Objectives in DER optimization

Purdue ME 597, Distributed Energy Resources

Kevin J. Kircher

Outline

Electricity economics

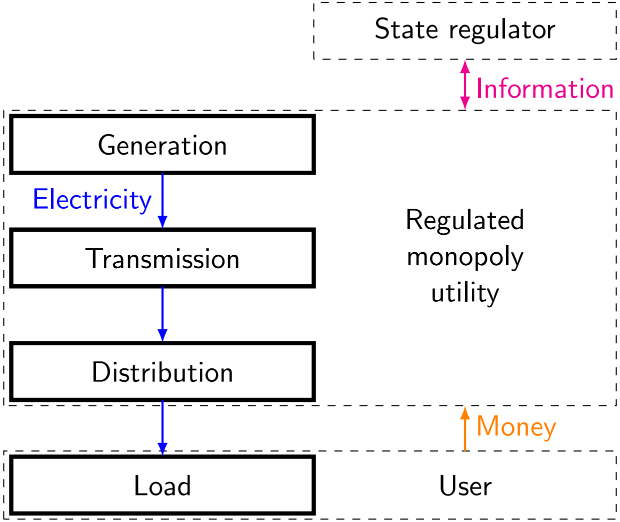
Optimization reminders

Linear objectives

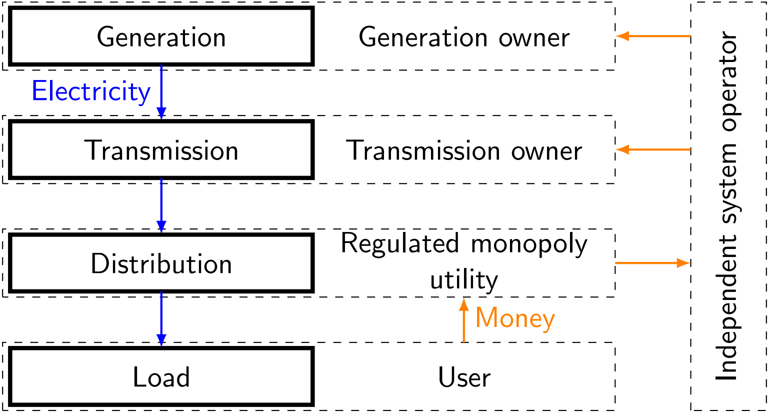
Nonlinear objectives

Reference tracking example

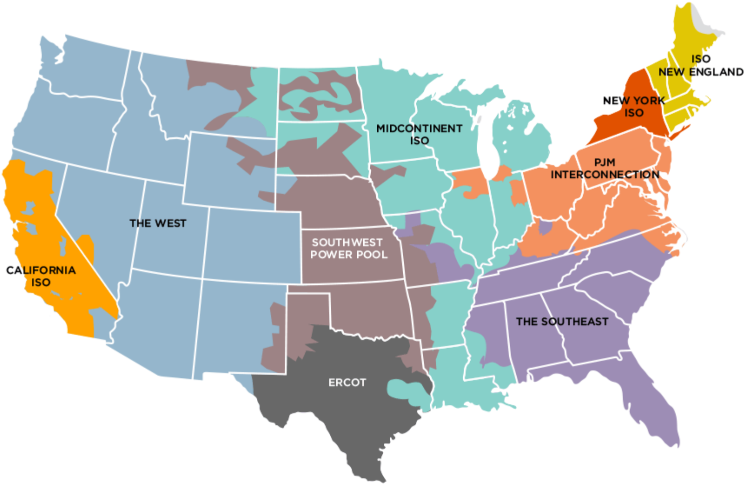
Vertically-integrated utilities



Competitive wholesale electricity markets

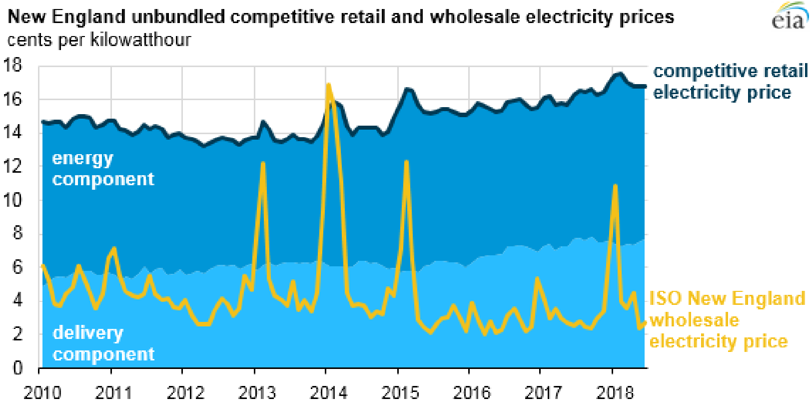


US Independent system operators (ISOs)



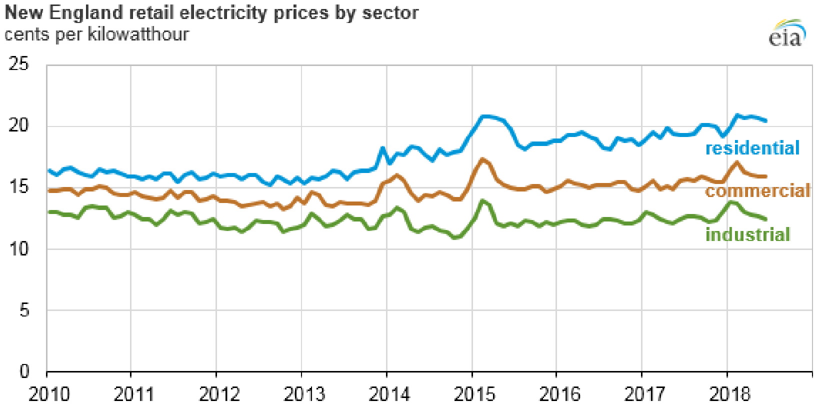
Sustainable FERC project: ISO RTO operating regions

Wholesale and retail electricity prices



US Energy Information Administration: New England's competitive electricity markets

Retail customer classes



US Energy Information Administration: New England's competitive electricity markets

Outline

Affine functions in optimization

- $f : \mathbf{R}^n \to \mathbf{R}$ is
  - **linear** if $f(x) = a^\top x$ for some $a \in \mathbf{R}^n$
  - **affine** if $f(x) = a^\top x + b$ for some $a \in \mathbf{R}^n$, $b \in \mathbf{R}$
- all linear functions are affine (with $b = 0$), but not vice versa
- affine functions are both convex and concave
- $f(x) = a^\top x + b$ is nondecreasing in x_i if $a_i \geq 0$
  (and nonincreasing in x_i if $a_i \leq 0$)
- minimizing $a^\top x + b$ is equivalent to minimizing $a^\top x$
  (additive constants in objectives don't influence solutions)

Composition rules

curvature:

- CVX(AFF) $=$ CVX
- CVXND(CVX) $=$ CVX
- CVXNI(CCV) $=$ CVX
- CCV(AFF) $=$ CCV
- CCVND(CCV) $=$ CCV
- CCVNI(CVX) $=$ CCV

monotonicity:

- ND(ND) $=$ ND
- NI(NI) $=$ ND
- NI(ND) $=$ NI
- ND(NI) $=$ NI

CVX(CVX) $\neq$ CVX in general

- consider $g, h : \mathbf{R} \to \mathbf{R}$ with $g(y) = -y$, $h(x) = x^2$
- h is convex
- g is affine (hence convex) but *nonincreasing*
- define $f : \mathbf{R} \to \mathbf{R}$ by $f(x) = g(h(x)) = -x^2$
- f is not convex
  (need g nondecreasing)

DER modeling

- most DERs can be modeled via

$$
\begin{aligned}
x(k+1) &= ax(k) + b(u(k) + w(k)) \\
\underline{x}(k) &\leq x(k) \leq \overline{x}(k) \\
\underline{u}(k) &\leq u(k) \leq \overline{u}(k) \\
p(k) &= g(k, x(k), u(k))
\end{aligned}
$$

- state $x(k)$ (kWh) is stored energy
- action $u(k)$ (kW) is controlled charging power
- disturbance $w(k)$ (kW) is uncontrolled charging power
- a (-) and b (h) are discrete-time dynamics parameters
- $\underline{x}(k)$ and $\overline{x}(k)$ (kWh) are energy capacity limits
- $\underline{u}(k)$ and $\overline{u}(k)$ (kW) are power capacity limits
- g is affine in $x(k)$, $u(k)$ for some DERs, convex for others

A generic DER optimization problem

- choose
  - $x(0), \ldots, x(K)$
  - $u(0), \ldots, u(K-1)$
  - $p(0), \ldots, p(K-1)$
- to minimize a function of $p(0), \ldots, p(K-1)$
- subject to $x(0) = x_0$ and for $k = 0, \ldots, K-1$,
  - $x(k+1) = ax(k) + b(u(k) + w(k))$
  - $\underline{x}(k+1) \leq x(k+1) \leq \overline{x}(k+1)$
  - $\underline{u}(k) \leq u(k) \leq \overline{u}(k)$
  - $p(k) = g(k, x(k), u(k))$
- given x_0, a, b, and the $w(k)$, $\underline{x}(k)$, $\overline{x}(k)$, $\underline{u}(k)$, $\overline{u}(k)$, $g(k, \ , \ )$

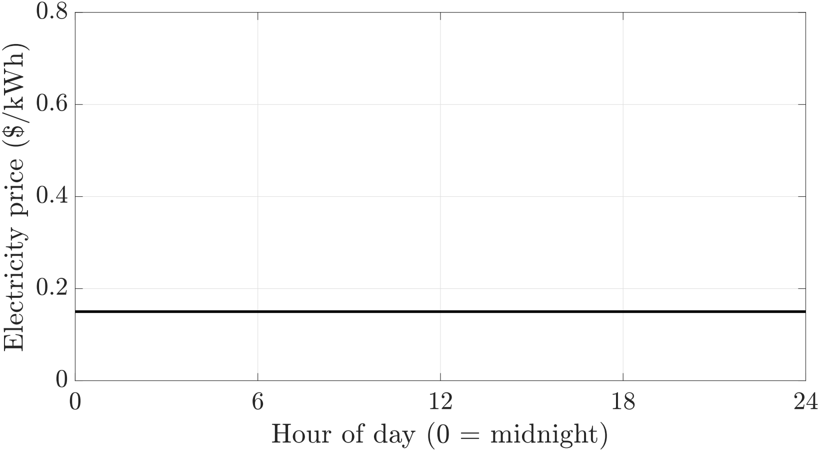
- for affine g, any objective that's convex in p is convex in x, u
- for convex g, we want convex *nondecreasing* objectives in p

Outline

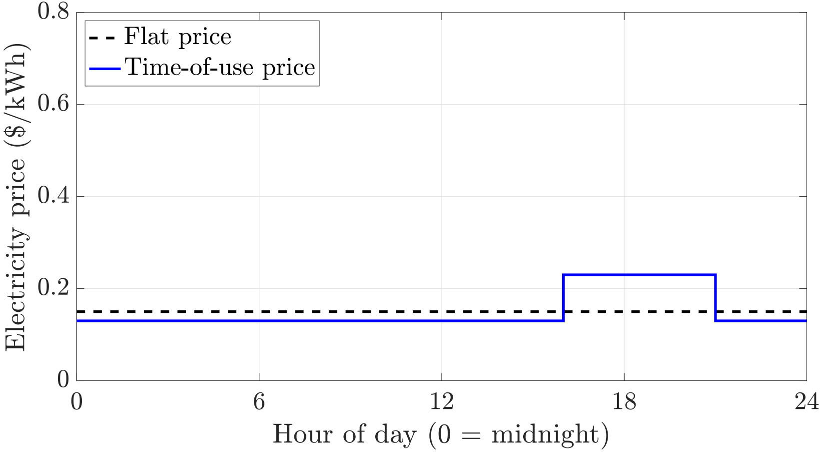
Energy and energy cost

- electrical energy: $\Delta t p(k)$
- electrical energy cost: $\Delta t \pi_e(k) p(k)$
- $\pi_e(k)$ (\$/kWh) is electrical energy price
- energy and energy cost are linear in $p(k)$
- energy is nondecreasing in $p(k)$
- energy cost is nondecreasing in $p(k)$ if $\pi_e(k) \geq 0$
  (but can be nonincreasing if price goes negative)

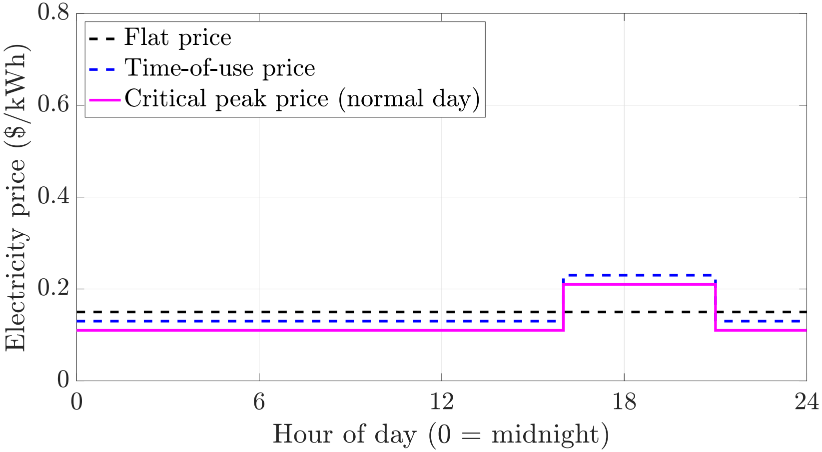
Flat energy price



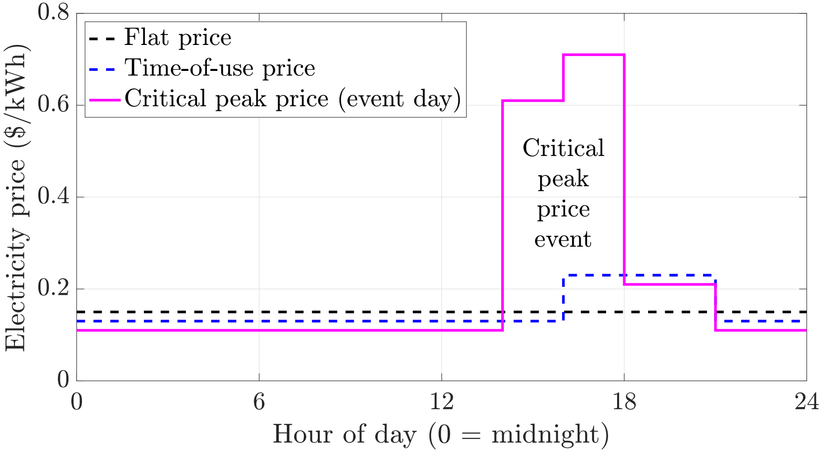
Time-of-use energy price



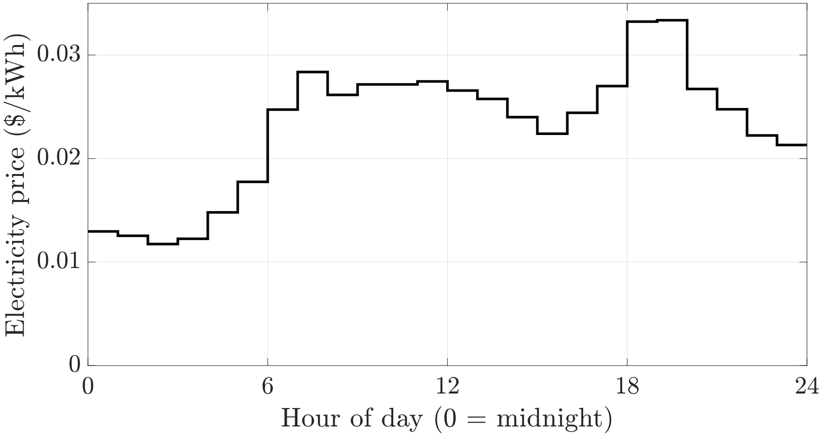
Critical peak price (normal day)



Critical peak price (event day)



Today's MISO wholesale energy price (note y-axis limits)



MISO market reports: Day-ahead pricing

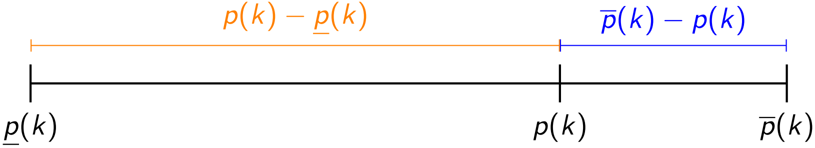
Pollution and pollution cost

- pollution (CO_2, CH_4, NO_x, SO_x, ...): $\Delta t \mu(k) p(k)$
- $\mu(k)$ (kg/kWh) is pollutant intensity of electricity
- pollution cost: $\Delta t \pi_g(k) \mu(k) p(k)$
- $\pi_g(k)$ (\$/kg) is pollutant price
- pollution and pollution cost are linear and nondecreasing in p (both $\mu(k)$ and $\pi_g(k)$ are ∼always nonnegative)

Demand flexibility

- flexibility is the capacity to adjust electrical power on call
- grid operators may pay DERs to **plan** and/or **use** flexibility
    - ◇ day-ahead, plan a baseline power trajectory
    - ◇ in real time, adjust power away from baseline if needed

Upward and downward flexibility



- power typically has limits: $p(k) \in [\underline{p}(k), \overline{p}(k)]$
- upward flexibility: $\overline{p}(k) - p(k)$
- downward flexibility: $p(k) - \underline{p}(k)$
- upward and downward flexibility are both affine in $p(k)$
- upward flexibility is nonincreasing in $p(k)$
- downward flexibility is nondecreasing in $p(k)$
- multiplying by nonnegative prices preserves monotonicity

Sums of linear objectives

- a sum of linear objectives is linear with a new price
- example: sum of energy and pollution costs,

$$\begin{aligned} &\Delta t \pi_e(k) p(k) + \Delta t \pi_g(k) \mu(k) p(k) \\ &= \Delta t \pi(k) p(k) \end{aligned}$$

with new price $\pi(k) = \pi_e(k) + \pi_g(k)\mu(k)$

Aggregate objectives

- suppose DERs $i = 1, \ldots, n$ each use power $p_i(k)$
- define the aggregate power

$$p(k) = p_1(k) + \cdots + p_n(k) + v(k),$$

  where $v(k)$ is power to/from uncontrolled devices
- aggregate power is affine and nondecreasing in the $p_i(k)$

$\Longrightarrow$ curvature, monotonicity attributes in $p(k)$ extend to the $p_i(k)$

Cumulative objectives

- discounted cost: $\gamma^k \Delta t \pi(k) p(k)$ for discount rate $\gamma \in [0,1]$ (undiscounted if $\gamma = 1$)
- cumulative discounted cost is linear in p:

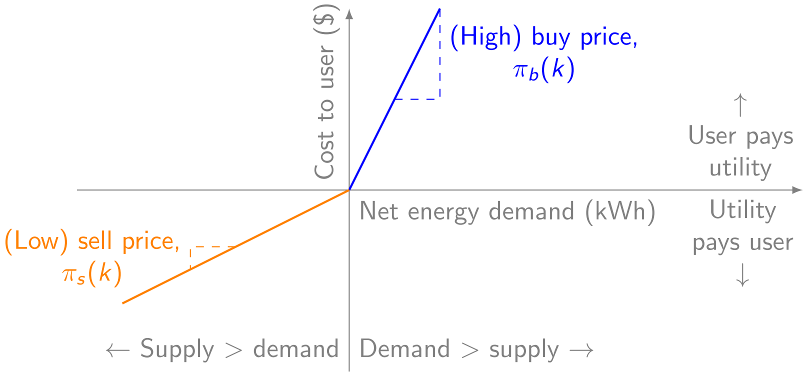
$$\begin{aligned} &\Delta t \left[\gamma^0 \pi(0) p(0) + \cdots + \gamma^{K-1} \pi(K-1) p(K-1)\right] \\ &= a^\top p \end{aligned}$$

where

$$p = \begin{bmatrix} p(0) \\ \vdots \\ p(K-1) \end{bmatrix}, \quad a = \Delta t \begin{bmatrix} \gamma^0 \pi(0) \\ \vdots \\ \gamma^{K-1} \pi(K-1) \end{bmatrix}$$

Outline

Energy cost with reduced net metering

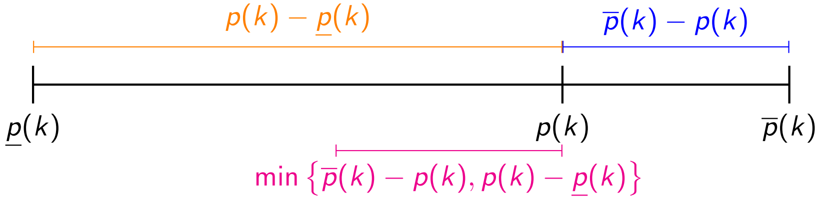


- $p(k)$ is **net** power demand (demand − supply)
- energy cost with reduced net metering ($\pi_b(k) < \pi_s(k)$) is

$$\Delta t \max \{\pi_b(k) p(k), \pi_s(k) p(k)\}$$

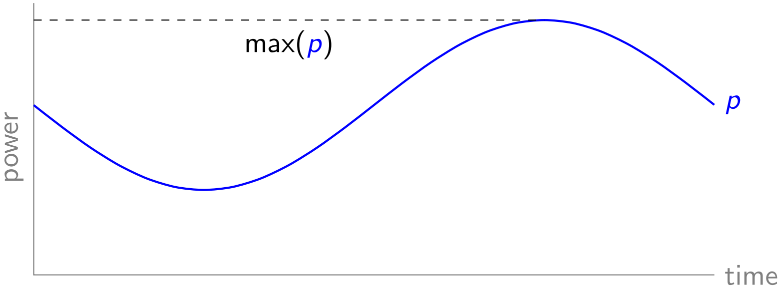
- convex, nondecreasing

Symmetric demand flexibility



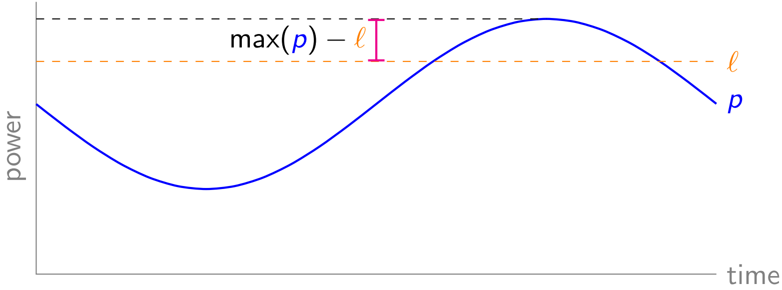
- some grid operators separate upward and downward flexibility
- others pay for *symmetric* capacity to adjust power up or down
- symmetric flexibility is concave:
    - ⋄ min is concave
    - ⋄ CCV(AFF) = CCV

Peak demand



- demand peaks drive sizing of electricity infrastructure
- peak demand cost: $\pi_d \max(p)$ with price π_d (\$/kW)
- convex nondecreasing in p if $\pi_d \geq 0$ since $\max(\cdot) =$ CVXND
- price π_d is typically ~10 to 50 \$/kW for monthly peaks
- per day, that's ~(10 to 50)/30 = 0.33 to 1.67 \$/kW

Peak demand with target demand limit

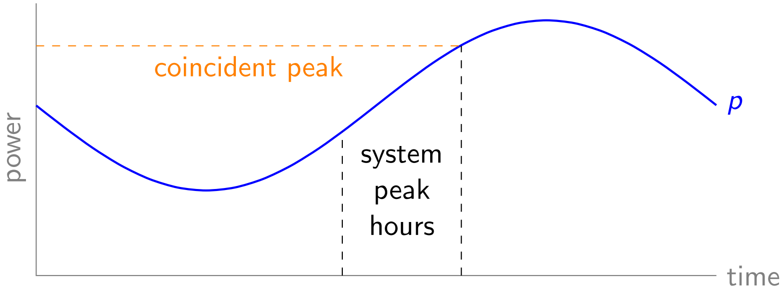


- penalize excess of peak $\max(p)$ above target demand limit ℓ
- no penalty if $\max(p) \leq \ell$: minimize

$$\pi_d \max\{0, \max(p) - \ell\}$$

- convex nondecreasing in p if $\pi_d \geq 0$

Coincident peak demand



- most peak demand charges penalize the **user's** peak
- what matters for infrastructure sizing is the **system** peak
- coincident peak demand charges
  - attempt to remedy this disconnect
  - penalize user's peak demand over system peak hours

Coincident peak demand charge

- coincident peak demand charge is

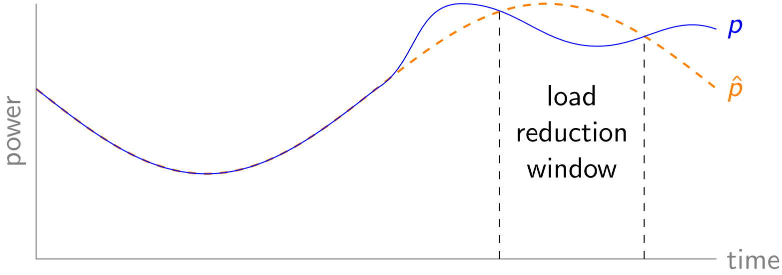
$$\pi_d \max(Cp)$$

- $C \in \mathbf{R}^{K \times K}$ picks out system peak hours:

$$C = \begin{bmatrix} c_0 & & \\ & \ddots & \\ & & c_{K-1} \end{bmatrix} \text{ with } c_k = \begin{cases} 1 & \text{if } t_k \text{ is a peak time} \\ 0 & \text{otherwise} \end{cases}$$

- convex nondecreasing in p if $\pi_d \geq 0$

Load reduction



- reduce load $p(k)$ below baseline $\hat{p}(k)$ during window?
  earn revenue $\Delta t \hat{\pi}(k)(\hat{p}(k) - p(k))$
- no revenue or penalty if $p(k) \geq \hat{p}(k)$

$\Longrightarrow$ maximize $\Delta t \hat{\pi}(k) \max\{0, \hat{p}(k) - p(k)\}$

⋆ price $\hat{\pi}(k)$ is **2 $/kWh** in California

California Public Utilities Commission: Emergency Load Reduction Program

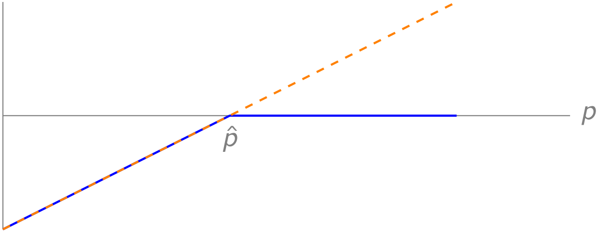
Maximizing load reduction revenue is nonconvex

- to maximize load reduction revenue, minimize

$$-\Delta t \hat{\pi}(k) \max\{0, \hat{p}(k) - p(k)\}$$

- $\max\{0, \hat{p}(k) - p(k)\} = \text{CVX(AFF)} = \text{CVX}$
- so $-\Delta t \hat{\pi}(k) \max\{0, \hat{p}(k) - p(k)\} = -\text{CVX} = \text{CCV}$
- minimizing a concave function yields a nonconvex problem

Load reduction: Convex approximation



- instead of minimizing the concave function

$$-\Delta t \hat{\pi}(k) \max\{0, \hat{p}(k) - p(k)\},$$

minimize 'nearest' convex function, $\Delta t \hat{\pi}(k)(p(k) - \hat{p}(k))$

- approximation is affine in $p(k)$ and, if $\hat{\pi}(k) \geq 0$, nondecreasing
- approximation adds fictitious penalty if $p(k) > \hat{p}(k)$

Reference tracking

- $r = (r(0), \ldots, r(K-1)) \in \mathbf{R}^K$ is a reference or target to track
- mean absolute tracking error:

$$\frac{1}{K} \sum_{k=0}^{K-1} |p(k) - r(k)| = \frac{\|p - r\|_1}{K}$$

- root mean square tracking error:

$$\sqrt{\frac{1}{K} \sum_{k=0}^{K-1} (p(k) - r(k))^2} = \frac{\|p - r\|_2}{\sqrt{K}}$$

- maximum absolute tracking error:

$$\max_{k=0,\ldots,K-1} |p(k) - r(k)| = \|p - r\|_\infty$$

- all convex since norms are convex

Linear reformulations

- most nonlinearities in these slides come from max, min, or $| \ |$
- CVX can reformulate these with linear objectives/constraints
  - minimize $\max\{x, y\}$ $\iff$ minimize z subject to $x \leq z$, $y \leq z$
  - maximize $\min\{x, y\}$ $\iff$ maximize z subject to $x \geq z$, $y \geq z$
  - minimize $|x|$ $\iff$ minimize y subject to $x \leq y$, $-x \leq y$
- so linear programming can solve most DER problems (great news; linear programming solvers are fast and robust)
- the only 'truly' nonlinear objective in these slides is $\|p - r\|_2$

Summary: DER objectives to **minimize** over p

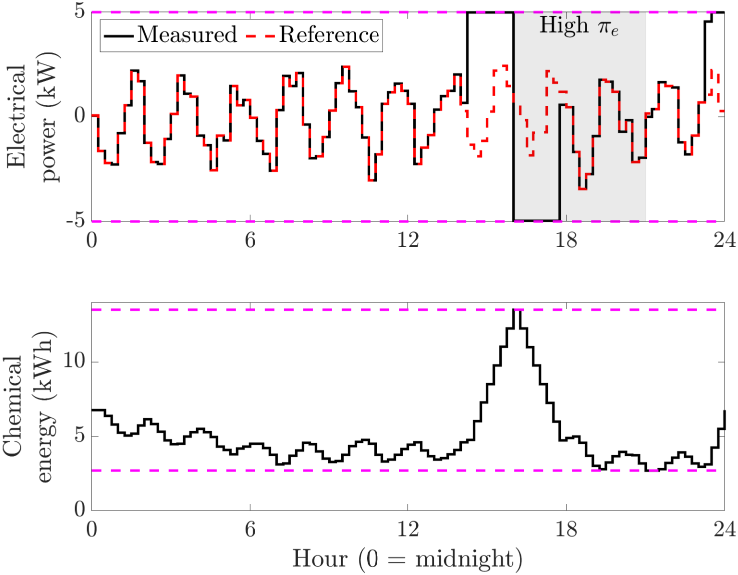
objective	curvature	monotonicity
energy	AFF	ND
energy cost	AFF	ND
energy cost (negative price)	AFF	NI
energy cost (reduced net metering)	CVX	ND
pollution	AFF	ND
pollution cost	AFF	ND
reference tracking error	CVX	not monotone
peak demand	CVX	ND
$-$(upward flexibility)	AFF	ND
$-$(downward flexibility)	AFF	NI
$-$(symmetric flexibility)	CVX	not monotone
$-$(load reduction)	CCV	ND
$-$(load reduction approximation)	AFF	ND

Outline

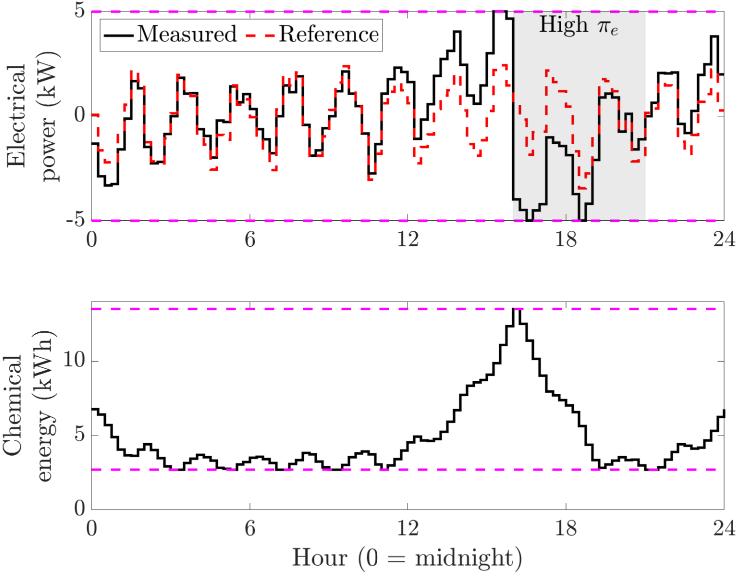
Problem setup

- stationary battery with 5 kW, 13.5 kWh capacities
- one-day horizon, 15-minute time step
- 1-norm objective: minimize $\Delta t \pi_e^\top p + \pi_t \left\| p - r \right\|_1 / K$
- 0.2 \$/kWh energy price from 4 to 9 PM; 0.1 otherwise
- tracking error price π_t hand-tuned to 0.2 \$/kW
- in 2-norm and ∞-norm examples,
    - ◇ energy cost constrained to equal 1-norm energy cost
    - ◇ objective: minimize $\left\| p - r \right\|_2$ or $\left\| p - r \right\|_\infty$
- in all examples, final energy constrained to equal initial energy

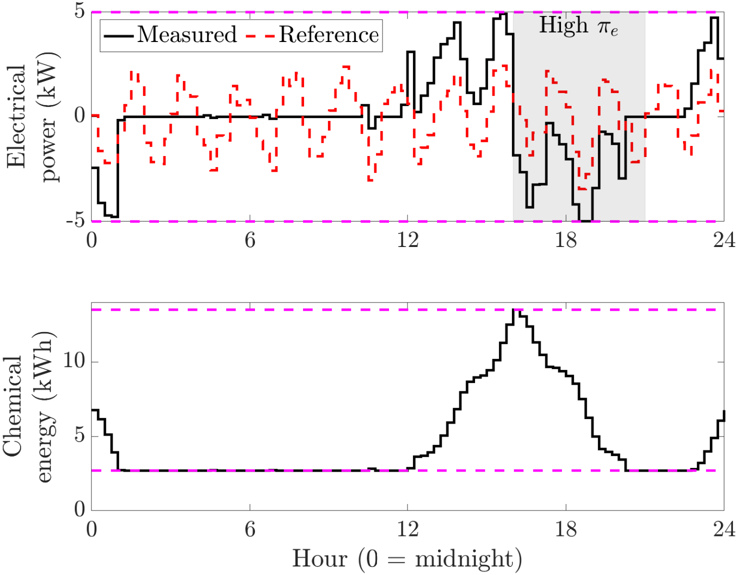
1-norm results



2-norm results



∞-norm results



Reference tracking error histograms