Homework 5: Solar energy

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Directions:

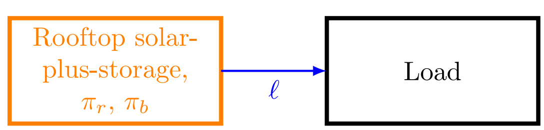
- Students may work individually or in groups, but each student must upload their own solutions to Gradescope by **11:59 PM ET on Sunday, March 2**.
- Use any outside resources you want, but **cite your sources**. (If you really want to learn the material, I recommend seriously attempting the problems yourself before looking for outside help.)
- The TA will grade each problem or subproblem quickly on a three-tier scale:
  - No credit for a solution that's mostly unreadable or missing.
  - Half credit for a serious attempt that's not easy to read or is substantially incorrect.
  - Full credit for a solution that's clearly readable and nearly or completely correct.

Problems:

1. (15%) At what local time will the sun set in West Lafayette on March 8? Show your work. *Hint: What is the sun's elevation angle at sunset?*

2. This question compares the overall capital costs of installing rooftop solar photovoltaics with energy storage ('solar-plus-storage') and utility-scale solar-plus-storage. To explore this question, imagine adding new load to a power grid that currently has just enough generation, transmission, and distribution capacity to meet load at the grid's peak times. Adding new load pushes the grid's peak load beyond its existing capacity by an amount ℓ (W).

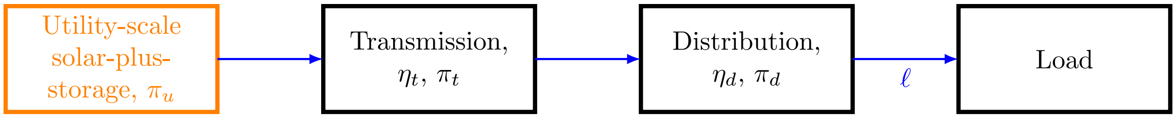
   In the rooftop configuration pictured below, new rooftop solar-plus-storage directly meets the new load ℓ at grid peak times.



   The rooftop configuration costs $\pi_r \ell$, where π_r (\$/W) is the installed cost of rooftop solar-plus-storage, normalized by its capacity to produce power at grid peak times. The rooftop configuration also adds value $\pi_b \ell$ by providing the capability to power potentially vital services during power outages. The price π_b (\$/W) is the value of 1 Watt of emergency backup capacity. The total cost to society of the rooftop configuration is therefore

$$\underbrace{\pi_r \ell}_{\text{solar-plus-storage cost}} - \underbrace{\pi_b \ell}_{\text{blackout value}} = (\pi_r - \pi_b)\ell.$$

   In the utility-scale configuration pictured below, new utility-scale solar-plus-storage indirectly meets the new load ℓ by way of a transmission network and a distribution network.



(a) (10%) Show that the utility-scale configuration costs

$$\left(\frac{\pi_u + \pi_t}{\eta_t} + \pi_d\right)\frac{\ell}{\eta_d},$$

where π_u (\$/W) is the installed cost of utility-scale solar-plus-storage, normalized by the system's capacity to produce power at system peak times, π_t (\$/W) is the price of new transmission capacity, π_d (\$/W) is the price of new distribution capacity, η_t is the transmission network efficiency, and η_d is the distribution network efficiency.

(b) (10%) Find an expression for the break-even rooftop solar-plus-storage price $\pi_r^\star$, defined such that if $\pi_r = \pi_r^\star$, then the costs of the rooftop and utility-scale configurations are equal.

(c) (15%) Assume $\eta_t = 97\%$, $\eta_d = 96\%$, $\pi_t = 0.5$ \$/W, $\pi_d = 1$ \$/W, and $\pi_b = 0.55$ \$/W (roughly the price of a backup generator [~20 kW for ~\$11k]). Plot $\pi_r^\star$ vs. π_u, with π_u ranging from 0 to 5 \$/W. Mark the region where the rooftop configuration costs less than the utility-scale configuration. *(Hint: What does it mean if $\pi_r < \pi_r^\star$?)* What do you take away from this plot? Are there assumptions or input data in this problem that, if changed, would significantly change your take-away(s)? Explain.

3. Download the Matlab or Python files in the Github repository solar and open the script `simulateSolar`.

(a) (15%) Fill in the missing code in the function `solarAngles`, which should calculate the solar elevation and azimuth angles (in degrees) at a given location (specified by the latitude and longitude angles) over a given time span (specified as a `datetime` object). Show your code here.

(b) (15%) Fill in the missing code in the function `surfaceIrradiance`, which should calculate the total, beam, and diffuse irradiance on a surface at a given orientation (specified by its azimuth and tilt angles) given solar angles and irradiance measurements from a standard weather station.

(c) (10%) In the `solar power supply` section of `simulateSolar`, fill in the code to calculate `solarEta`, the temperature-dependent solar array efficiency.

(d) (10%) In the `electricity costs` section of `simulateSolar`, fill in the code to calculate `c3`, the annual electricity cost with reduced net metering.