Batteries and electric vehicles

Purdue ME 597, Distributed Energy Resources

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Outline

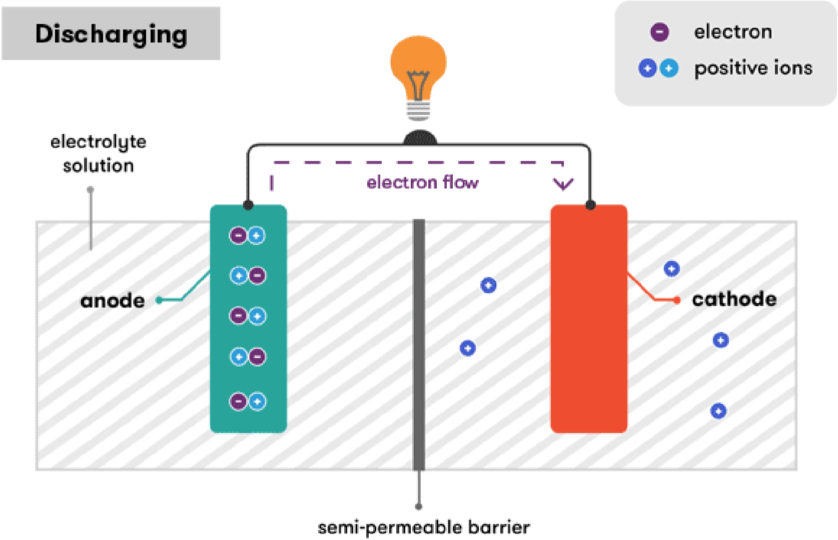
Battery basics

- batteries
  - ◇ convert between electrical and chemical potential energy
  - ◇ are also called electrochemical energy storage
- most batteries today are lithium-ion
  - ◇ personal electronics (phones, laptops, tablets, headphones, . . .)
  - ◇ electric vehicles (cars, trucks, bikes, scooters, skateboards, . . .)
  - ◇ stationary applications (home batteries, grid-scale storage, . . .)
  - ⋆ invented 1980s, commercialized 1991, Nobel Prize 2019

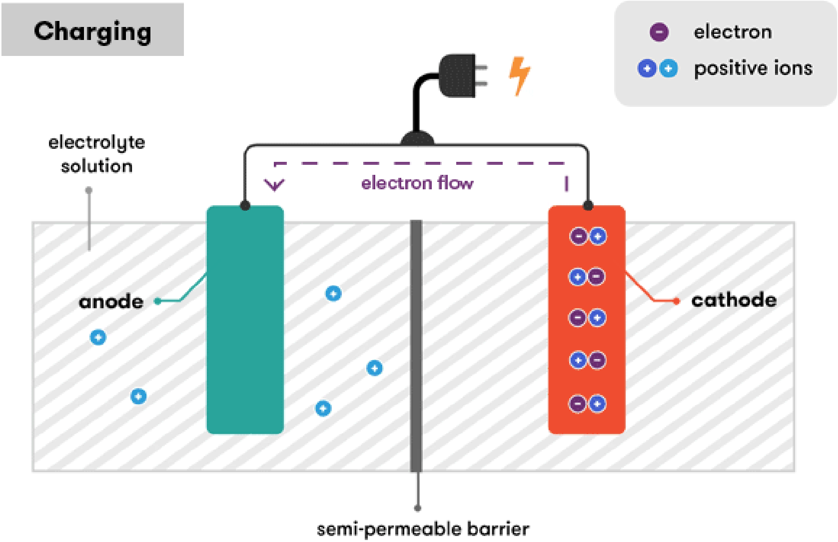
Battery components

- **anode** emits electrons and ions during discharge
  (typically graphite in lithium-ion batteries)
- **cathode** absorbs electrons and ions during discharge
  (lithium/metal oxides)
- **electrolyte** allows movement of ions but not electrons
  (polymer gels or mixes of salts, solvents, additives)
- **separator** blocks electrons, prevents anode-cathode contact
  (plastics such as polyethylene or polypropylene)

Discharging a battery



Australian Academy of Science: How a battery works



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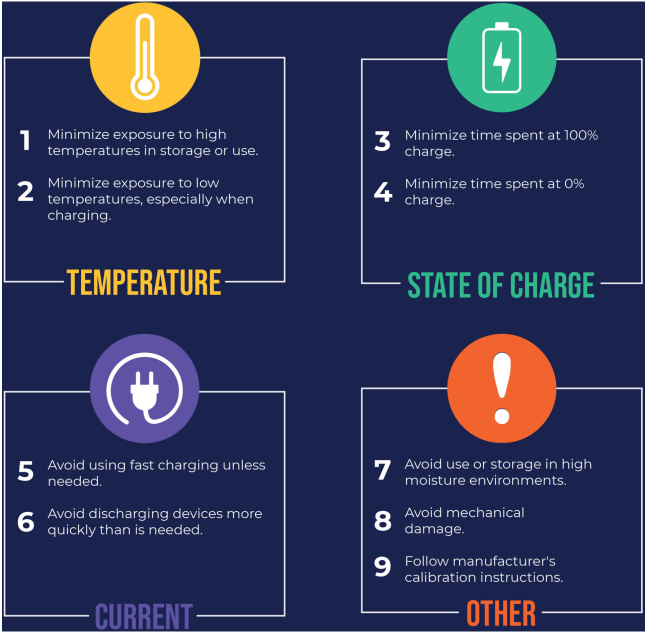


University of Michigan: Tips for extending the lifetime of lithium-ion batteries

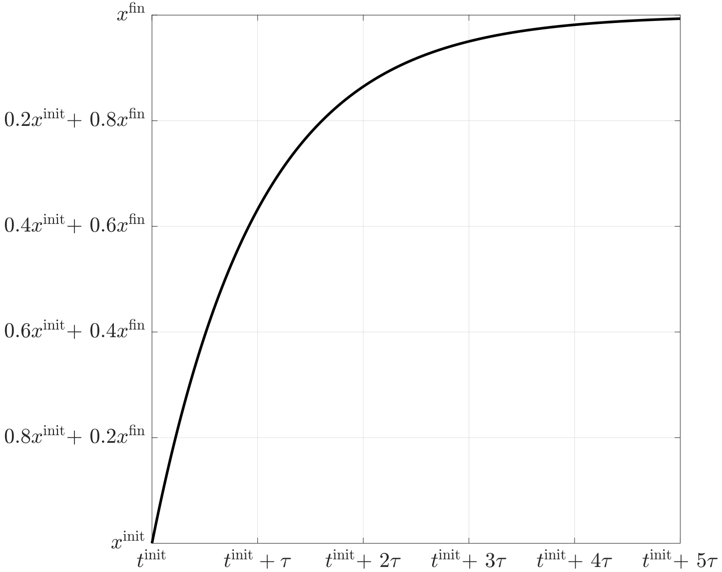
A simple battery model

- a simple model of a battery's energy dynamics is

$$\frac{dx(t)}{dt} = -\frac{x(t)}{\tau} + p^{\text{chem}}(t)$$

- $x(t) \in \mathbf{R}$ (kWh) is the stored chemical potential energy
- $\tau > 0$ (h) is the self-dissipation time constant
  (for an ideal battery with no self-dissipation, $\frac{dx(t)}{dt} = p^{\text{chem}}(t)$)
- $p^{\text{chem}}(t) \in \mathbf{R}$ (kW) is the chemical charging power
  (or discharging if $p^{\text{chem}}(t) < 0$)

Energy evolution with constant p^{chem}



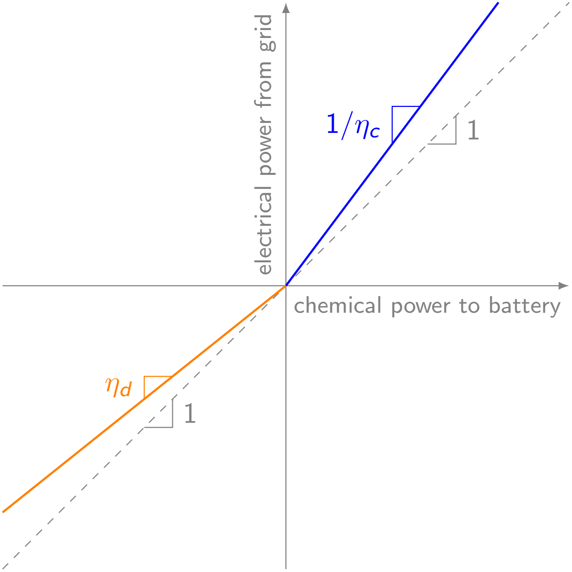
Chemical and electrical power

- the electrical charging (or discharging) power is

$$p(t) = \begin{cases} p^{\text{chem}}(t)/\eta_c & \text{if } p^{\text{chem}}(t) \geq 0 \\ \eta_d p^{\text{chem}}(t) & \text{if } p^{\text{chem}}(t) < 0 \end{cases}$$
$$= \max\left\{ p^{\text{chem}}(t)/\eta_c, \eta_d p^{\text{chem}}(t) \right\}$$

- η_c, $\eta_d \in (0, 1]$ are the charging and discharging efficiencies

Chemical and electrical power (continued)



Discrete-time battery model

- with uniform time step Δt and piecewise constant $p^{\text{chem}}(t)$,

$$x(k+1) = ax(k) + (1-a)\tau p^{\text{chem}}(k),$$

  where

$$a = e^{-\Delta t/\tau}$$

- **homework**: show this

Battery constraints

- batteries have energy and power constraints:

$$0 \le x(k) \le \overline{x}$$
$$-\overline{p}_d \le p(k) \le \overline{p}_c$$

- $\overline{x} \ge 0$ (kWh) is the chemical energy capacity
- $\overline{p}_c \ge 0$ (kW) is the electrical charging power capacity
- $\overline{p}_d \ge 0$ (kW) is the electrical discharging power capacity
- in terms of p^{chem}, the power constraints are

$$-\frac{\overline{p}_d}{\eta_d} \le p^{\text{chem}}(k) \le \eta_c \overline{p}_c$$

Outline

Stationary battery parameters

the stationary battery model has six parameters:

1. chemical energy capacity $\overline{x} \geq 0$ (kWh)
2. electrical charging power capacity $\overline{p}_c \geq 0$ (kW)
3. electrical discharging power capacity $\overline{p}_d \geq 0$ (kW)
4. self-dissipation time constant $\tau > 0$ (h)
5. charging efficiency $\eta_c \in [0, 1]$
6. discharging efficiency $\eta_d \in [0, 1]$

Typical values of the self-dissipation time constant τ

- unused, a typical battery might lose ~1 to 3% energy per day
- if $p^{\text{chem}}(t) = 0$ for all t and $x(0) = x_0$, then $x(t) = e^{-t/\tau} x_0$
- after one day, $t = 24$ h and $x(t)/x_0 \approx 0.97$ to 0.99
- so a typical time constant is

$$\tau = \frac{-t}{\ln(x(t)/x_0)} = \frac{-24 \text{ h}}{\ln(0.97 \text{ to } 0.99)} \approx 800 \text{ to } 2400 \text{ h}$$

Typical values of the energy and power capacities

- choosing appropriate capacities is a design problem
- sometimes, hardware limitations impose constraints
- for example, stationary lithium-ion batteries typically have
  - ◇ similar charging and discharging power capacities ($\overline{p}_c \approx \overline{p}_d$)
  - ◇ one to four hours of storage ($\overline{x}/\overline{p}_c \approx 1$ to 4 h)
- also, a building's wiring may limit current or voltage:

$$p_c = IV \approx (15 \text{ to } 25 \text{ A})(240 \text{ V}) = 3.6 \text{ to } 6 \text{ kW}$$

- one popular home battery has
  - ◇ $\overline{p}_c = \overline{p}_d = 5$ kW
  - ◇ $\overline{x} = 13.5$ kWh (so $\overline{x}/\overline{p}_c = 2.7$ h)

Round-trip efficiency

- consider a battery with no self-dissipation ($\tau = \infty$)
- if we charge at electrical power $p_{\text{in}} > 0$ at time k, then

$$x(k+1) = x(k) + \Delta t \eta_c p_{\text{in}}$$

- suppose we then return the battery to its initial state $x(k)$:

$$\begin{aligned} & x(k+2) = x(k) \\ \iff & x(k+1) - \Delta t p_{\text{out}}/\eta_d = x(k) \\ \iff & x(k) + \Delta t \eta_c p_{\text{in}} - \Delta t p_{\text{out}}/\eta_d = x(k) \\ \iff & \eta_c p_{\text{in}} = p_{\text{out}}/\eta_d \end{aligned}$$

- so we get electrical power $p_{\text{out}} = \eta_c \eta_d p_{\text{in}}$ back out
- the product $\eta_c \eta_d$ is called the **round-trip efficiency**

Typical values of the charging and discharging efficiencies

- a typical battery has round-trip efficiency $\eta_c \eta_d \approx 0.9$
- usually, charging and discharging efficiencies are similar
- so $\eta_c \approx \eta_d \approx \sqrt{0.9} \approx 0.95$

Summary: Instantiating a stationary battery model

- set $\tau \approx 800$ to 2400 h
- set $\eta_c \approx \eta_d \approx 0.95$
- either
  - ◇ choose an appropriate energy capacity $\overline{x}$ for the application
  - ◇ set $\overline{p}_c \approx \overline{p}_d \approx \overline{x}/(1 \text{ to } 4 \text{ h})$

  or
  - ◇ set $\overline{p}_c \approx \overline{p}_d = IV$ with appropriate I, V for building's wiring
  - ◇ set $\overline{x} \approx (1 \text{ to } 4 \text{ h})\overline{p}_c$

Outline

EV battery dynamics

- for modeling purposes, EVs are just batteries that move
- they have the same energy dynamics as stationary batteries:

$$x(k+1) = ax(k) + (1-a)\tau p^{\text{chem}}(k)$$

- if an EV drives $d(k)$ km between t_k and $t_{k+1} = t_k + \Delta t$, then

$$p^{\text{chem}}(k) = -\frac{\alpha(k)d(k)}{\Delta t}$$

- $\alpha(k)$ (kWh/km) is the energy intensity of driving

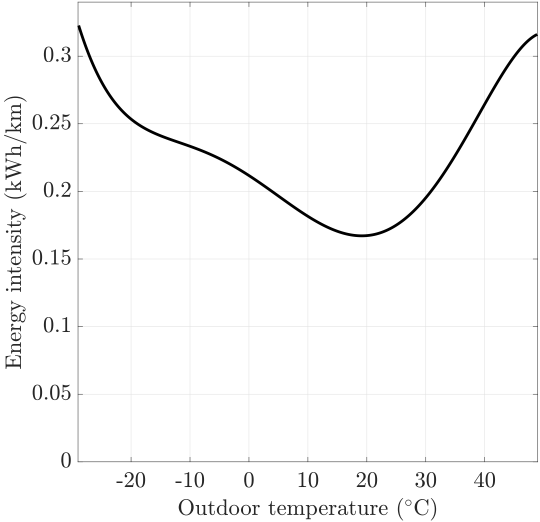
The energy intensity of driving an EV

- the energy intensity $\alpha(k)$ (kWh/km) depends on the vehicle's
  - ◇ speed
  - ◇ weight
  - ◇ shape
  - ◇ drivetrain efficiency

  as well as the
  - ◇ terrain (hills)
  - ◇ weather (cabin heating/cooling, battery thermal management)
- a typical EV energy intensity is 0.15 to 0.4 kWh/km

⋆ electric bikes use ~0.005 kWh/km

EVs use more energy when it's cold or hot outside



Yuksel and Michalek (2015): *Effects of Regional Temperature on Electric Vehicle Efficiency, Range, and Emissions in the United States*

An energy intensity model

$$\alpha(k) = c_0 + c_1 T(k) + \cdots + c_5 T(k)^5$$

- $\alpha(k)$ is the energy intensity at time k, in kWh/km
- $T(k)$ is the outdoor temperature at time k, in $^\circ$**F**
- the coefficients (in the corresponding units) are

c_0	0.3950
c_1	-0.0022
c_2	9.1978×10^{-5}
c_3	-3.9249×10^{-6}
c_4	5.2918×10^{-8}
c_5	-2.0659×10^{-10}

Yuksel and Michalek (2015): *Effects of Regional Temperature on Electric Vehicle Efficiency, Range, and Emissions in the United States*

EV charging and discharging constraints

- when unplugged, EVs
  - can't charge
  - functionally, have no discharge power limit (400+ kW)
- when plugged in,
  - most EVs can charge but not discharge
  - EVs with bidirectional charging can do both
- define the indicator variable

$$z(k) = \begin{cases} 1 & \text{if the EV is plugged in over time step } k \\ 0 & \text{otherwise} \end{cases}$$

- then the charging and discharging constraints become

$$\begin{cases} -\overline{p}_d \leq p(k) \leq \overline{p}_c & \text{if } z(k) = 1 \\ p(k) \leq 0 & \text{if } z(k) = 0 \end{cases}$$

with $\overline{p}_d = 0$ for EVs without bidirectional charging

Typical EV parameter values

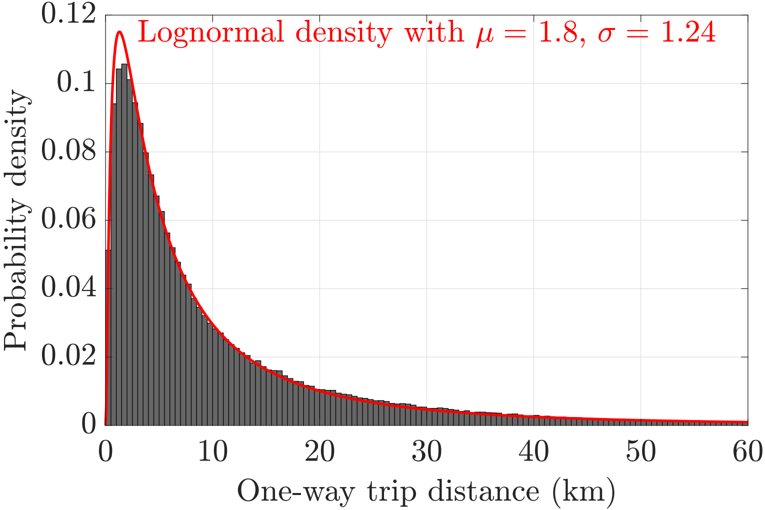
- τ, η_c, and η_d are typically similar to stationary batteries
- typical values of $\overline{x}$ are 55, 80, 100, or 130 kWh
- for Level 1 charging (∼120 V/15 A), $\overline{p}_c \approx 1.8$ kW
- for Level 2 charging (∼240 V/48 A), $\overline{p}_c \approx 11.5$ kW
- for EVs without bidirectional charging, $\overline{p}_d = 0$
- for EVs with bidirectional charging, $\overline{p}_d \approx \overline{p}_c$

Typical US driving patterns

- the average household has 1.9 private vehicles (cars, pickup trucks, SUVs, or vans)
- the average private vehicle
  - drives 26 miles (41.6 km) per day
  - drives 7.9 miles (12.6 km) per trip (one way, not round trip)
  - so takes about 3.3 trips per day
- most trips happen between 6 AM and 8 PM

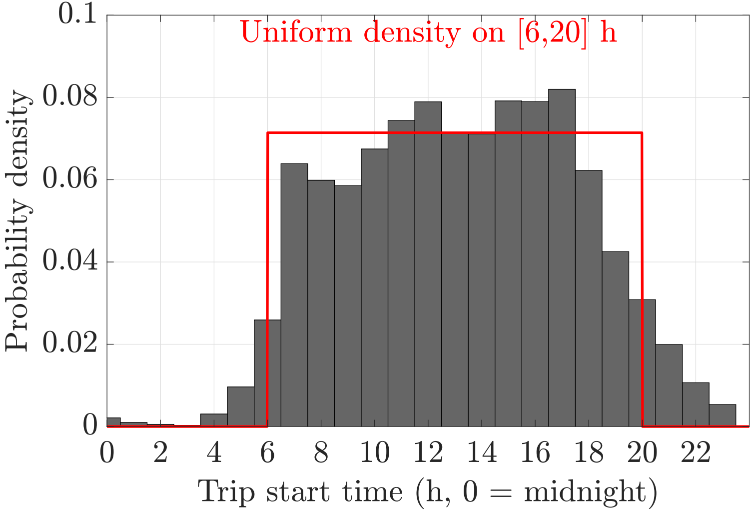
US DOT (2017): *National Household Travel Survey*

One-way trip distance distribution for private vehicles



US DOT (2017): *National Household Travel Survey*

Trip start time distribution for private vehicles



US DOT (2017): *National Household Travel Survey*

Outline

1. When plugged in, charge at maximum until full

- if $z(k) = 1$ (meaning the EV is plugged in), set

$$p^{\text{chem}}(k) = \min\left\{\eta_c \overline{p}_c, \frac{\overline{x} - ax(k)}{(1-a)\tau}\right\}$$

- if the battery is nearly full, this places $x(k+1)$ at $\overline{x}$:

$$x(k+1) = ax(k) + (1-a)\tau \frac{\overline{x} - ax(k)}{(1-a)\tau} = \overline{x}$$

2. When energy gets low, charge at maximum until full

- define the minimum acceptable energy $\underline{x}$ (kWh)
- initialize $y(k)$, an indicator of 'charging mode,' at $y(k-1)$
- if $z(k) = 0$ or $x(k) = \overline{x}$, set $y(k) = 0$
- if $z(k) = 1$ and $x(k) < \underline{x}$, set $y(k) = 1$
- if $y(k) = 1$, set

$$p^{\text{chem}}(k) = \min\left\{\eta_c \overline{p}_c, \frac{\overline{x} - a x(k)}{(1-a)\tau}\right\}$$

3. When energy gets low, charge steadily to meet deadline

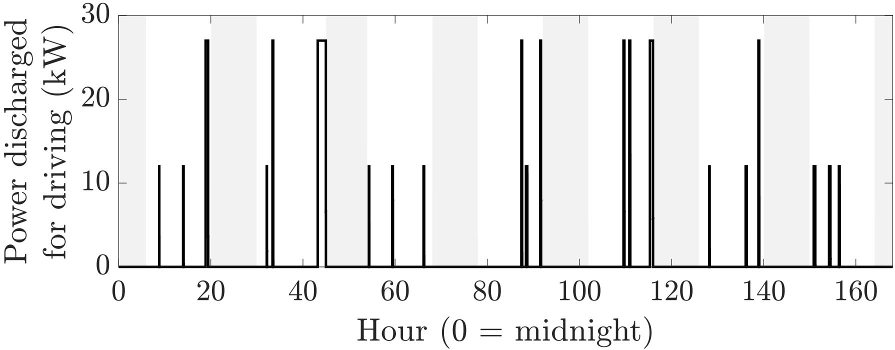
- suppose the user wants energy $x^\star$ stored by deadline $k^\star > k$
- charging at constant power p_0^{chem} from time k to $k^\star$ gives

$$
\begin{aligned}
x(k+1) &= ax(k) + (1-a)\tau p_0^{\text{chem}} \\
x(k+2) &= ax(k+1) + (1-a)\tau p_0^{\text{chem}} \\
&= a[ax(k) + (1-a)\tau p_0^{\text{chem}}] + (1-a)\tau p_0^{\text{chem}} \\
&= a^2 x(k) + (1+a)(1-a)\tau p_0^{\text{chem}} \\
&\vdots \\
x(k^\star) &= a^{k^\star - k} x(k) + (1 + a + \cdots + a^{k^\star - k - 1})(1-a)\tau p_0^{\text{chem}}
\end{aligned}
$$

- so if $y(k) = 1$, set

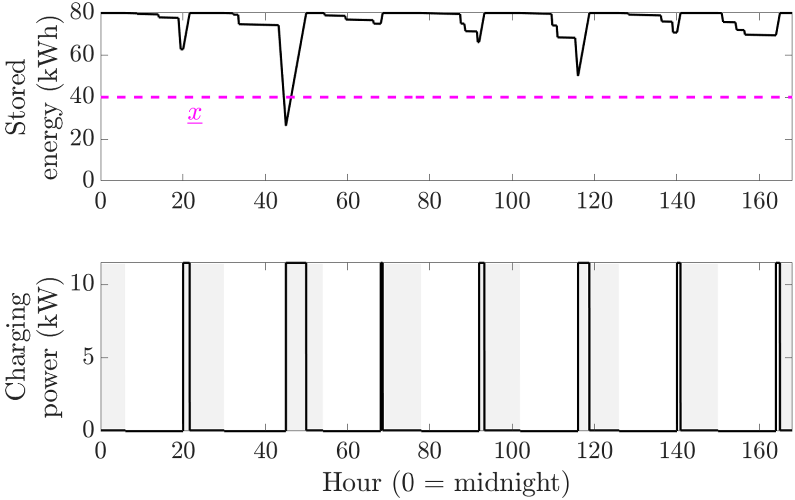
$$
p^{\text{chem}}(k) = \min\left\{ \eta_c \overline{p}_c, \frac{x^\star - a^{k^\star - k} x(k)}{(1 + a + \cdots + a^{k^\star - k - 1})(1-a)\tau} \right\}
$$

Example

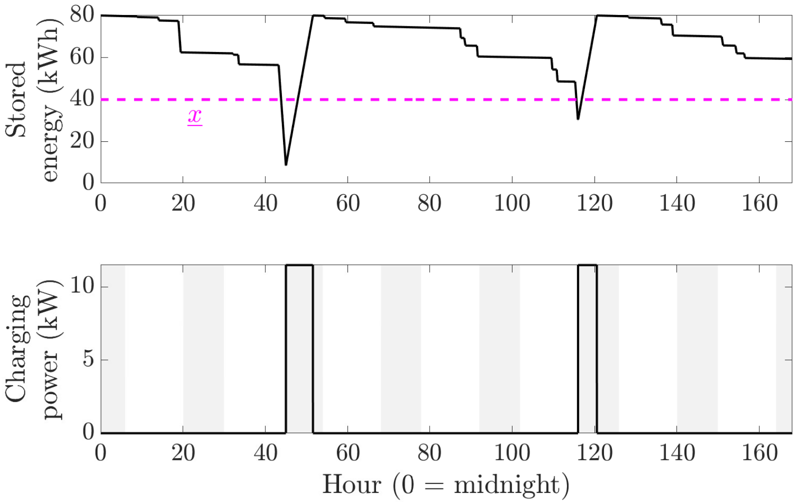


- EV with 80 kWh battery over 7 days, $\alpha = 0.3$ kWh/km
- EV is plugged in during shaded periods
- short trips are at 40 km/h, long trips at 90

Policy 1 tops off battery every night



Policy 2 charges full blast when $x < \underline{x}$



Policy 3 spreads charging out over whole night