Homework 2: Batteries and electric vehicles

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Directions:

- Students may work individually or in groups, but each student must upload their own solutions to Gradescope by **11:59 PM ET on Wednesday, February 5**.
- Use any outside resources you want, but **cite your sources**. (If you really want to learn the material, I recommend seriously attempting the problems yourself before looking for outside help.)
- The TA will grade each problem quickly on a three-tier scale:
  - Zero points for a solution that's mostly unreadable or missing.
  - One point for a serious attempt that's not easy to read or is substantially incorrect.
  - Two points for a solution that's clearly readable and nearly or completely correct.

Problems:

1. (Refer to 'Linear dynamical systems' lecture slides.)

   (a) Show that with uniform time step Δt and piecewise constant $p^{\text{chem}}(t)$, the continuous-time battery model
   $$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -\frac{x(t)}{\tau} + p^{\text{chem}}(t).$$
   can be written in discrete time as
   $$x(k+1) = ax(k) + (1-a)\tau p^{\text{chem}}(k),$$
   where $a = e^{-\Delta t/\tau}$.

   (b) In the special case of a battery with no self-dissipation, the continuous-time model simplifies to
   $$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = p^{\text{chem}}(t).$$
   Show that with uniform time step Δt and piecewise constant $p^{\text{chem}}(t)$, a discrete-time version of this model is $x(k+1) = x(k) + \Delta t p^{\text{chem}}(k)$.

2. (Short calculations; take care with units.)

   (a) The charge state of a battery, initially at 80% of its energy capacity, drops to 50% of its energy capacity after 30 days unplugged and unused. What is the battery's self-dissipation time constant?

   (b) Suppose an electric vehicle has an energy intensity of $\alpha = 0.3$ kWh/km and a comparable gasoline vehicle gets $\beta = 25$ miles per gallon. If burning one gallon of gasoline causes $\gamma = 26$ pounds of CO_2 emissions (including upstream emissions associated with oil extraction and processing), what is the break-even CO_2 intensity of electricity μ (in units of g/kWh) at which the two vehicles cause the same CO_2 emissions per unit distance driven? By what percent would the EV reduce CO_2 emissions from driving with the US-average CO_2 intensity of electricity, 345 g/kWh? With the average CO_2 intensity of electricity in your home state or country? What factors not considered here might complicate this analysis?

(c) Suppose someone commutes two miles each way, five days per week, riding an electric bike with an energy intensity of 5 Wh/km. If they work 50 weeks per year and electricity costs 0.15 $/kWh, how much do they spend on bike electricity per year? Compare this to the annual fuel cost from the same commute in an automobile that gets 30 miles per gallon with a fuel price of 3 $/gallon.

3. (Refer to 'Batteries and electric vehicles' lecture slides.) Download the files in the Github repository electric-vehicles. Fill in the missing code from the functions `simulatePolicy1`, `simulatePolicy2`, and `simulatePolicy3`. Given the inputs in the `simulateEV` script, these functions should return trajectories of the EV battery's stored chemical energy and electrical charging power. Show the missing lines of code here. Show the graphs here that `simulateEV` draws in figures 1 through 4.