Thermal modeling of buildings: Part 1

Purdue ME 597, Distributed Energy Resources

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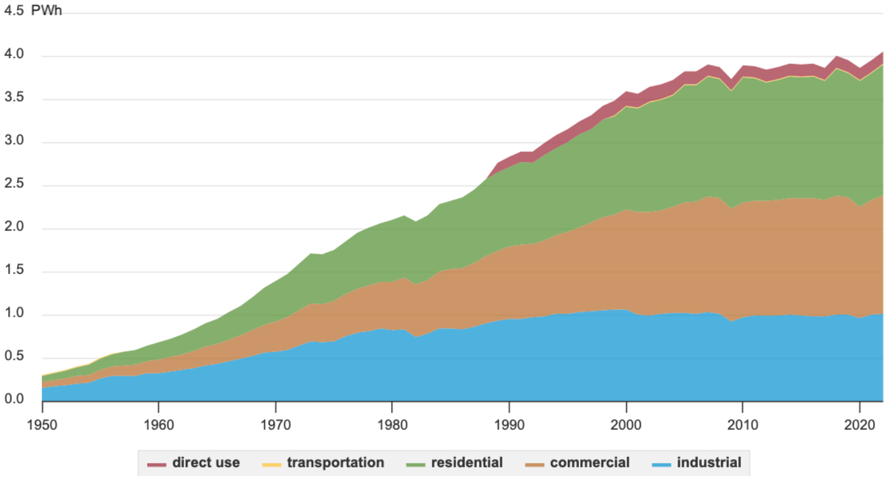
Outline

Background

First-order thermal circuits

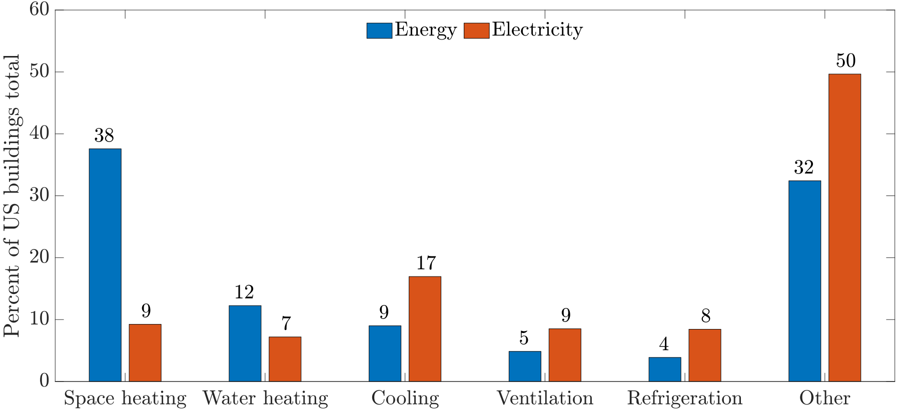
Thermal circuit model parameters

Buildings dominate US electricity use



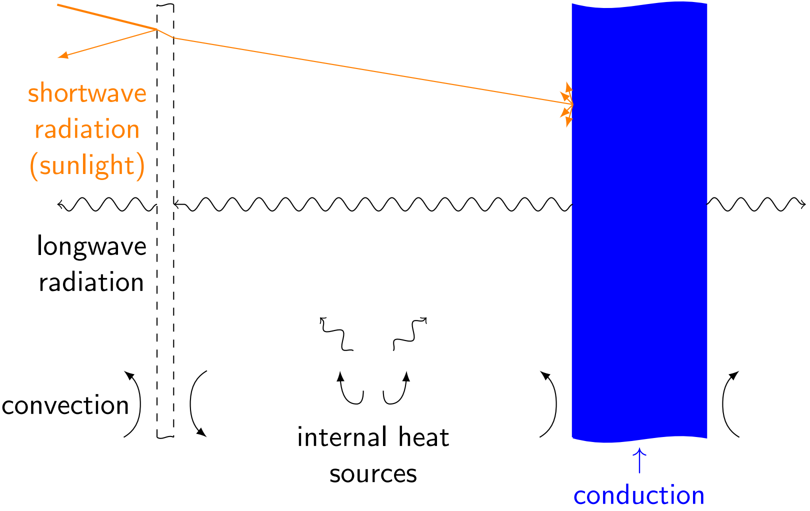
Energy Information Administration: Use of electricity

HVAC&R dominates building energy and electricity use



Detailed thermal modeling is hard

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \dots$$



Software for detailed thermal modeling

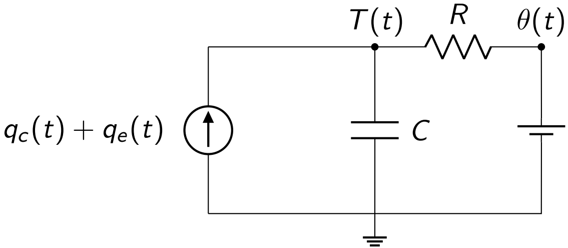
- EnergyPlus (US DOE, open source)
- TRNSYS (**Tr**a**n**sient **Sy**stem **S**imulation, commercial)
- Modelica buildings library (US DOE, open source)
- these programs are broad and powerful, but can
  - require many parameters, some hard to specify or fit
  - be slow (numerical solution of coupled nonlinear PDEs)
  - be difficult to integrate with other software
- our focus: simpler models for DER design and control

Outline

Thermal circuit models capture dominant physics

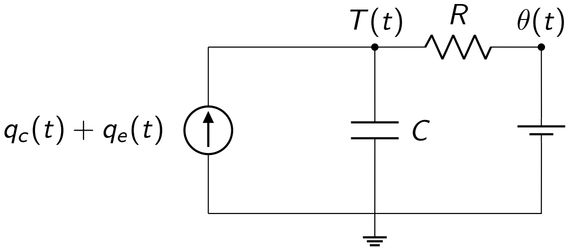
- thermal circuits work by analogy to electrical circuits
  - temperature $\leftrightarrow$ voltage
  - heat $\leftrightarrow$ charge
  - thermal resistance $\leftrightarrow$ electrical resistance
  - thermal capacitance $\leftrightarrow$ electrical capacitance
- basic idea: temperature differences drive heat flow
  (like voltage differences drive charge flow [current])

Simplest thermal circuit: 1R1C



- $T(t)$ (°C) is indoor temperature
- $\theta(t)$ (°C) is boundary (often outdoor air) temperature
- R (°C/kW) is thermal resistance between T and θ
- C (kWh/°C) is indoor thermal capacitance
- $q_c(t)$ (kW) is thermal power from controlled equipment
- $q_e(t)$ (kW) is thermal power from exogenous sources (sunlight, body heat, plug loads, ...)

1R1C continuous-time dynamics



- Ohm's law: current through resistor is $(T(t) - \theta(t))/R$
- rate of charge accumulation on capacitor is $C\mathrm{d}T(t)/\mathrm{d}t$
- Kirchhoff's Current Law (KCL) at node labeled $T(t)$:

$$\text{current inflow} = \text{current outflow}$$

$$q_c(t) + q_e(t) = \frac{T(t) - \theta(t)}{R} + C\frac{\mathrm{d}T(t)}{\mathrm{d}t}$$

1R1C discrete-time dynamics

- rearranging continuous-time dynamics gives

$$\frac{\mathrm{d}T(t)}{\mathrm{d}t} = \frac{1}{RC}\left[-T(t) + \theta(t) + R\left(q_c(t) + q_e(t)\right)\right],$$

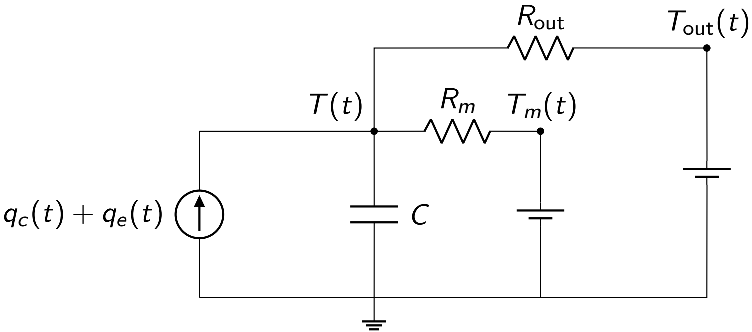
  a first-order linear ODE

- with uniform time step Δt and piecewise constant θ, q_c, q_e,

$$T(k+1) = aT(k) + (1-a)R\left(q_c(k) + w(k)\right)$$

  where $a = e^{-\Delta t/(RC)}$ and $w(k) = q_e(k) + \theta(k)/R$

A 2R1C thermal circuit



- $T_m(t)$ (°C) is thermal mass temperature
- $T_{out}(t)$ (°C) is outdoor air temperature
- R_m (°C/kW) is thermal resistance between T and T_m
- R_{out} (°C/kW) is thermal resistance between T and T_{out}

Reducing a 2R1C thermal circuit to 1R1C

- KCL at node labeled $T(t)$ gives 2R1C dynamics:

$$C\frac{\mathrm{d}T(t)}{\mathrm{d}t} = \frac{T_m(t) - T(t)}{R_m} + \frac{T_{\text{out}}(t) - T(t)}{R_{\text{out}}} + q_c(t) + q_e(t)$$

- a bit of algebra shows this is equivalent to the 1R1C model

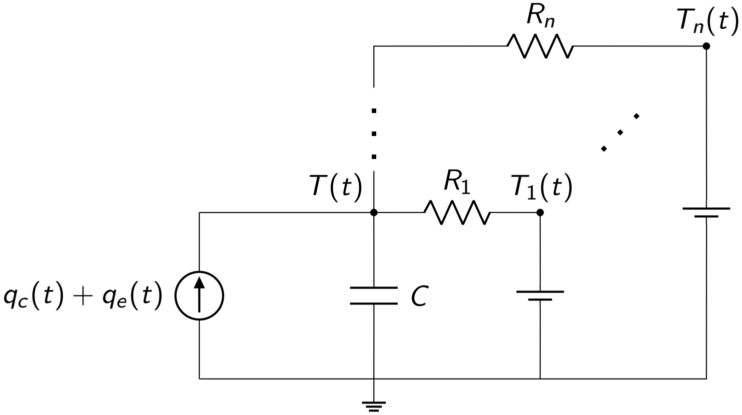
$$C\frac{\mathrm{d}T(t)}{\mathrm{d}t} = \frac{\theta(t) - T(t)}{R} + q_c(t) + q_e(t)$$

with

$$R = \frac{R_m R_{\text{out}}}{R_m + R_{\text{out}}}$$

$$\theta(t) = \frac{R_{\text{out}} T_m(t) + R_m T_{\text{out}}(t)}{R_m + R_{\text{out}}}$$

An nR1C thermal circuit



$$\text{KCL} \implies C\frac{\mathrm{d}T(t)}{\mathrm{d}t} = \sum_{i=1}^{n} \frac{T_i(t) - T(t)}{R_i} + q_c(t) + q_e(t)$$

Reducing an nR1C thermal circuit to 1R1C

can show that the nR1C model is equivalent to the 1R1C model

$$C \frac{\mathrm{d}T(t)}{\mathrm{d}t} = \frac{\theta(t) - T(t)}{R} + q_c(t) + q_e(t)$$

with

$$R = \frac{R_1 \cdots R_n}{R_2 \cdots R_n + \cdots + R_1 \cdots R_{n-1}}$$

$$\theta(t) = \frac{R_2 \cdots R_n T_1(t) + \cdots + R_1 \cdots R_{n-1} T_n(t)}{R_2 \cdots R_n + \cdots + R_1 \cdots R_{n-1}}$$

Outline

Thermal circuit parameters

thermal circuit models have three types of parameter:

1. thermal capacitance $C > 0$ (kWh/$^\circ$C)
2. thermal resistance $R > 0$ ($^\circ$C/kW)
3. exogenous thermal power $q_e(t)$ (kW, a time series)

Typical thermal capacitance values

- can model thermal capacitance as $C = \rho c_p V$, where
  - $\rho = 1.293$ kg/m^3
  - $c_p = 2.792 \times 10^{-4}$ kWh/(kg °C)
  - $V = A_f h$ is enclosed volume
    (A_f is total floor area on all stories; h is height of one story)
- this models air at standard temperature and pressure
- often, 'shallow' thermal mass couples tightly to indoor air

$$\implies C \approx (10 \text{ to } 15)\rho c_p A_f h = (0.0036 \text{ to } 0.0054 \text{ kWh}/[^\circ\text{Cm}^3])A_f h$$

- example: for a ~200 m^2 house with ceiling heights of ~3 m,

$$C \approx 2.2 \text{ to } 3.2 \text{ kWh}/^\circ\text{C}$$

Thermal resistance from weather and energy data

- if $T(t) \approx \hat{T}$ and $q_e(t) \approx \hat{q}_e$ are ~constant over some period, then

$$C\cancelto{\sim 0}{\frac{\mathrm{d}T(t)}{\mathrm{d}t}} \approx \frac{\theta(t) - \hat{T}}{R} + q_c(t) + \hat{q}_e \implies q_c(t) \approx \frac{\hat{T} - \theta(t)}{R} - \hat{q}_e$$

- suppose we guess a δ (°C) such that if $\hat{T} - \theta(t) \approx \delta$, then $q_c(t) \approx 0$
- this gives an estimate of $\hat{q}_e$:

$$\hat{T} - \theta(t) \approx \delta \implies q_c(t) \approx \frac{\delta}{R} - \hat{q}_e \approx 0 \implies \hat{q}_e \approx \frac{\delta}{R}$$

- it also gives a simplified expression for $q_c(t)$:

$$q_c(t) \approx \frac{\hat{T} - \delta - \theta(t)}{R} = \frac{\theta_h - \theta(t)}{R}$$

- $\theta_h = \hat{T} - \delta$ is the **heating balance outdoor temperature**

R from weather and energy data (continued)

- define the cumulative heat demand $H = \int \max\{0, q_c(t)\}\, \mathrm{d}t$
- with $q_c(t) \approx \frac{\theta_h - \theta(t)}{R}$,

$$H \approx \int \max\left\{0, \frac{\theta_h - \theta(t)}{R}\right\} \mathrm{d}t = \frac{\Theta_h}{R}$$

- Θ_h ($^\circ$Ch) is the number of heating degree-hours at HBOT θ_h:

$$\Theta_h = \int \max\{0, \theta_h - \theta(t)\}\, \mathrm{d}t$$

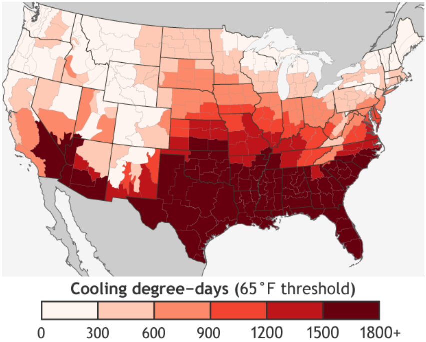
- with heater efficiency η, the heater input energy is $E = H/\eta$ and

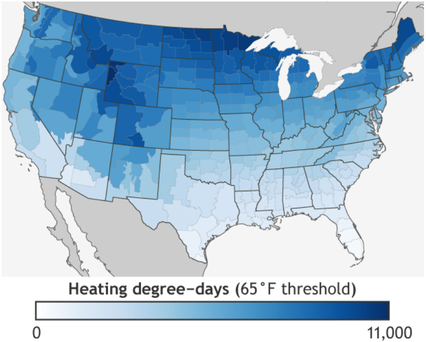
$$R \approx \frac{\Theta_h}{\eta E}$$

Heating and cooling degree-days

$$\mathrm{HDD}(\theta_h) = \left(\frac{1\ \mathrm{day}}{24\ \mathrm{h}}\right) \int \max\{0, \theta_h - \theta(t)\}\, \mathrm{d}t = \left(\frac{1\ \mathrm{day}}{24\ \mathrm{h}}\right) \Theta_h$$

$$\mathrm{CDD}(\theta_c) = \left(\frac{1\ \mathrm{day}}{24\ \mathrm{h}}\right) \int \max\{0, \theta(t) - \theta_c\}\, \mathrm{d}t = \left(\frac{1\ \mathrm{day}}{24\ \mathrm{h}}\right) \Theta_c$$





Climate.gov: Annual average cooling and heating degree-days (1901-2000)

Summary: R from weather and energy data

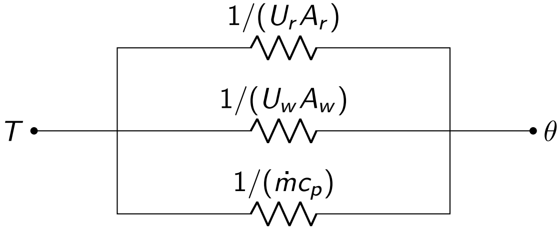
1. get typical heating temperature setpoint $\hat{T}$
2. estimate indoor-outdoor temperature difference δ where $q_c \approx 0$ (typically 6 to 10 °C for badly- to well-insulated detached houses)
3. define heating balance outdoor temperature $\theta_h = \hat{T} - \delta$
4. get historical HDD(θ_h) (°C·day) over a heating period
5. get historical heater input energy E (kWh) over the same period
6. estimate heater efficiency η
7. set

$$R \approx \left(\frac{24\text{ h}}{1\text{ day}}\right)\frac{\text{HDD}(\theta_h)}{\eta E}$$

⋆ can find benchmark E data in RECS and CBECS

⋆ bonus: time-average of $q_e(t)$ should be $\sim\delta/R$

Thermal resistance from first principles



- can model thermal resistance as $R = 1/(U_r A_r + U_w A_w + \dot{m} c_p)$
- U (kW/[$^\circ$Cm^2]) is thermal transmittance
- A (m^2) is outward-facing surface area
- subscript r means roof, w means vertical wall/window assemblies
- $\dot{m}$ (kg/h) is mass flow rate of outdoor air infiltration

A typical wall assembly

The Gold Hive: How (and why) to install rockwool insulation

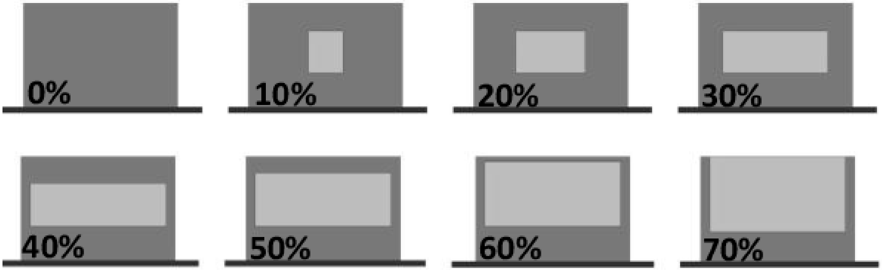
- typical thermal transmittances (note units of **W**, not **kW**) are
  - ◇ $\sim$1, 2.5, or 5 **W**/(°Cm^2) for windows with 3, 2, or 1 pane(s)
  - ◇ $\sim$0.2 to 0.8 **W**/(°Cm^2) for framed, insulated, opaque walls
  - ◇ $\sim$0.2 to 0.5 **W**/(°Cm^2) for framed, insulated, opaque roofs
- for a wall assembly whose surface area is $100\lambda\%$ windows,

$$U_w = \lambda U_{\text{window}} + (1 - \lambda) U_{\text{wall}}$$

- a typical wall assembly has $\lambda \approx 0.1$ to 0.3

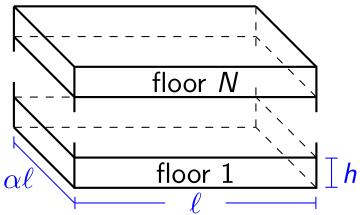
$\Longrightarrow$ overall $U_w \approx 0.7$ to 1.3 **W**/(°Cm^2)

Window-to-wall ratio λ



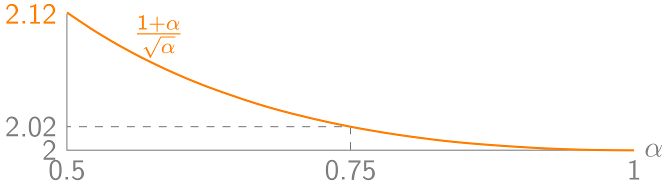
El-Deeb (2013): Combined effect of window-to-wall ratio and wall composition on energy consumption

Wall area

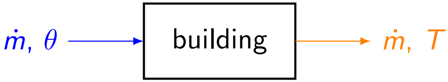


floor aspect ratio: α
total floor area: $A_f = \alpha N \ell^2$
$\implies \ell = \sqrt{A_f/(\alpha N)}$
total wall area: $A_w = 2(1+\alpha)N\ell h$
$\implies A_w = 2(1+\alpha)h\sqrt{NA_f/\alpha}$

- roof area is $A_r = A_f/N$
- $(1+\alpha)/\sqrt{\alpha} \approx 2$, so $A_w \approx 4h\sqrt{NA_f}$



Infiltration of outdoor air



- outdoor air enters through gaps in doors, windows, etc.
- indoor air exits at ∼same mass flow rate $\dot{m}$
- this mass transfer entails heat transfer at rate $\dot{m}c_p(\theta - T)$
- with ν (1/h) air changes per hour, $\dot{m} = \nu\rho A_f h$
  (1 air change = outdoor air fills building's volume $A_f h$ once)
- typically, $\nu \approx 0.3$ to 0.9 per hour

Handley (1973): Home ventilation rates: A literature study, Younes (2011): Air infiltration through building envelopes: A review

Summary: Thermal resistance from first principles

- with total floor area A_f and N stories, each of height h,

$$
\begin{aligned}
1/R \approx\ & (2 \text{ to } 5 \times 10^{-4}\ \text{kW}/[^\circ\text{Cm}^2])A_f/N \\
& + (2.8 \text{ to } 5.2 \times 10^{-3}\ \text{kW}/[^\circ\text{Cm}^2])h\sqrt{NA_f} \\
& + (1.5 \text{ to } 2.9 \times 10^{-4}\ \text{kW}/[^\circ\text{Cm}^3])A_f h
\end{aligned}
$$

- example: for a ~200 m^2 house with 2 stories and ~3 m ceilings,

$$R \approx 2.5\ ^\circ\text{C/kW}$$

Typical exogenous thermal power values

- can decompose $q_e(t)$ into heat transfer from
  1. sun
  2. plugged-in devices (lights, electronics, electric cooking, ...)
  3. other (body heat, fossil-fueled cooking, wood fireplaces, ...)
- #2 may be measured
  (typical time-average $\approx$ 250 W + (2.65 W/m^2)A_f)
- #3 is often small (e.g., $\sim$100 W per body)
- check: from HDD analysis, time-average of $q_e(t)$ should be $\sim\delta/R$

Thermal power from sunlight through windows (#1)

- can approximate the total solar thermal power from all windows as

$$q_{\mathrm{sun}}(t) \approx \left(1.5 \text{ to } 1.7 \sqrt{\mathrm{kW}}/\mathrm{m}\right) c\lambda h \sqrt{N A_f S_{\mathrm{tot}}^{-}(t)}.$$

- $S_{\mathrm{tot}}^{-}(t)$ (kW/m^2) is the total solar irradiance on a horizontal surface
- $c \in [0,1]$ (typically 0.25 to 0.8) is the **solar heat gain coefficient** (the fraction of incident solar irradiance that a window transmits)

⋆ can simulate shading from blinds, trees, etc. by reducing c

- more on this when we learn about solar