Solar energy

Purdue ME 597, Distributed Energy Resources

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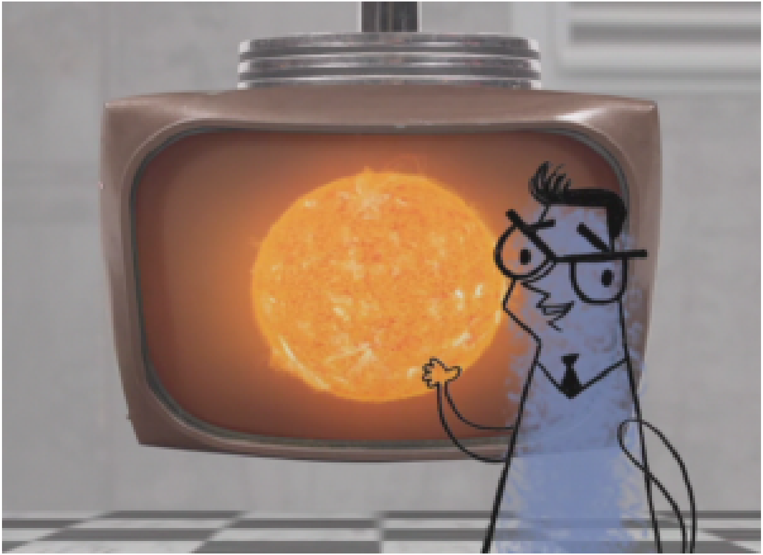
Outline

The sun is (?) a mass of incandescent gas

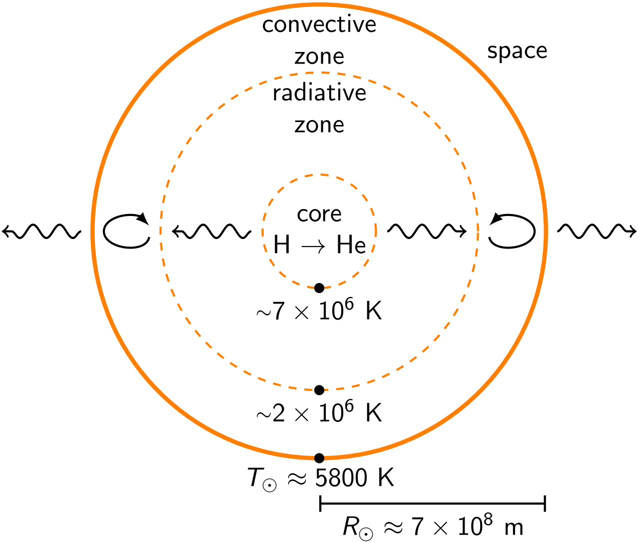
They Might Be Giants (1993): Why Does the Sun Shine?

The sun is a miasma of incandescent **plasma**

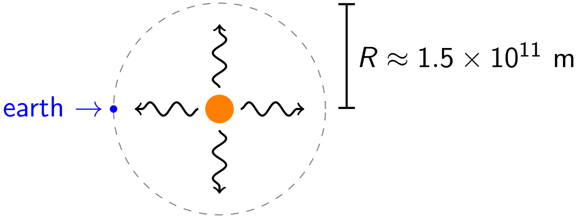


They Might Be Giants (2009): Why Does the Sun Really Shine?

The sun


convective
zone
space
radiative
zone
core
H → He
~7 × 10^6 K
~2 × 10^6 K
T☉ ≈ 5800 K
R☉ ≈ 7 × 10^8 m

Sunlight in space



- as a black body at $T_\odot \approx 5800$ K, sun's surface radiates

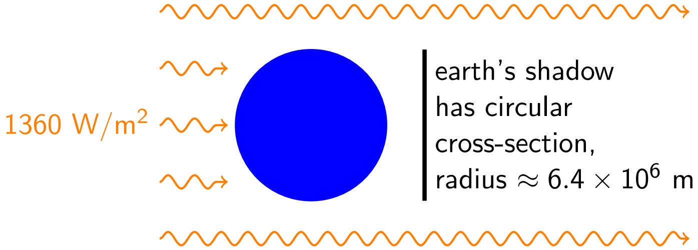
$$\sigma A_\odot T_\odot^4 = 4\sigma\pi R_\odot^2 T_\odot^4 \approx 4 \times 10^{26}\ \mathrm{W}$$

- at earth's distance from sun, irradiance is

$$S_0 := \frac{\sigma A_\odot T_\odot^4}{4\pi R^2} \approx \frac{4 \times 10^{26}\ \mathrm{W}}{4\pi(1.5 \times 10^{11}\ \mathrm{m})^2} \approx 1360\ \mathrm{W/m^2}$$

- **solar constant** S_0 is irradiance at top of earth's atmosphere

How much solar power 'hits' the earth?



- solar power incident on earth's upper atmosphere:

$$(1360 \text{ W/m}^2)\pi(6.4 \times 10^6 \text{ m})^2 \approx 1.7 \times 10^{17} \text{ W}$$

- $\sim$10,000 times humanity's $\sim 1.8 \times 10^{13}$ W used for all purposes

(Almost) all energy on earth is solar energy

- hydro $\leftarrow$ solar (via the water cycle)
- wind $\leftarrow$ solar (via hot air rising)
- biomass $\leftarrow$ solar (via photosynthesis)
- fossil fuels $\leftarrow$ biomass (via rotting underground) $\leftarrow$ solar

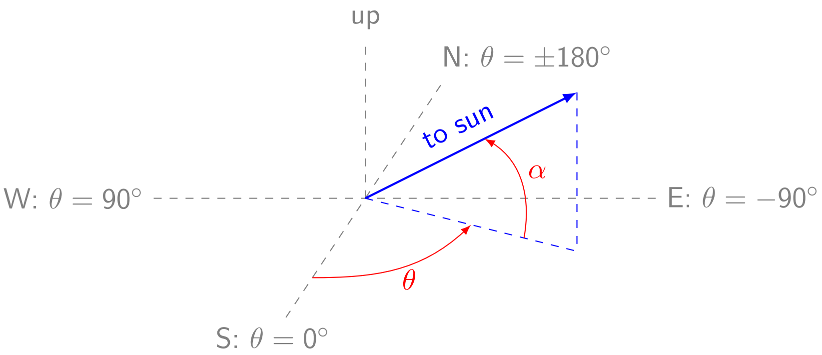
but

- nuclear $\not\leftarrow$ solar
- geothermal $\leftarrow$ nuclear (via reactions underground)

and solar $\leftarrow$ nuclear

so really, all (?) energy on earth is **nuclear** energy

Outline



- origin is at observer on earth's surface
- observer sees sun at **elevation** α and **azimuth** θ
- $\alpha = 0^\circ$ at sunrise/sunset
- $\theta = 0^\circ$ if sun is due south, increases clockwise

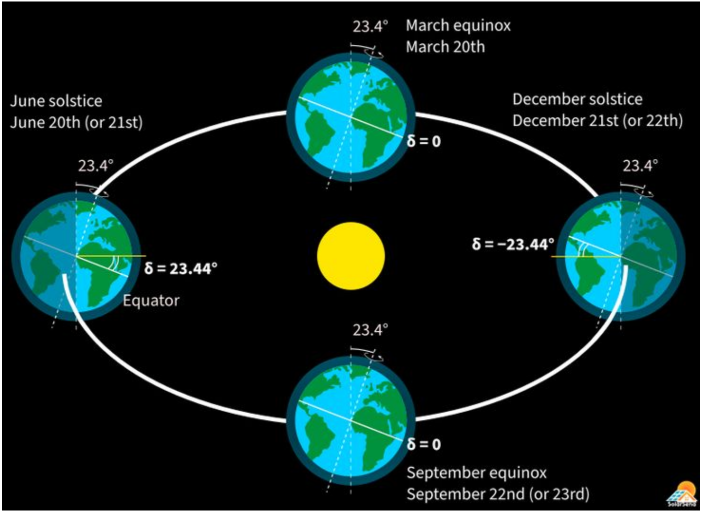
Elevation and azimuth formulas

$$\alpha = 90^\circ - \mathrm{acos}[\sin(\phi_0)\sin(\delta) + \cos(\phi_0)\cos(\delta)\cos(\lambda)]$$
$$\theta = \mathrm{atan2}[\cos(\delta)\sin(\lambda), \sin(\phi_0)\cos(\delta)\cos(\lambda) - \cos(\phi_0)\sin(\delta)]$$

- ϕ_0 is observer's **latitude**
- δ and λ are **declination** and **hour** angles
- acos returns values in $[0^\circ, 90^\circ]$; atan2 in $[-180^\circ, 180^\circ]$

Zhang (2021): A solar azimuth formula based on the subsolar point and atan2

Declination angle



$$\delta \approx 23.45^\circ \sin(\gamma), \;\; \gamma = \frac{360^\circ(d-81)}{365}, \;\; \text{day } \# \; d \text{ is 1 on Jan 1}$$

Hour angle

- the empirical **equation of time**,

$$\tau \approx (0.165\ \text{h})\sin(2\gamma) - (0.126\ \text{h})\cos(\gamma) - (0.025\ \text{h})\sin(\gamma),$$

  converts Greenwich Mean Time t_{gm} to local **solar time** (solar time, $t_{\text{gm}} + \tau$, equals 12 h when sun is highest)

- the hour angle,

$$\lambda = \lambda_0 + (15\ ^\circ/\text{h})(t_{\text{gm}} + \tau - 12\ \text{h}),$$

  is defined such that

  - λ equals observer's **longitude** λ_0 when sun is highest
  - λ increases by 360° every 24 h

Outline

The solar resource

The sun's position in the sky

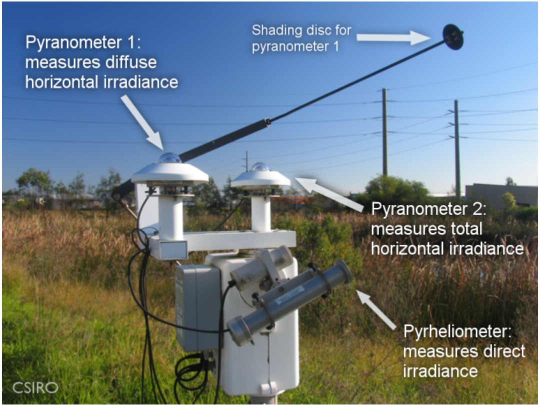
Incident irradiance on surfaces

Solar photovoltaics

Sunlight at earth's surface

- on a clear day, $\sim$75% of S_0 reaches earth's surface
  - $\sim$70% transmitted through atmosphere (**beam**)
  - $\sim$5% scattered to earth by air, dust, water vapor (**diffuse**)
  - $\sim$20% absorbed by atmosphere
  - $\sim$5% scattered back to space
- as cloud cover increases,
  - less of S_0 reaches surface (as little as $\sim$10%)
  - beam % of total surface sunlight falls, diffuse % rises

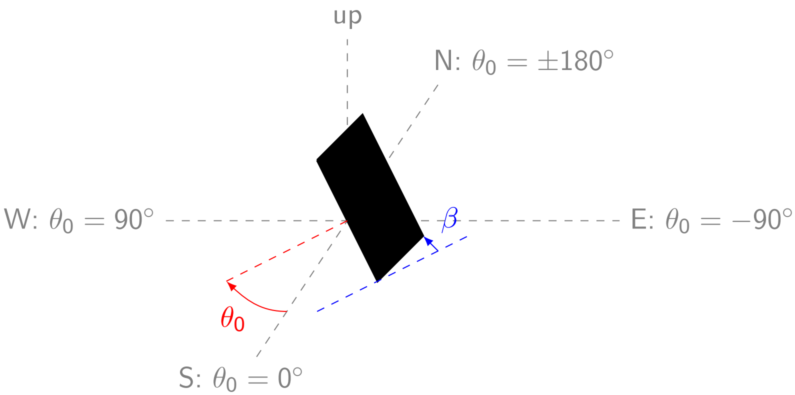
Sunlight measurements



- weather data services often provide one or more of
  - ◇ beam irradiance $S_b^\perp$ on surface $\perp$ to sunbeam
  - ◇ diffuse irradiance S_d^- on horizontal surface
  - ◇ total irradiance S_{tot}^- on horizontal surface
  - ◇ beam irradiance S_b^- on horizontal surface

CSIRO: Perfect day for solar power

Surface tilt and azimuth



- surface **tilt** β and **azimuth** θ_0 define its orientation
- $\beta = 0°$ for horizontal surfaces, 90° for vertical
- θ_0 follows same convention as sun's azimuth θ

Irradiance on arbitrarily oriented surface

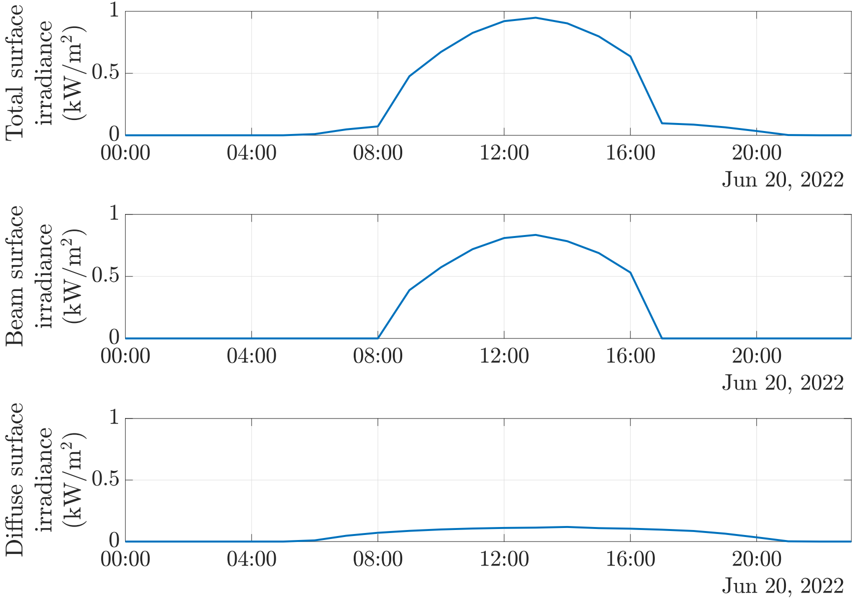
$$S_b = \begin{cases} 0 \text{ if } \alpha < 0^\circ \text{ or } 90^\circ < |\theta - \theta_0| < 270^\circ \\ S_b^\perp[\cos(\alpha)\sin(\beta)\cos(\theta - \theta_0) + \sin(\alpha)\cos(\beta)] \text{ else} \end{cases}$$

$$S_d \approx S_d^-$$

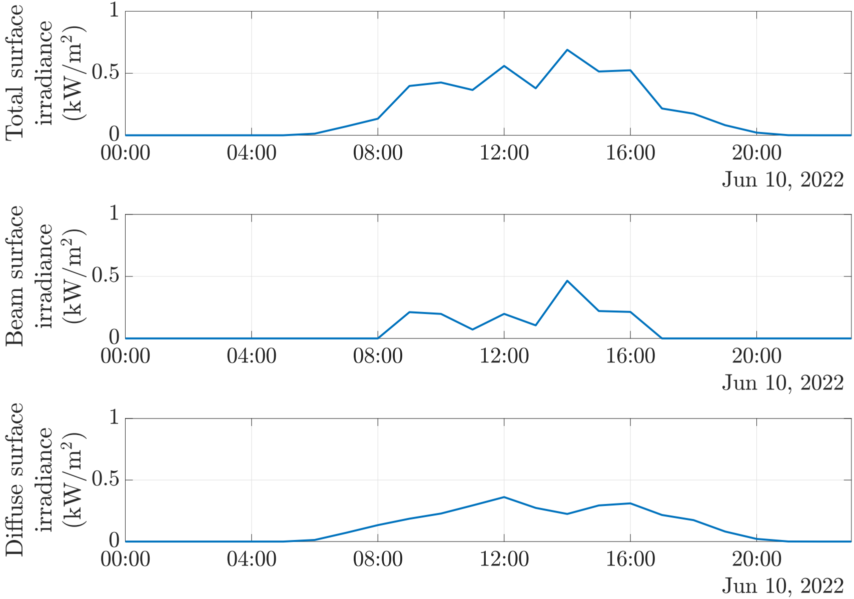
$$S_{tot} = S_b + S_d$$

- S_b and S_{tot} are beam and total irradiance on surface
- $\alpha < 0^\circ$ means sun is down
- $90^\circ < |\theta - \theta_0| < 270^\circ$ means sun is behind surface

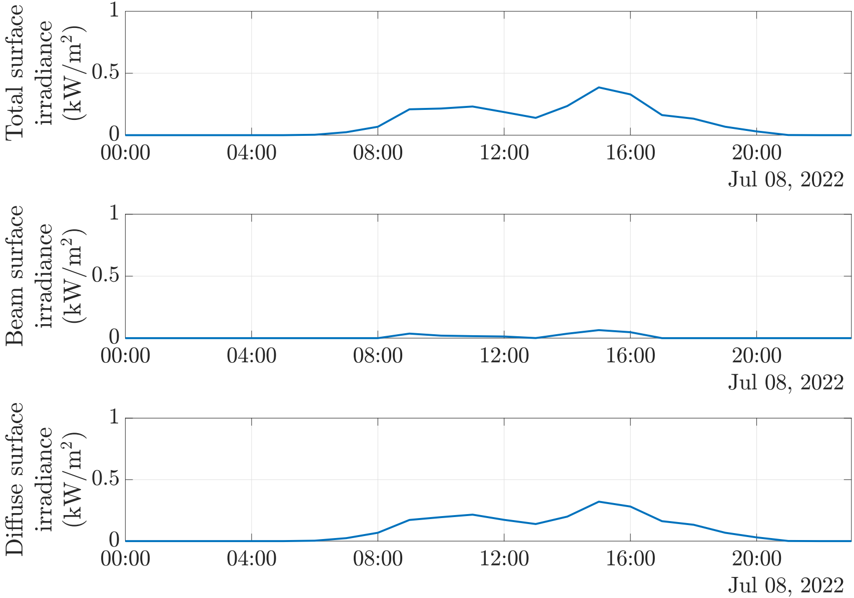
Clear summer day with $\beta = \phi_0$, $\theta_0 = 0^\circ$



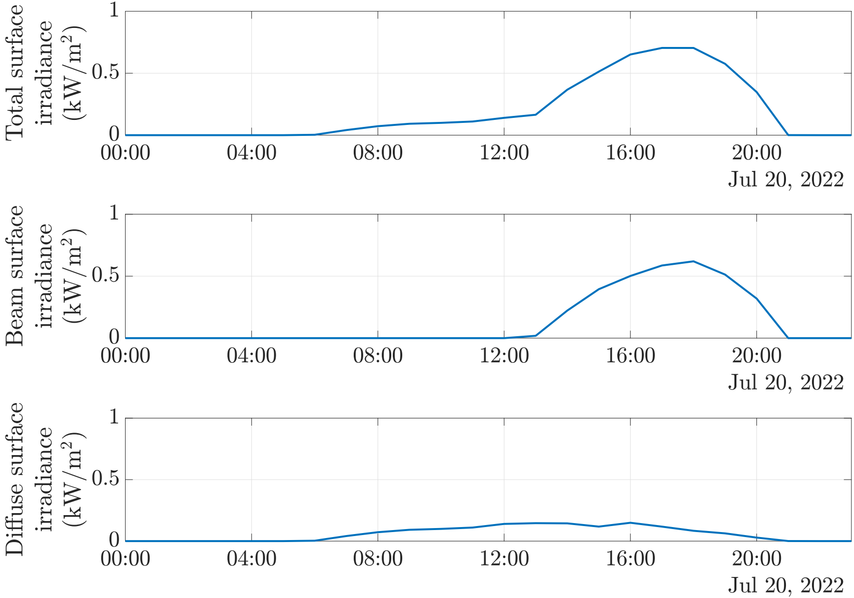
Partly cloudy summer day with $\beta = \phi_0$, $\theta_0 = 0^\circ$



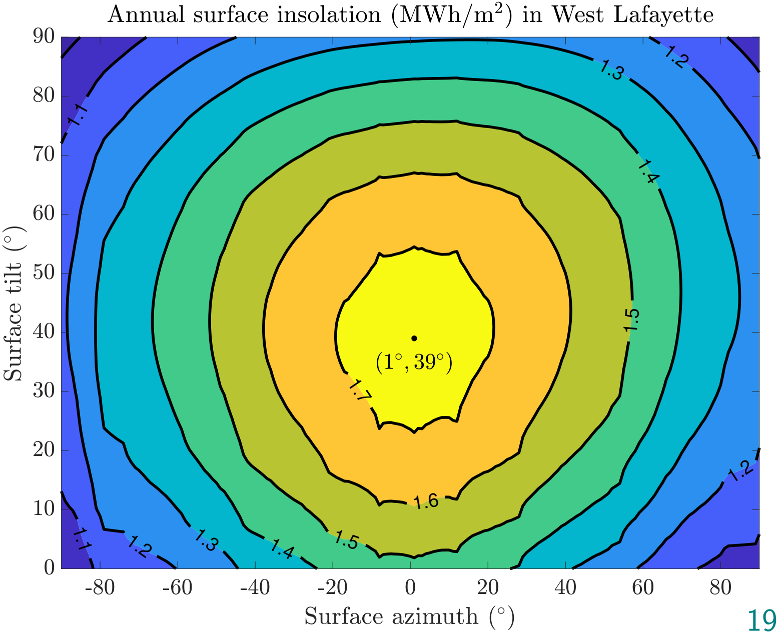
Cloudy summer day with $\beta = \phi_0$, $\theta_0 = 0^\circ$



Clear summer day with $\beta = \theta_0 = 90^\circ$



$\beta \approx \phi_0$ and $\theta_0 \approx 0^\circ$ maximize annual incident energy



Sunlight and windows

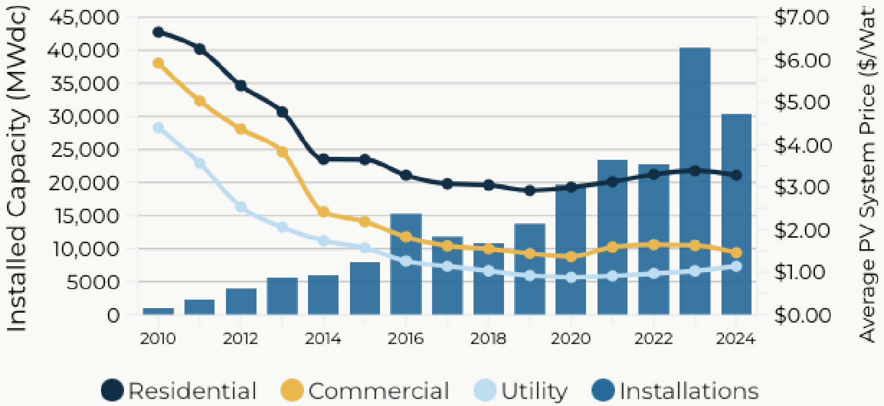
- most windows are vertical: $\beta = 90^\circ$
- with $\sin(\beta) = 1$ and $\cos(\beta) = 0$, incident irradiance reduces to

$$S_{\text{tot}} \approx \begin{cases} S_{\text{d}}^{-} \text{ if } \alpha < 0^\circ \text{ or } 90^\circ < |\theta - \theta_0| < 270^\circ \\ S_{\text{d}}^{-} + S_{\text{b}}^{\perp} \cos(\alpha) \cos(\theta - \theta_0) \text{ else} \end{cases}$$

- a window of area A transmits solar power cAS_{tot}
- $c \in [0, 1]$ is the window's **solar heat gain coefficient** (typically, $c \approx 0.25$ to 0.8)
- can simulate shading (from trees, blinds, ...) by adjusting c

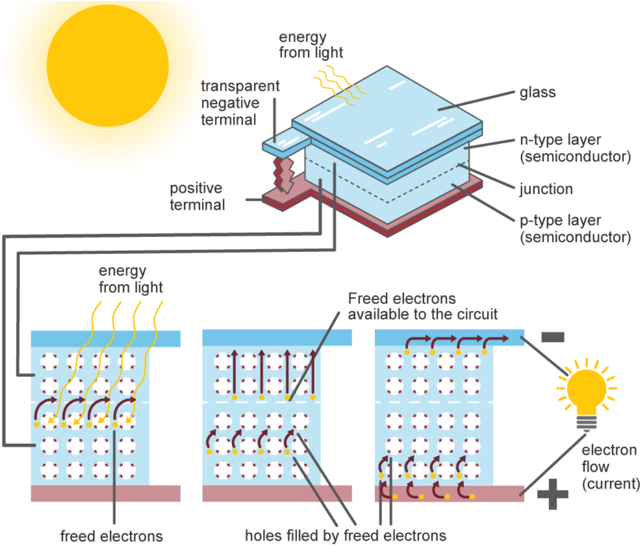
Outline

US costs and installations of solar photovoltaics



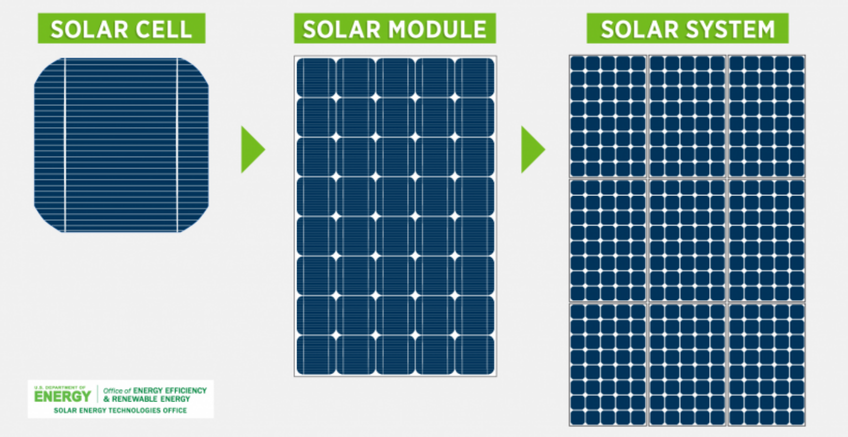
Solar Energy Industries Association (2024): Solar Industry Research Data

Solar photovoltaic cells



US Energy Information Administration: Photovoltaics and electricity

Solar photovoltaic modules and arrays



- typical cell size: ~6"×6", ~0.023 m^2, ~4.2 W peak
- typical module size: ~5'×3', ~1.4 m^2, ~250 W peak

US Department of Energy: PV Cells 101

Solar photovoltaic efficiency

$$\eta = \frac{\text{electric power output}}{\text{radiative power input}}$$

- whole-system efficiency includes cells, inverter, ...
- for typical solar arrays, $\eta \approx 15$ to 20%
- Shockley-Queisser limit: $\eta \leq \sim 33\%$ for any single-junction cell

Efficiency and temperature

- efficiency scales ~linearly with cell temperature T_c

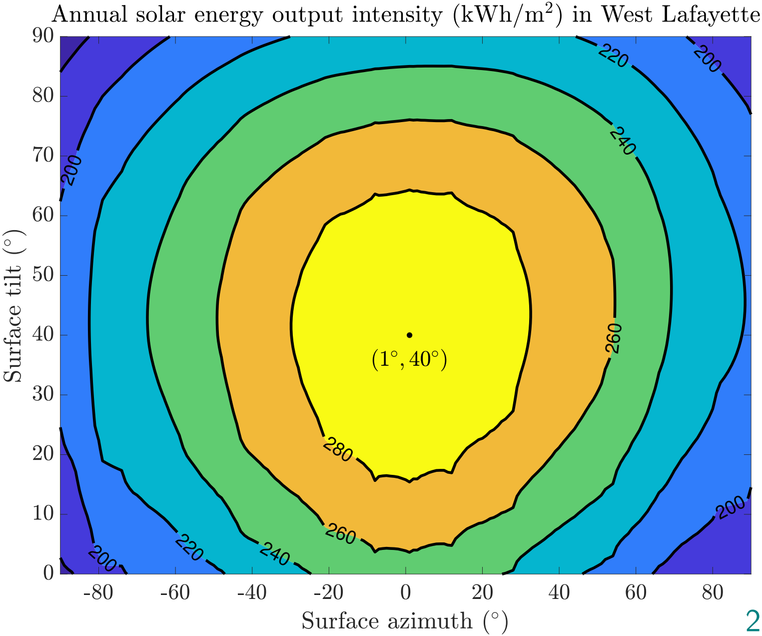
$$\eta \approx \tilde{\eta} \left(1 - \frac{T_c - \tilde{T}}{T_0 - \tilde{T}} \right)$$

- $\tilde{\eta}$ is efficiency at rated cell temperature $\tilde{T} \approx 25\ ^\circ\mathrm{C}$
- $T_0 \approx 270\ ^\circ\mathrm{C}$ is cell temperature at which generation stops
- cell temperature scales ~linearly with outdoor temperature T_a

$$T_c \approx T_a + (35\ ^\circ\mathrm{C m^2/kW}) S_{\mathrm{tot}}$$

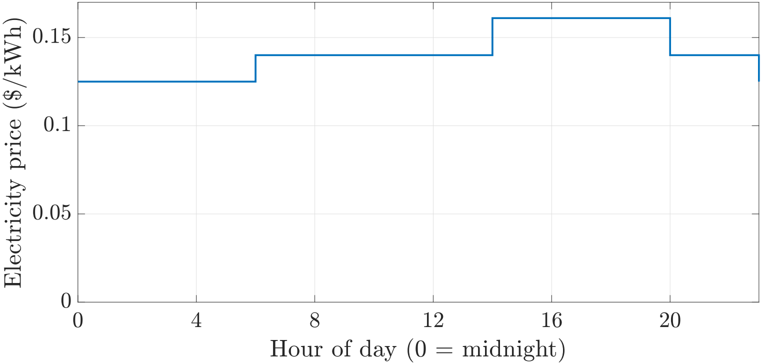
Dubey (2012): Temperature dependent photovoltaic efficiency; PV Education

$\beta \approx \phi_0$ and $\theta_0 \approx 0^\circ$ maximize annual energy output

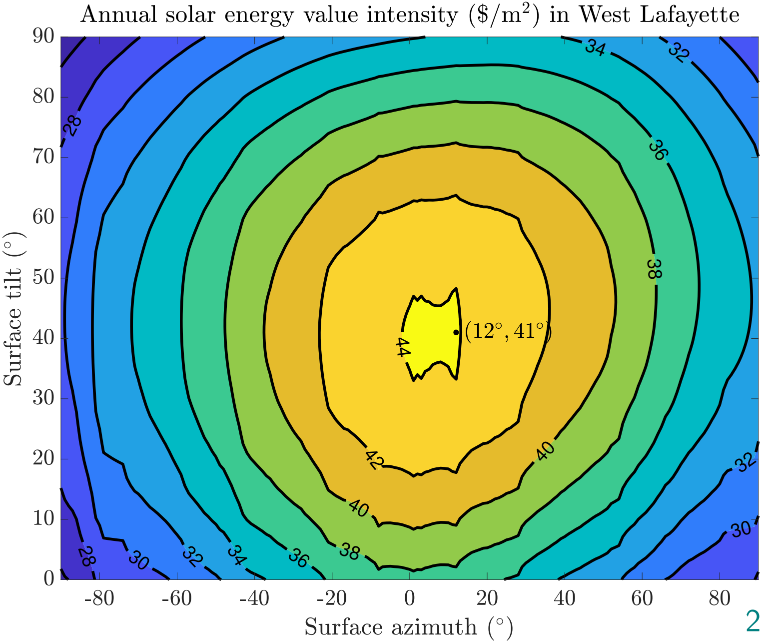


Time-varying electricity prices

- most people buy electricity at a constant price
- some see 'time-of-use' prices with 2 or 3 tiers
- a few see hourly prices tied to wholesale markets



Time-of-use pricing shifts optimal panel orientation west



Net metering

- sometimes, rooftop solar supply exceeds building demand
- some utilities buy excess power at their electricity sale price
- others pay a lower price; some pay nothing at all