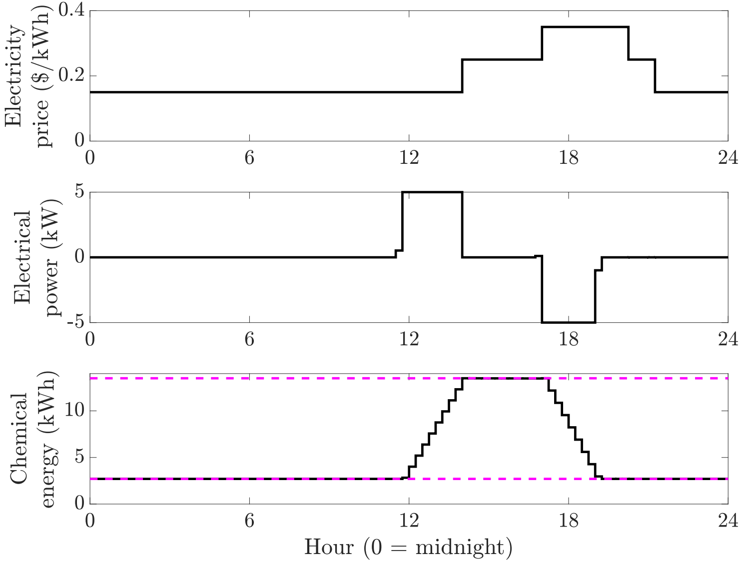
Constant energy price, $x(K) = x(0)$, no initial condition



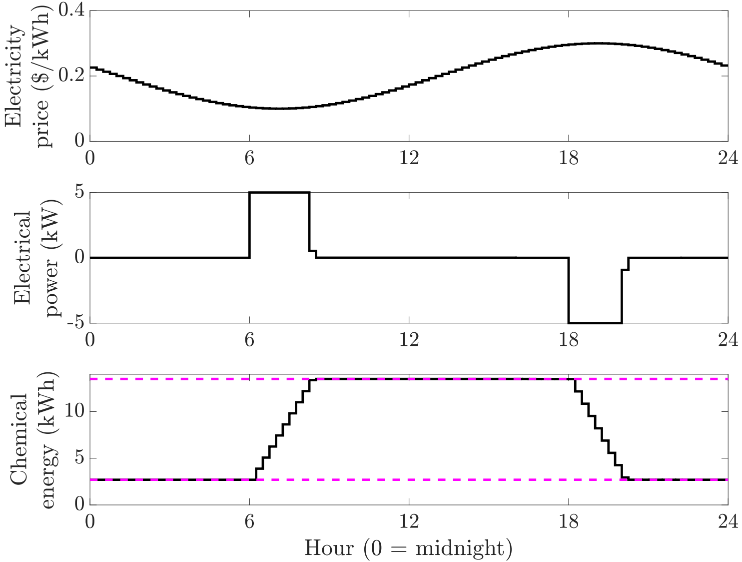
Two-tiered energy price



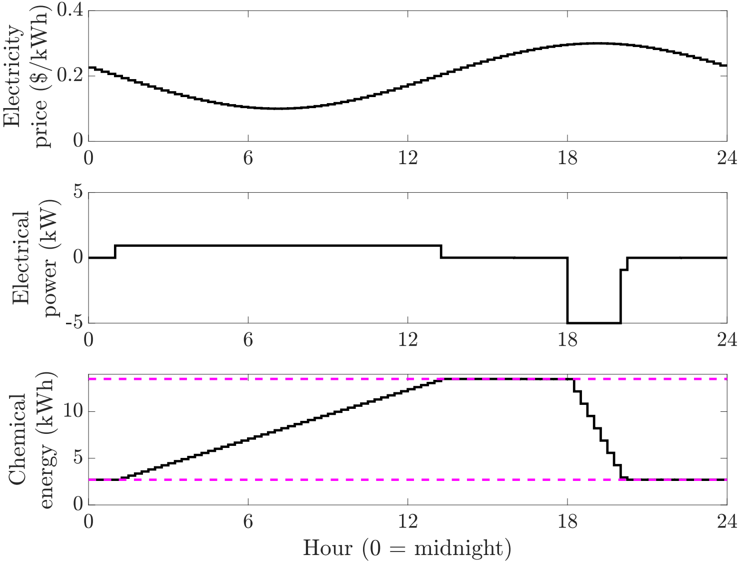
Three-tiered energy price



Real-time energy price



Real-time energy price, objective $\Delta t \pi^\top p + \pi_d \max(p)$



Outline

```
pie(1) = -pie(1);
cvx_begin
  variables x(K+1,1) pChem(K,1)
  minimize( dt*pie'*max(pChem/etac, etad*pChem) )
  subject to
    x(K+1) == x(1)
    x(2:K+1) == a*x(1:K) + (1-a)*tau*pChem
    xMin <= x(2:K+1) <= xMax
    -pdMax/etad <= pChem <= etac*pcMax
cvx_end
```

```
Disciplined convex programming error:
  Illegal affine combination of convex and/or concave
terms detected.
```

Summary: DER objectives to **minimize** over p

objective	curvature	monotonicity
energy	AFF	ND
energy cost	AFF	ND
energy cost (negative price)	AFF	NI
energy cost (reduced net metering)	CVX	ND
pollution	AFF	ND
pollution cost	AFF	ND
reference tracking error	CVX	not monotone
peak demand	CVX	ND
−(upward flexibility)	AFF	ND
−(downward flexibility)	AFF	NI
−(symmetric flexibility)	CVX	not monotone
−(load reduction)	CCV	ND
−(load reduction approximation)	AFF	ND

Options for dealing with nonmonotone objectives

approximate electrical power as affine in the basic variables

1. assume perfect charging and discharging efficiencies:

$$\eta_d = \eta_c = 1 \implies p = p^{\text{chem}}$$

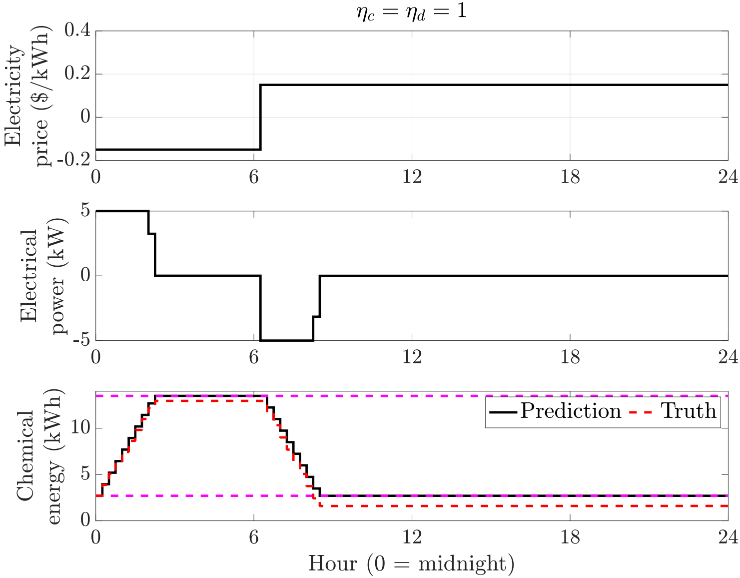
2. choose charging and discharging powers separately:

$$p = p_c - p_d, \ 0 \leq p_c \leq \overline{p}_c, \ 0 \leq p_d \leq \overline{p}_d$$
$$x(k+1) = ax(k) + (1-a)\tau(\eta_c p_c(k) - p_d(k)/\eta_d)$$

Perfect charging and discharging efficiencies

- choose
  - ◇ $x(0), \dots, x(K)$
  - ◇ $p(0), \dots, p(K-1)$
- to minimize $\Delta t \pi^\top p$
- subject to $x(K) = x(0)$ and for $k = 0, \dots, K-1$,
  - ◇ $x(k+1) = ax(k) + (1-a)\tau p(k)$
  - ◇ $\underline{x} \le x(k+1) \le \overline{x}$
  - ◇ $-\overline{p}_d \le p(k) \le \overline{p}_c$
- given Δt, π, x_0, a, τ, $\underline{x}$, $\overline{x}$, $\overline{p}_c$, $\overline{p}_d$

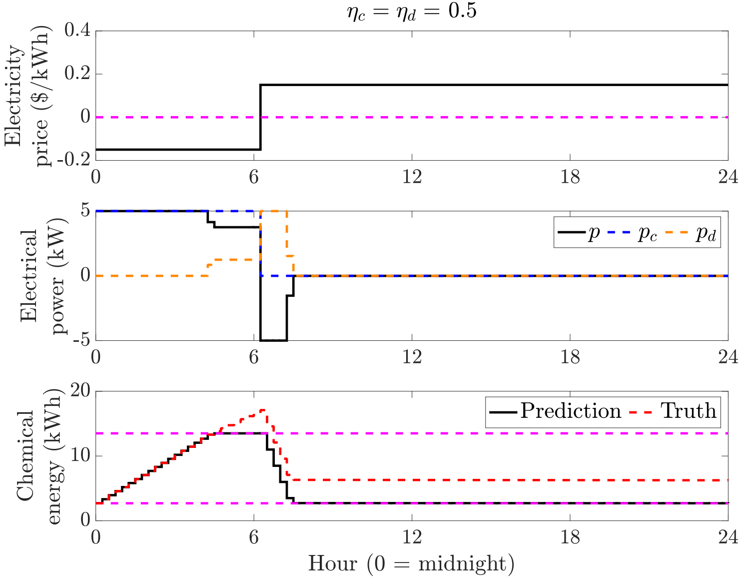
Overestimation of stored energy



Separate charging and discharging powers

- choose
  - $x(0), \ldots, x(K)$
  - charging electrical powers $p_c(0), \ldots, p_c(K-1)$
  - discharging electrical powers $p_d(0), \ldots, p_d(K-1)$
- to minimize $\Delta t \pi^\top (p_c - p_d)$
- subject to $x(K) = x(0)$ and for $k = 0, \ldots, K-1$,
  - $x(k+1) = ax(k) + (1-a)\tau(\eta_c p_c(k) - p_d(k)/\eta_d)$
  - $\underline{x} \leq x(k+1) \leq \overline{x}$
  - $0 \leq p_c(k) \leq \overline{p}_c$
  - $0 \leq p_d(k) \leq \overline{p}_d$
- given Δt, π, x_0, a, τ, η_c, η_d, $\underline{x}$, $\overline{x}$, $\overline{p}_c$, $\overline{p}_d$

(Unphysical) simultaneous charging and discharging



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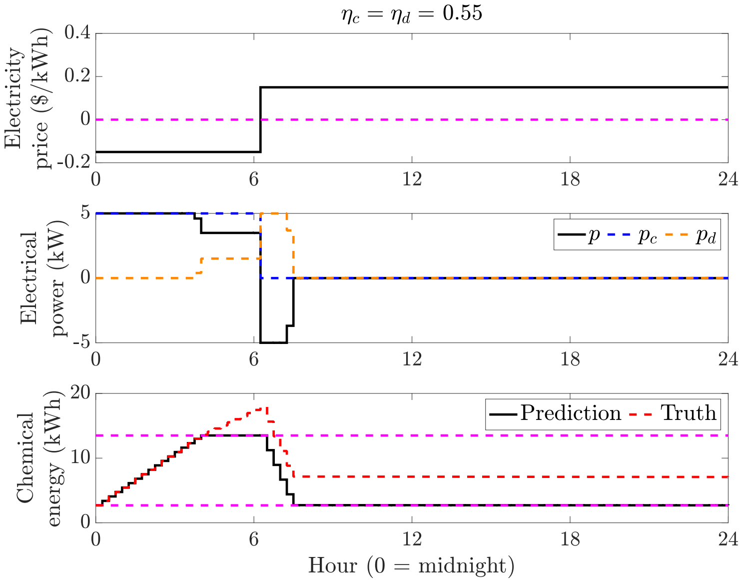

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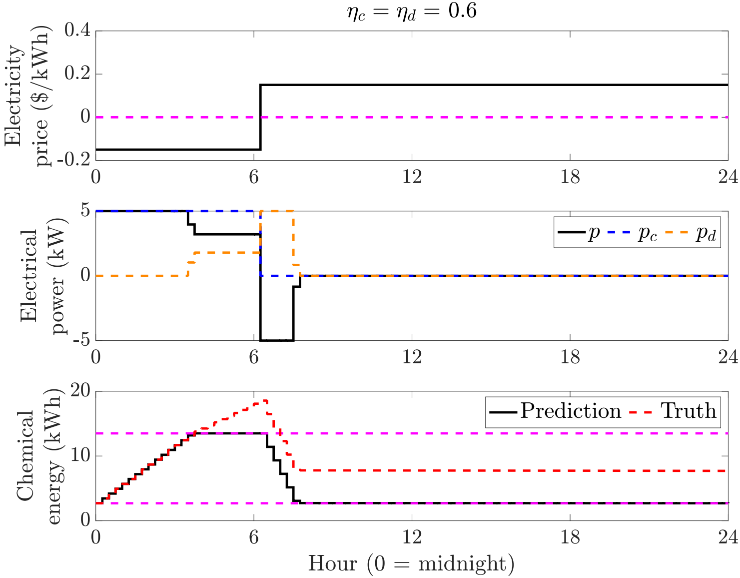

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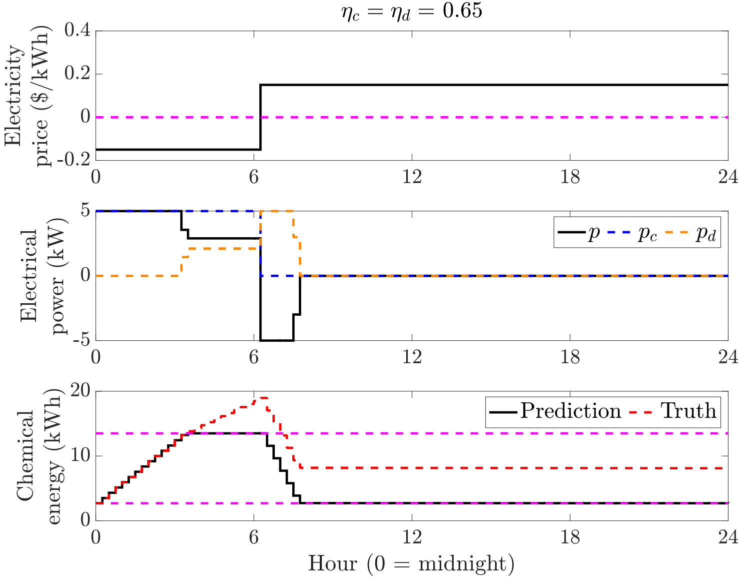

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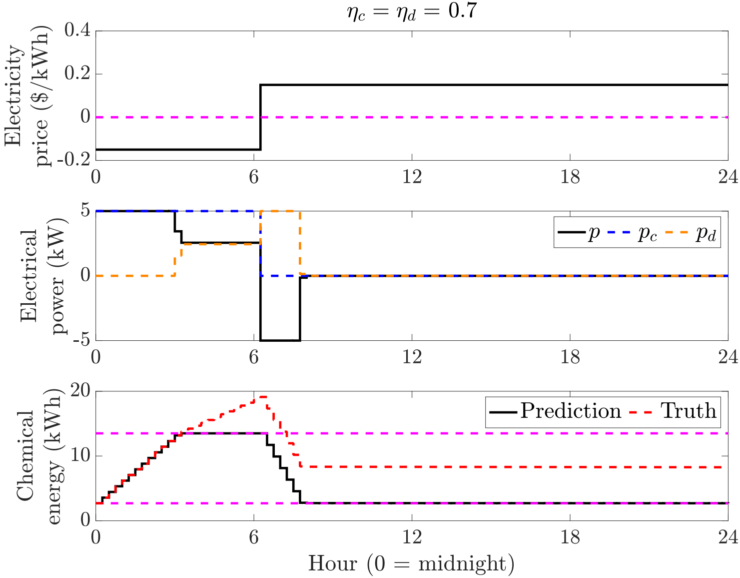

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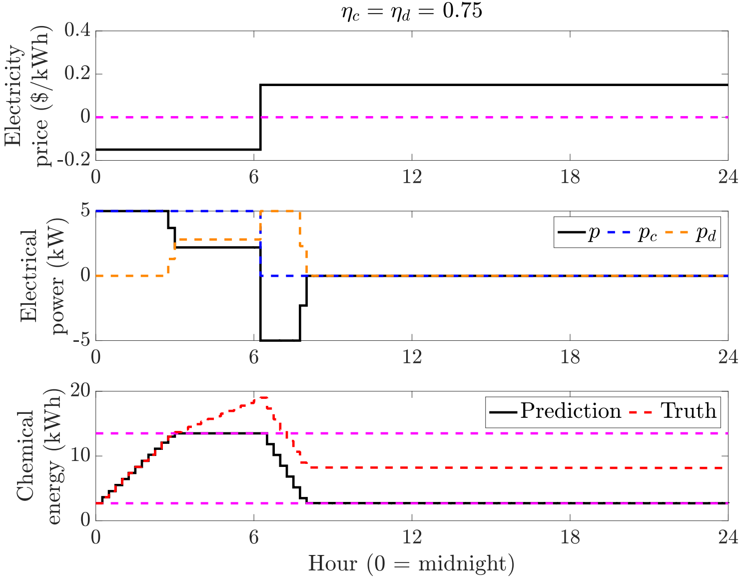

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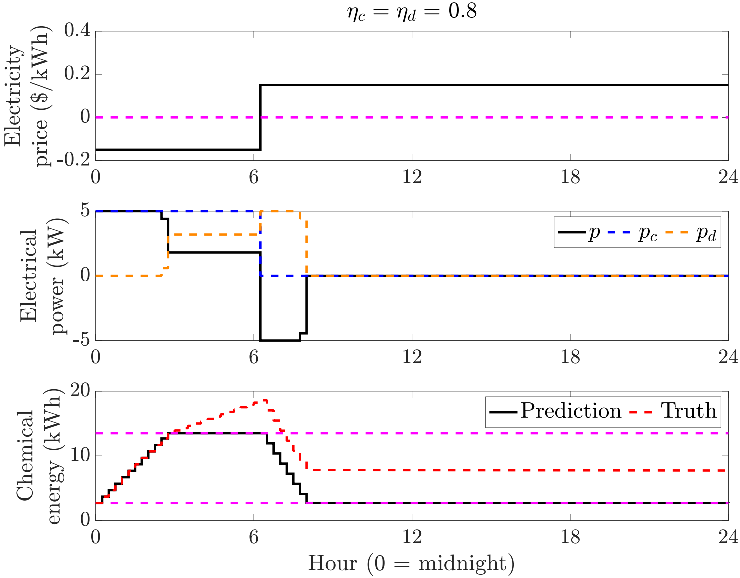

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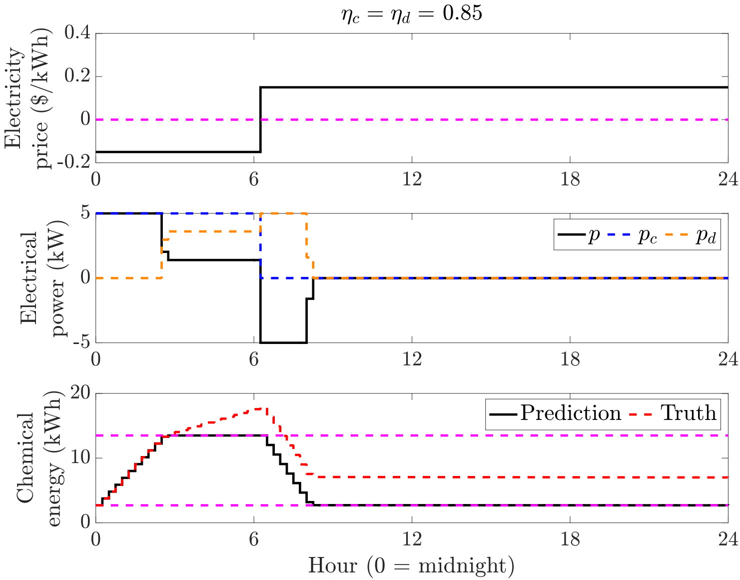
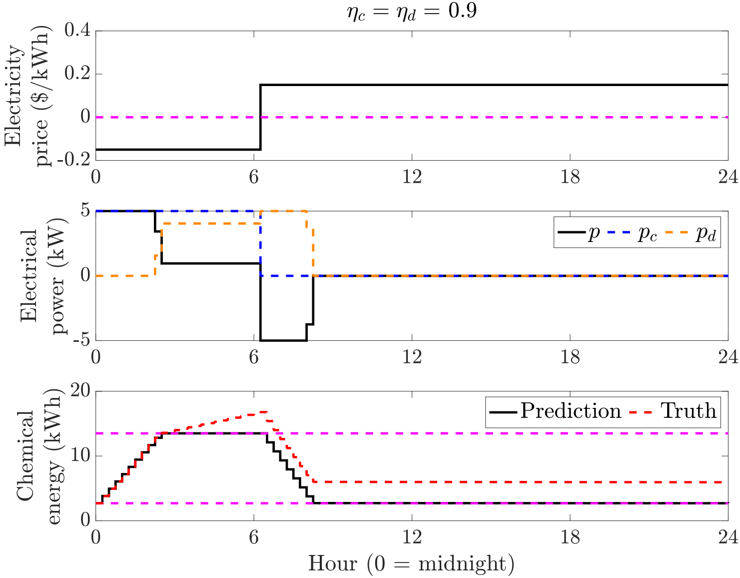


(Unphysical) simultaneous charging and discharging



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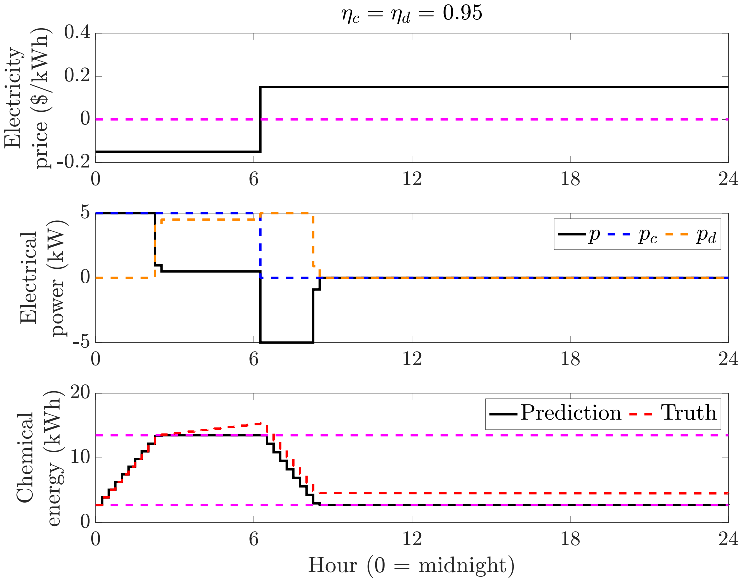
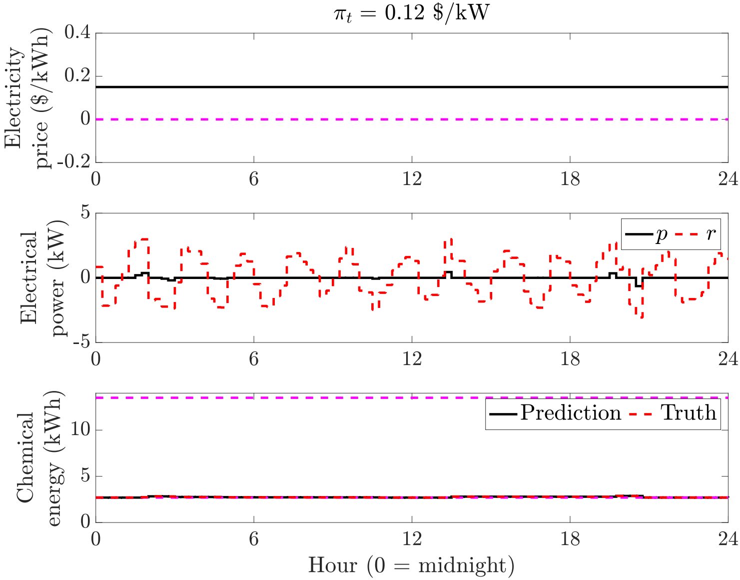


Reference tracking, objective $\Delta t \pi^\top p + \frac{\pi_t}{\sqrt{K}} \|p - r\|_2$



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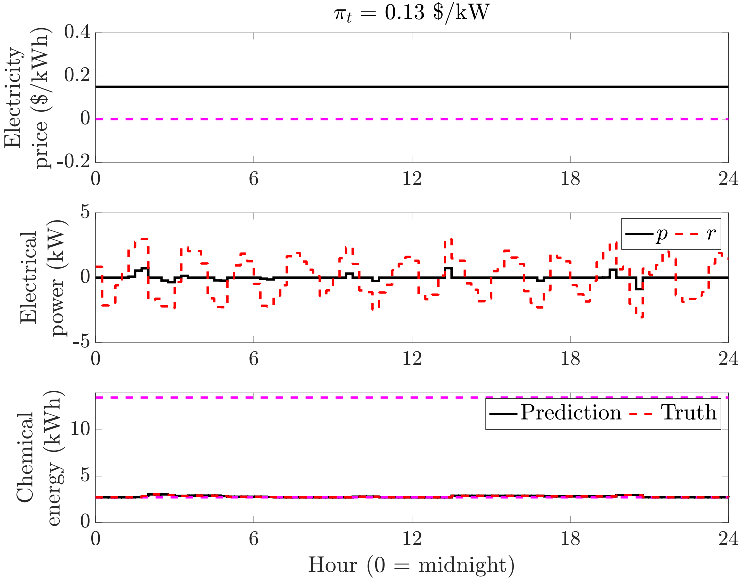

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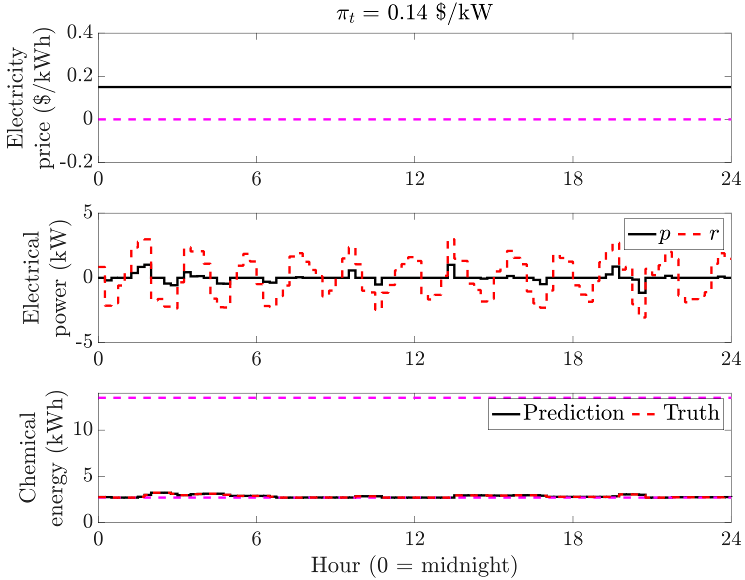

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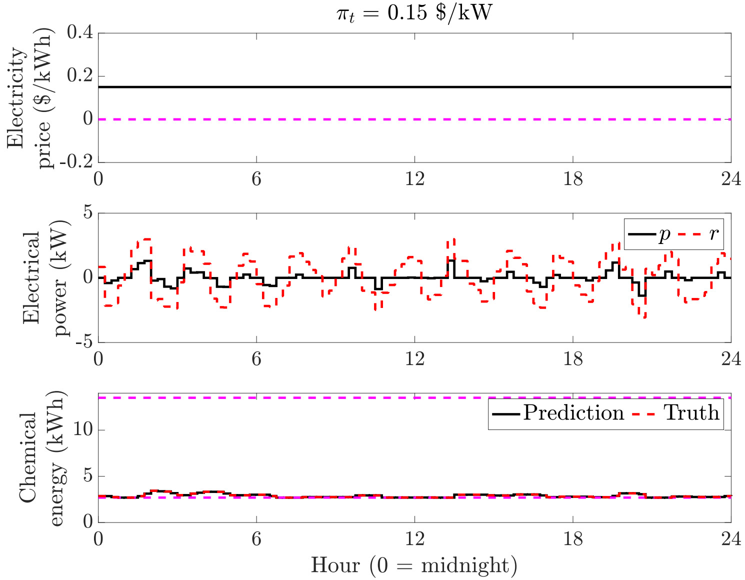

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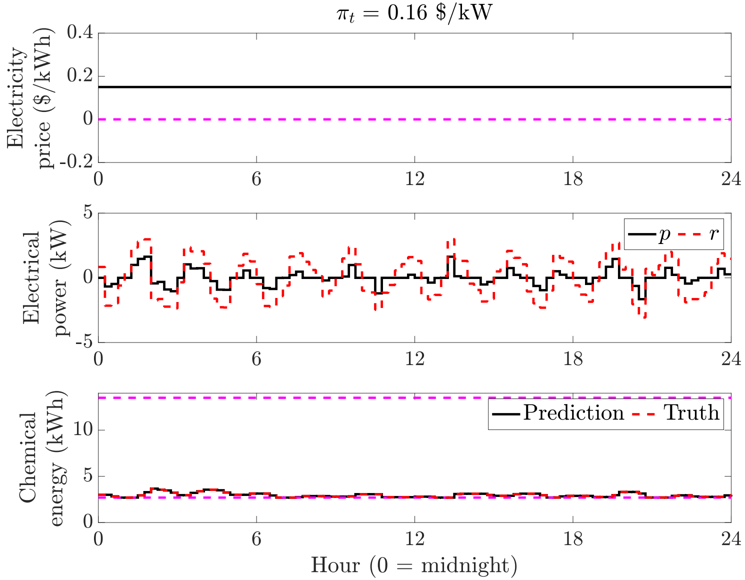

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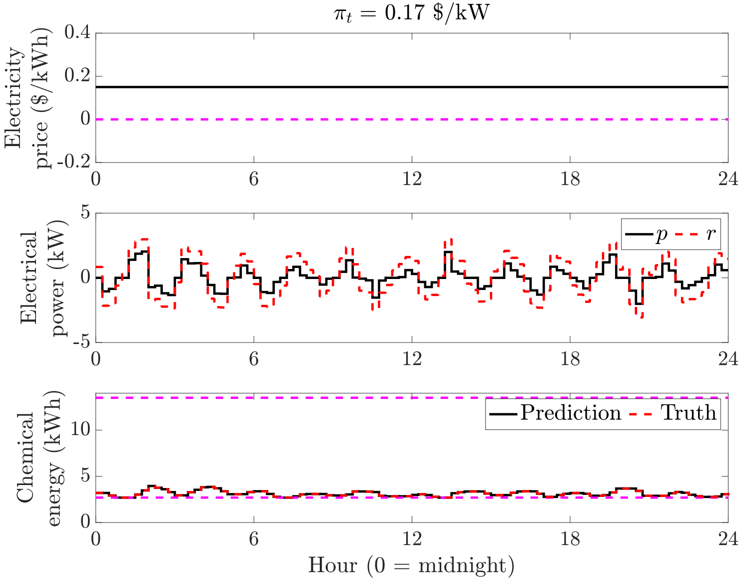
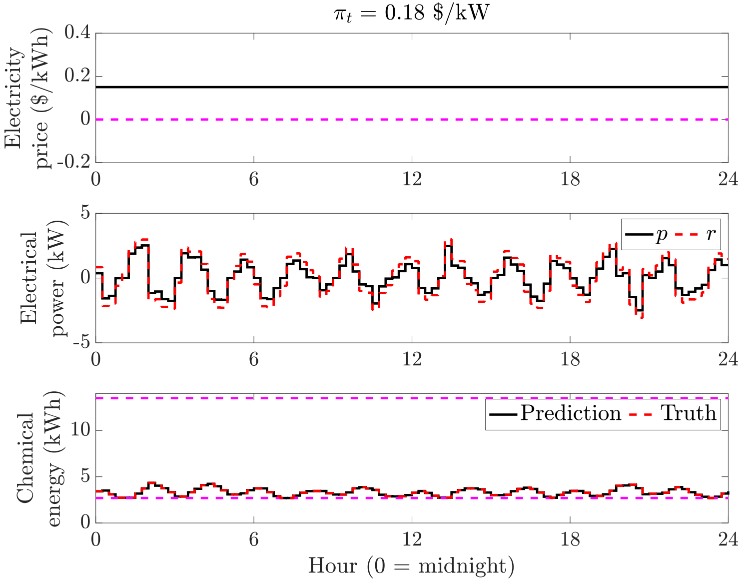
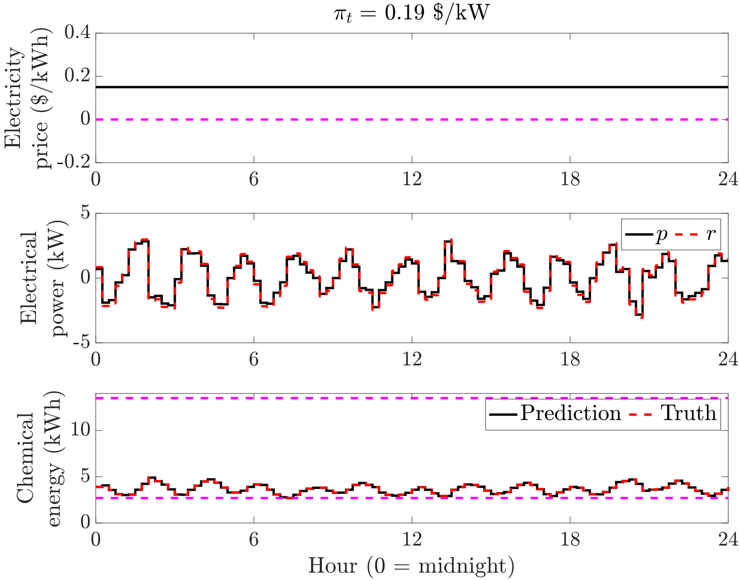


Reference tracking, objective $\Delta t \pi^\top p + \frac{\pi_t}{\sqrt{K}} \|p - r\|_2$



Reference tracking, objective $\Delta t \pi^\top p + \frac{\pi_t}{\sqrt{K}} \|p - r\|_2$



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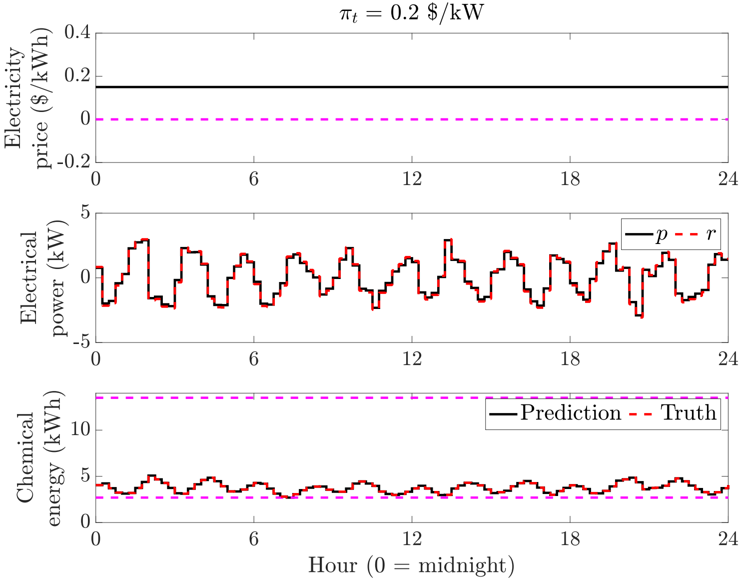

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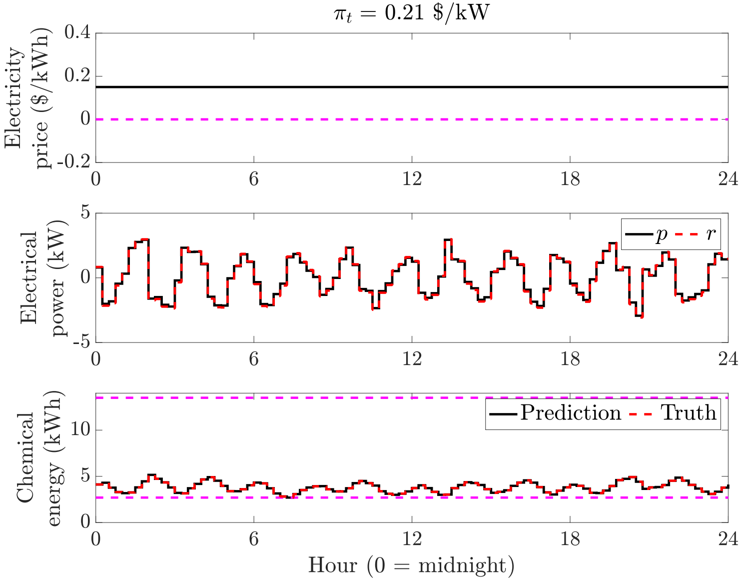

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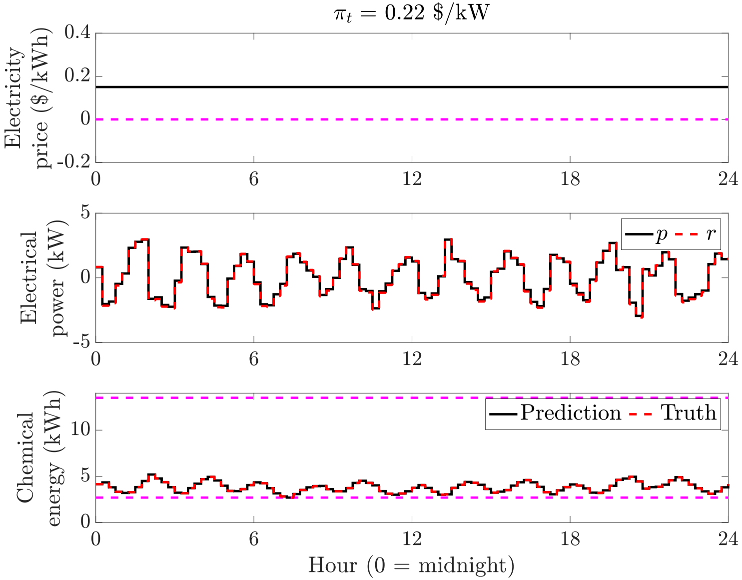
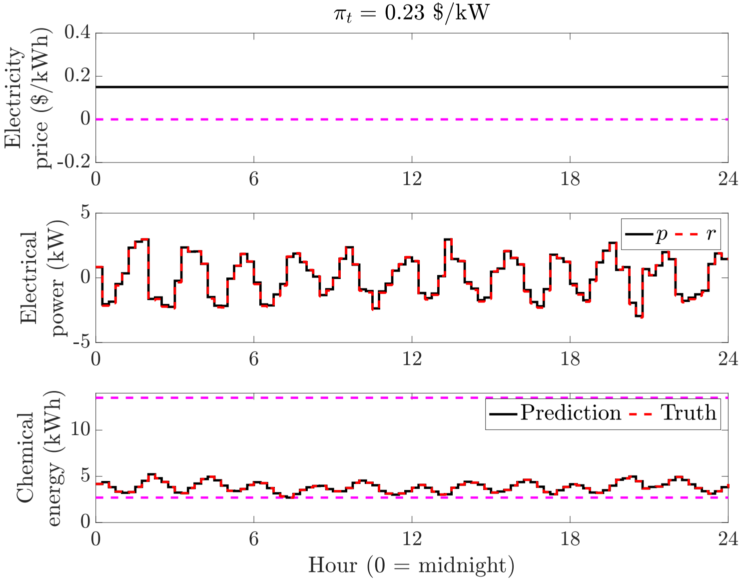


Reference tracking, objective $\Delta t \pi^\top p + \frac{\pi_t}{\sqrt{K}} \|p - r\|_2$



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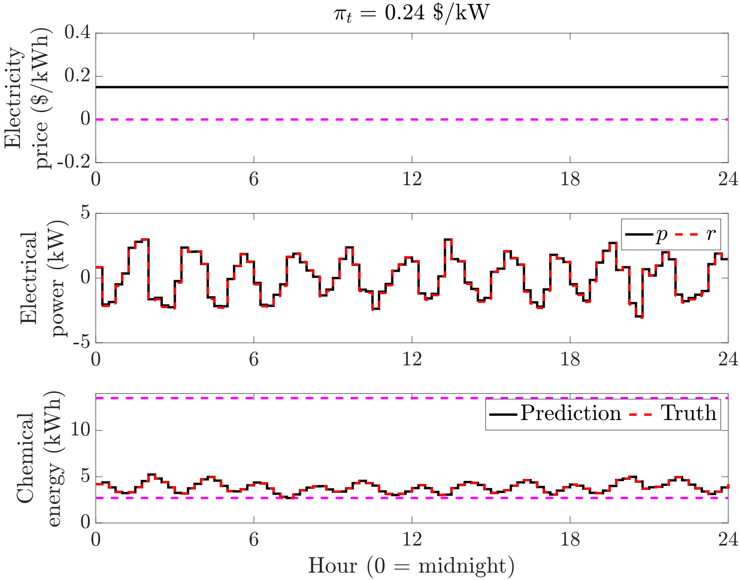
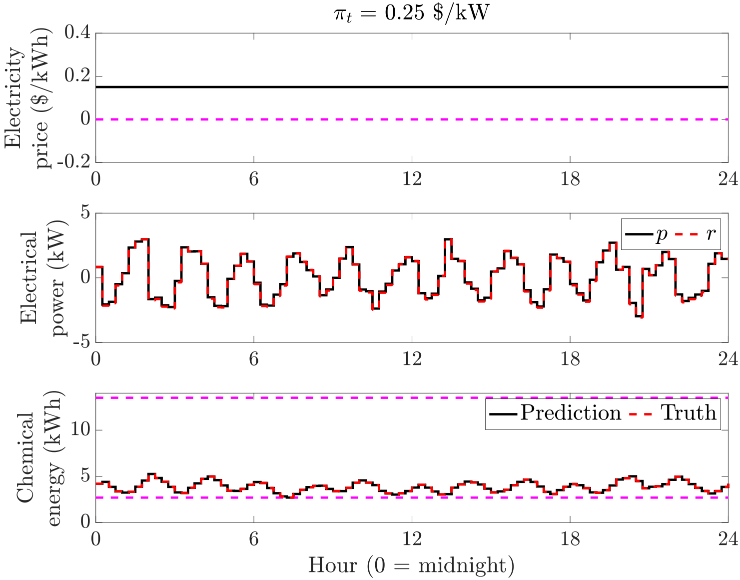


Reference tracking, objective $\Delta t \pi^\top p + \frac{\pi_t}{\sqrt{K}} \|p - r\|_2$



Outline

Weighing money now vs. money later

- most people would
  - prefer \$1,000 now to \$1,000 next year
  - trade \$1,000 now for $\$1{,}000(1+\rho)$ next year for some $\rho > 0$
- the **present value** of money m that will arrive next year is

$$\frac{m}{1+\rho}$$

- the present value of money m_n that will arrive n years out is

$$\frac{m_n}{(1+\rho)^n}$$

Net present cost

if a project

- costs c_0 to deploy today
- will last N years
- will (net-)cost $c_1, \ldots, c_N$ to operate in years $1, \ldots, N$
- will have salvage value s in year N

then its **net present cost** is

$$c_0 + \frac{c_1}{1+\rho} + \cdots + \frac{c_N - s}{(1+\rho)^N}$$

Net present cost with constant annual costs

- if $c_1 = \cdots = c_N$, then the net present cost is

$$c_0 + c_1 \left(\frac{1}{1+\rho} + \cdots + \frac{1}{(1+\rho)^N} \right) - \frac{s}{(1+\rho)^N}$$
$$= c_0 + \lambda c_1 - \frac{s}{(1+\rho)^N}$$

- can show (from partial sum of geometric series) that

$$\lambda = \frac{1}{1+\rho} + \cdots + \frac{1}{(1+\rho)^N} = \frac{1-(1+\rho)^{-N}}{\rho}$$

Net present cost of a battery

- can model total installed cost as $c_0 = \begin{cases} 0 & \bar{x} = 0 \\ \alpha\bar{x} + \beta & \bar{x} > 0 \end{cases}$
  - ◇ $\bar{x}$ (kWh) is the energy capacity
  - ◇ α (\$/kWh) is the energy capacity price
  - ◇ β (\$) is a fixed cost associated with installation labor, etc.

  (for one popular home battery, $\alpha \approx 520$ \$/kWh, $\beta \approx \$4500$)
- can model salvage value as $s = \gamma c_0$ for some $\gamma \in [0, 1]$
- with these models, net present cost (for $\bar{x} > 0$) becomes

$$\alpha\bar{x} + \beta + \lambda c_1 - \frac{\gamma(\alpha\bar{x} + \beta)}{(1+\rho)^N} = \pi_x\bar{x} + \lambda c_1 + \beta\left[1 - \frac{\gamma}{(1+\rho)^N}\right]$$

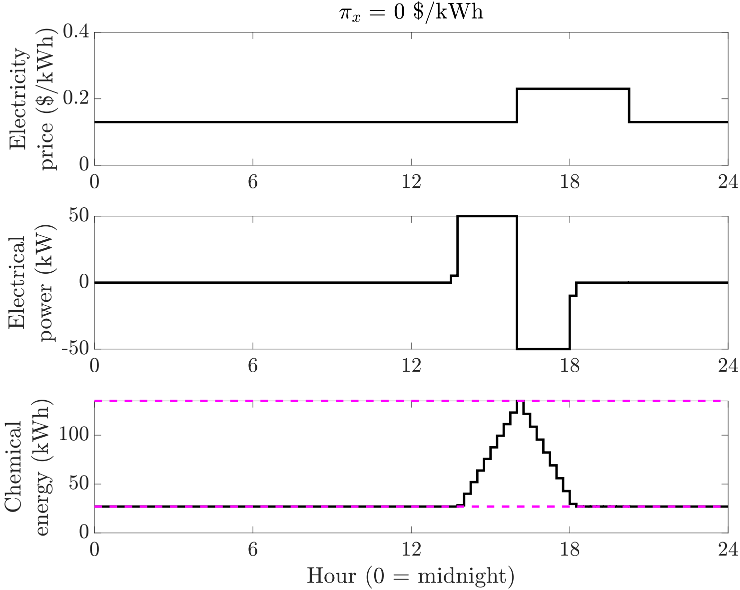
where $\pi_x = \alpha[1 - \gamma/(1+\rho)^N]$

$\Longrightarrow$ to minimize net present cost, minimize $\pi_x\bar{x} + \lambda c_1$

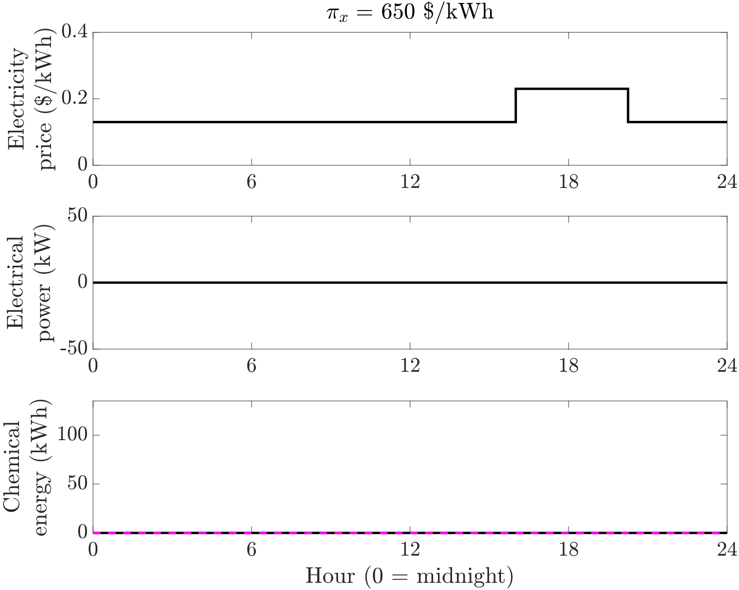
Joint sizing and operation example

- model energy and power capacities as $\underline{x} = \mu \overline{x}$, $\overline{p}_d = \overline{p}_c = \overline{x}/\nu$ (for example, $\mu \approx 0.2$, $\nu \approx 2.7$ h)
- define biggest feasible energy capacity E (kWh)
- choose
  - $x(0), \ldots, x(K)$
  - $p^{\text{chem}}(0), \ldots, p^{\text{chem}}(K-1)$
  - $\overline{x}$
- to minimize $\pi_x \overline{x} + 365 \lambda \Delta t \pi^\top \max\left\{p^{\text{chem}}/\eta_c, \eta_d p^{\text{chem}}\right\}$
- subject to $x(K) = x(0)$ and for $k = 0, \ldots, K-1$,
  - $x(k+1) = a x(k) + (1-a)\tau p^{\text{chem}}(k)$
  - $\mu \overline{x} \leq x(k+1) \leq \overline{x}$
  - $-\overline{x}/(\nu \eta_d) \leq p^{\text{chem}}(k) \leq \eta_c \overline{x}/\nu$
  - $0 \leq \overline{x} \leq E$
- given π_x, λ, Δt, π, η_c, η_d, x_0, a, τ, μ, ν, E

Operating profile with low energy capacity price π_x



Operating profile with high energy capacity price π_x



Optimal size vs. energy capacity price π_x