Homework 3: Buildings

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Directions:

- Students may work individually or in groups, but each student must upload their own solutions to Gradescope by **11:59 PM ET on Wednesday, February 12**.
- Use any outside resources you want, but **cite your sources**. (If you really want to learn the material, I recommend seriously attempting the problems yourself before looking for outside help.)
- The TA will grade each problem or subproblem quickly on a three-tier scale:
  - No credit for a solution that's mostly unreadable or missing.
  - Half credit for a serious attempt that's not easy to read or is substantially incorrect.
  - Full credit for a solution that's clearly readable and nearly or completely correct.

Problems:

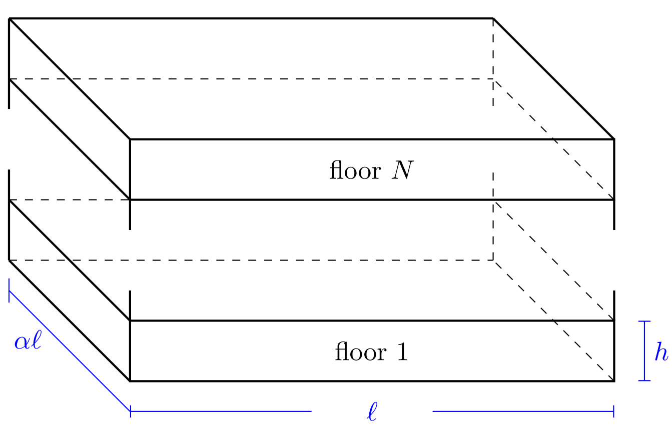
1. (15%) Consider a 1R1C thermal circuit with continuous-time dynamics

$$C\frac{\mathrm{d}T(t)}{\mathrm{d}t} = \frac{\theta(t) - T(t)}{R} + q_c(t) + q_e(t).$$

   Show that with uniform time step Δt and piecewise constant θ, q_c, and q_e, the equivalent discrete-time dynamics are $T(k+1) = aT(k) + (1-a)R\left(q_c(k) + w(k)\right)$ with $a = e^{-\Delta t/(RC)}$ and $w(k) = q_e(k) + \theta(k)/R$.

2. (15%) Suppose a detached single-family house in West Lafayette, Indiana has a heating temperature setpoint of 21 °C, needs no heating when the indoor-outdoor temperature difference is less than 6 °C, and has an 80% efficient natural gas furnace that uses 71 MMBtu of input chemical potential energy per year for space heating. Estimate the thermal resistance between the indoor and outdoor air, in units of °C/kW. *Hint: Download data from degreedays.net.*

3. (Is it cheaper to heat short or tall buildings?) Consider a box-shaped building:



(a) (5%) Write the total vertical surface area A_w and the roof area A_r in terms of the number of stories N, the total floor area A_f (which includes all stories), and (possibly) the height h of each story. Use the approximation $(1+\alpha)/\sqrt{\alpha} \approx 2$.

(b) (10%) The rate of steady-state heat transfer per unit indoor-outdoor temperature difference through the building envelope can be written as

$$\alpha\sqrt{N} + \frac{\beta}{N}.$$

Find expressions for the coefficients α and β in terms of A_f, h, and the thermal transmittances U_w and U_r of the vertical surfaces and roof. Start by writing the rate of steady-state heat transfer through each surface as $UA(T-\theta)$, where $T-\theta$ is the indoor-outdoor temperature difference.

(c) (10%) Suppose $A_f = 200\ \text{m}^2$, $h = 3\ \text{m}$, $U_w = 1.5\ \text{W}/(°\text{C}\cdot\text{m}^2)$, and $U_r = 0.7\ \text{W}/(°\text{C}\cdot\text{m}^2)$. Calculate $\alpha\sqrt{N} + \beta/N$ for $N = 1$, 2, 3, and 4. Based on your calculations, how much less (as a percentage) would you expect to pay to heat a house with the best value of N on your plot, relative to the worst value of N? What factors not considered here might change this analysis?

4. Download the files in the Github repository buildings and open the script `simulate2R2C`.

   (a) (10%) Fill in the missing code in the cell `2R2C system matrices`, which should generate the continuous-time and discrete-time dynamics and input matrices.

   (b) (15%) Fill in the missing code in the function `perfectTrackingControl`, which should implement near-perfect setpoint tracking control (but saturate the HVAC thermal power at equipment capacity limits if necessary).

   (c) (15%) Fill in the missing code in the function `thermostaticControl`, which should implement thermostatic control. Report the cumulative heat demand (in kWh) under each control policy.

   Show the outputs of your code here.

5. (5%, graded for completion, either no credit or full credit.) Write down the names of your project team members and the topic or topics your project will focus on.