Thermal storage and water heaters

Purdue ME 597, Distributed Energy Resources

Kevin J. Kircher

Outline

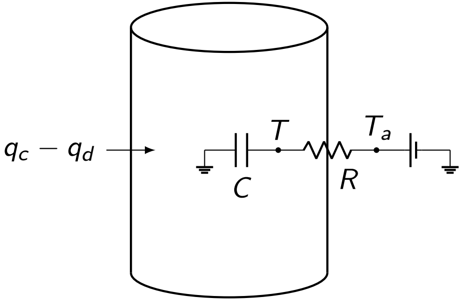
Three types of thermal storage

Domestic hot water use

Resistance water heaters

Heat-pump water heaters

Lumped sensible thermal storage



- C (kWh/°C) is thermal capacitance of storage medium
- R (°C/kW) is thermal resistance of tank wall
- q_c (kW) is charging thermal power
- q_d (kW) is discharging thermal power
- assumption: temperature is spatially uniform inside tank

Lumped sensible thermal storage (continued)

- suppose tank temperature satisfies $T \in [\underline{T}, \overline{T}]$
- choose an arbitrary reference temperature ('datum') T_{LS}
- define stored energy x (kWh) as zero when $T = T_{\text{LS}}$:

$$x = C(T - T_{\text{LS}}) \in [C(\underline{T} - T_{\text{LS}}), C(\overline{T} - T_{\text{LS}})]$$

- from conservation of energy,

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= \frac{T_a - T}{R} + q_c - q_d \\ &= \frac{T_a - (T_{\text{LS}} + x/C)}{R} + q_c - q_d \\ &= -\frac{x}{RC} + q_c + w, \text{ where } w = \frac{T_a - T_{\text{LS}}}{R} - q_d \end{aligned}$$

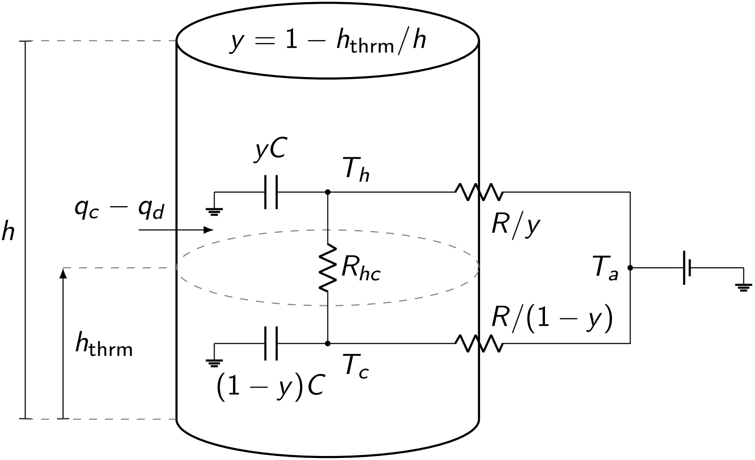
Lumped sensible thermal storage in a 'box of rocks'

Green Energy Times: Electric Thermal Storage

Lumped sensible thermal storage in a ‘box of rocks’

Steffes: Comfort Plus Forced Air

Stratified sensible thermal storage



- hot column sits on cold column, separated by thermocline
- T_h and T_c are constant; charging moves thermocline down

Stratified sensible thermal storage (continued)

- define column energies relative to (arbitrary) T_{SS}:

$$x_h = yC(T_h - T_{\mathrm{SS}}), \; x_c = (1-y)C(T_c - T_{\mathrm{SS}})$$

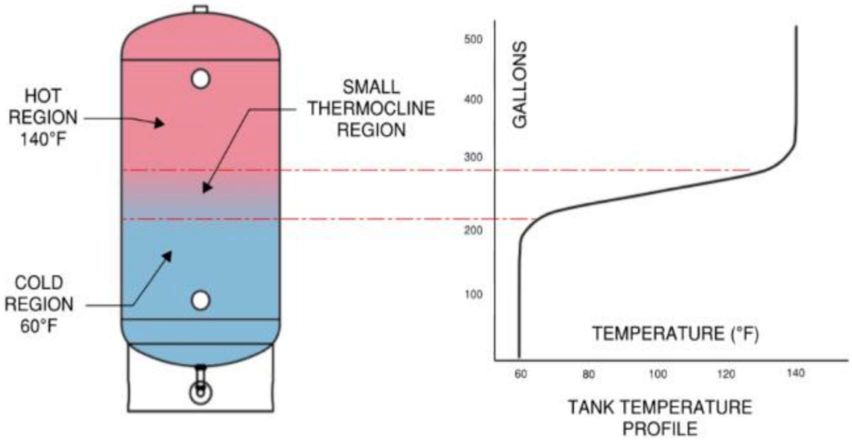
- then total energy is

$$\begin{aligned} x = x_h + x_c = \cdots &= C[y(T_h - T_c) + T_c - T_{\mathrm{SS}}] \\ &\in [C(T_c - T_{\mathrm{SS}}), C(T_h - T_{\mathrm{SS}})] \end{aligned}$$

- from conservation of energy,

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= \frac{(T_a - T_h)y}{R} + \frac{(T_a - T_c)(1-y)}{R} + q_c - q_d \\ &= \cdots = -\frac{x}{RC} + q_c + w \text{ where } w = \frac{T_a - T_{\mathrm{SS}}}{R} - q_d \end{aligned}$$

Stratified sensible thermal storage in hot water



Bonneville Power Administration (2022): Improving thermal energy storage to reduce installation costs for heat pump water heating systems

Latent thermal storage

- tank contains M kg of material that freezes and melts
- latent heat of fusion L (kWh/kg), melting point T_m (°C)
- liquid mass $m_\ell \in [0, M]$ (kg) increases as tank charges
- define energy relative to (arbitrary) T_L:

$$x = C(T_m - T_\mathrm{L}) + Lm_\ell \in [C(T_m - T_\mathrm{L}), C(T_m - T_\mathrm{L}) + LM]$$

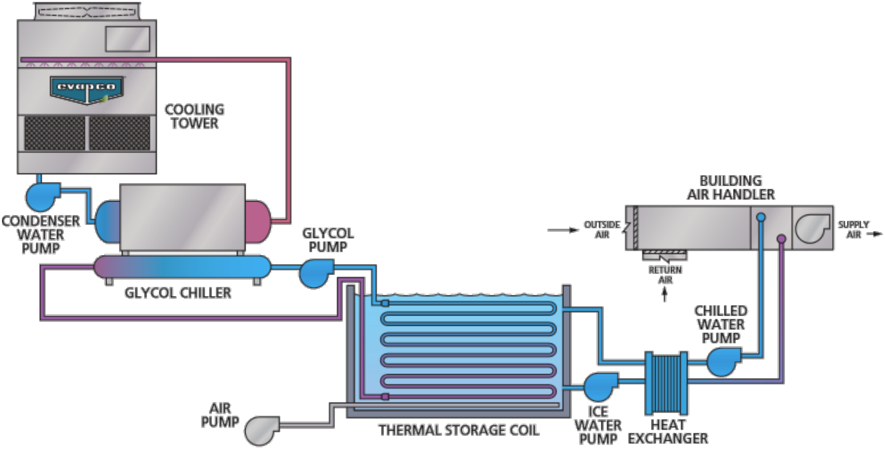
- from conservation of energy,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = q_c + w \text{ where } w = \frac{T_a - T_m}{R} - q_d$$

Latent thermal storage in ice

Evapco: Extra-Pak Ice Coils

Latent thermal storage in ice



Evapco: Extra-Pak Ice Coils

Summary

- all three storage types follow 'heat battery' model:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{x}{\tau} + q_c + w, \ \underline{x} \le x \le \overline{x}$$

- in discrete time with $a = e^{-\Delta t/\tau}$,

$$x(k+1) = \begin{cases} ax(k) + (1-a)\tau(q_c(k) + w(k)) & \text{if } \tau < \infty \\ x(k) + \Delta t(q_c(k) + w(k)) & \text{if } \tau = \infty \end{cases}$$

	Lumped sensible	Stratified sensible	Latent
x	$C(T - T_{\mathrm{LS}})$	$C[y(T_h - T_c) + T_c - T_{\mathrm{SS}}]$	$C(T_m - T_{\mathrm{L}}) + Lm_\ell$
$\overline{x}$	$C(\overline{T} - T_{\mathrm{LS}})$	$C(T_h - T_{\mathrm{SS}})$	$C(T_m - T_{\mathrm{L}}) + LM$
$\underline{x}$	$C(\underline{T} - T_{\mathrm{LS}})$	$C(T_c - T_{\mathrm{SS}})$	$C(T_m - T_{\mathrm{L}})$
τ	RC	RC	∞
w	$(T_a - T_{\mathrm{LS}})/R - q_d$	$(T_a - T_{\mathrm{SS}})/R - q_d$	$(T_a - T_m)/R - q_d$

One model can capture switches between storage types

- example: suppose tank goes stratified → lumped as it heats
- goal: choose T_{LS}, T_{SS}, $\underline{T}$ so model switches smoothly
- to get $w_{\mathrm{LS}} = w_{\mathrm{SS}}$, need $T_{\mathrm{LS}} = T_{\mathrm{SS}}$
  (might as well choose $T_{\mathrm{LS}} = T_{\mathrm{SS}} = T_a$ so $w_{\mathrm{LS}} = w_{\mathrm{SS}} = -q_d$)
- to get $\underline{x}_{\mathrm{LS}} = \overline{x}_{\mathrm{SS}}$, need $\underline{T} = T_h$
- with these choices, one model captures full operating range:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= -\frac{x}{\tau} + q_c - q_d \\ C(T_c - T_a) &\le x \le C(\overline{T} - T_a) \end{aligned}$$

- stratified ↔ lumped switch happens when $x = C(T_h - T_a)$

Charging and discharging efficiencies

- discharging typically takes little energy (just a pump or fan)
- charging typically uses a heat pump or resistance heat
- heat pump COPs depend on tank and ambient temperatures
- for resistance, COP ≈ 1

Typical parameter values

- thermal capacitance is $C = \rho c V$
- thermal resistance is $R = 1/(UA)$
- typically, $U \approx 0.0005$ to 0.001 kW/(°C m^2)
- for liquid water at 50 °C,
  - ◇ $\rho = 988$ kg/m^3
  - ◇ $c = 4.17$ kJ/(°C kg) $= 0.00116$ kWh/(°C kg)
  - ◇ $\rho c = 1.15$ kWh/(°C m^3)
  - ◇ $L = 334$ kJ/kg $= 0.09$ kWh/kg
- for a cylinder with radius r and height h,
  - ◇ $V = \pi r^2 h$
  - ◇ $A = 2\pi r h + 2\pi r^2 = 2V(1/r + 1/h)$

How much energy does a domestic water heater store?

- stratified sensible thermal storage capacity:

$$\overline{x} - \underline{x} = C(T_h - T_c)$$

- typical $V \approx 0.2$ to 0.3 m^3, so $C \approx 0.25$ to 0.35 kWh/°C
- typically, $T_c \approx 15$ °C
- below ∼49 °C, legionella bacteria can grow
- above ∼60 °C, water can scald
- with $T_h \approx 51$ °C, energy capacity ≈ 9 to 13 kWh

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Three types of thermal storage

Domestic hot water use

Resistance water heaters

Heat-pump water heaters

Typical hot water volume draws

- typical household draws ∼$(16n + 4)$ gal of hot water per day
  - ◇ ∼$16n$ gal for one shower or bath apiece from n occupants
  - ◇ ∼4 gal for everything else (like washing dishes and clothes)
- typical shower lasts ∼8 min, draws volumetric flow rate

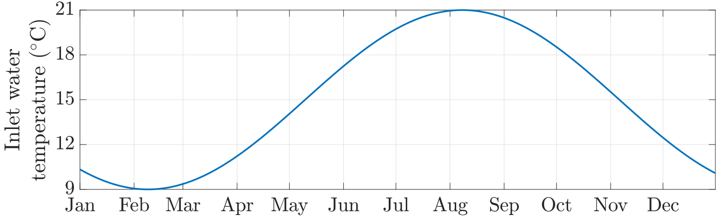
$$\dot{V} \approx 2 \text{ gal/min} = 0.45 \text{ m}^3/\text{h}$$

Florida Solar Energy Center (2015): Estimating daily domestic hot water use in North American homes

Typical hot water thermal power draws

- thermal power associated with $\dot{V}$ is $\rho c \dot{V}(T - T_{\text{in}})$
- inlet water temperature on day d of the year ($d = 1$ on Jan 1):

$$T_{\text{in}} \approx 15\ {}^\circ\text{C} + (6\ {}^\circ\text{C})\sin(d - 130)$$



- with $T \approx 51\ {}^\circ\text{C}$, $T_{\text{in}} \approx 15\ {}^\circ\text{C}$, typical shower draws

$$\sim \left(1.15\ \text{kWh}/({}^\circ\text{C m}^3)\right)\left(0.45\ \text{m}^3/\text{h}\right)(36^\circ\text{C}) \approx 19\ \text{kW}$$

Hendron (2010): Building America house simulation protocols.

Most heat withdrawals do not depend on tank temperature

- typical shower or bath has constant mixed water flow $\dot{m}^\star$
- typical bather wants constant mixed water temperature $T^\star$
- so

$$\frac{\overbrace{\dot{m}T + (\dot{m}^\star - \dot{m})T_{\text{in}}}^{\text{mixed water temperature}}}{\dot{m}^\star} = T^\star$$

$$\iff \dot{m}(T - T_{\text{in}}) + \dot{m}^\star T_{\text{in}} = \dot{m}^\star T^\star$$

$$\iff \underbrace{\dot{m}c(T - T_{\text{in}})}_{q_d} = \underbrace{\dot{m}^\star c(T^\star - T_{\text{in}})}_{\text{independent of } T}$$

$\implies$ heat withdrawal q_d is independent of tank temperature T
(if T decreases, bather can mix in more hot water, less cold)

Simulating hot water draws

- showers and baths dominate hot water use
- so generate *n* showers per day, each at ∼19 kW for ∼10 min
- and place them at random plausible times

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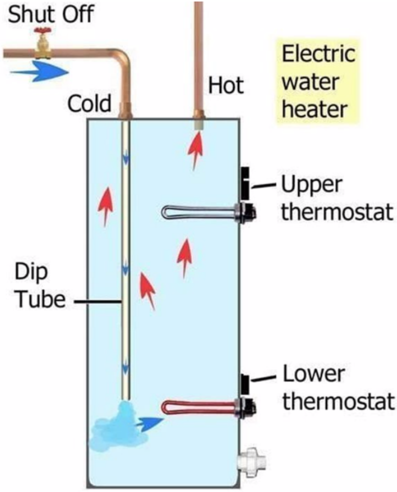
Electric resistance water heaters

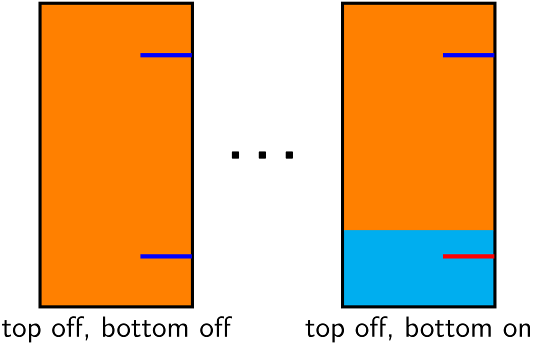


- typically designed to maintain stratification
- hot water drawn from top, cold added to bottom

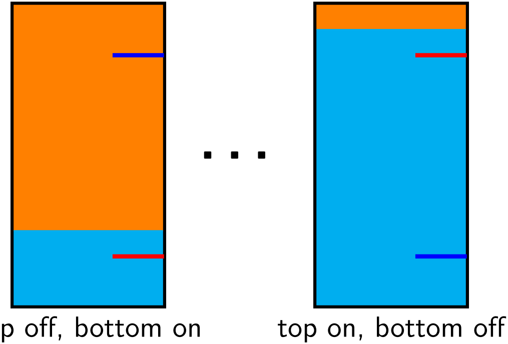
Resistance-only water heater controls

- water heater controls vary by manufacturer
- one approach:
  - ◇ one resistor near bottom, one near top
  - ◇ each resistor has a water temperature sensor
  - ◇ run bottom resistor if bottom temperature drops below setpoint
  - ◇ switch to top resistor if top temperature drops below setpoint
  - ◇ when top temperature returns to setpoint, switch to bottom
  - ◇ when bottom temperature returns to setpoint, turn bottom off

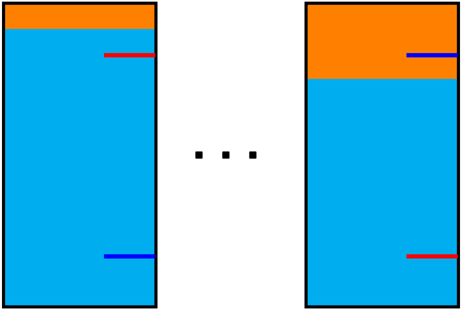
Resistance-only water heater controls (continued)



Resistance-only water heater controls (continued)



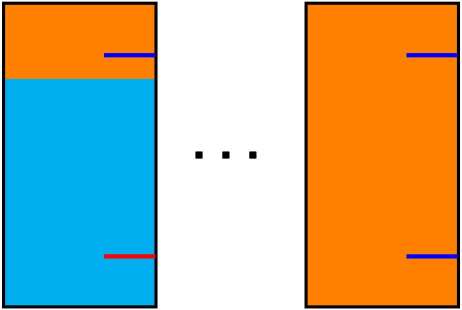
Resistance-only water heater controls (continued)



top on, bottom off

top off, bottom on

Resistance-only water heater controls (continued)



top on, bottom on

top off, bottom off

Simulating resistance-only water heater controls

- anytime tank is not fully charged, exactly one resistor runs

$\Longrightarrow$ model:

  - ⋄ try to place $x(k+1) = ax(k) + (1-a)\tau(q_c(k) + w(k))$ at $\overline{x}$
  - ⋄ but saturate $q_c(k)$ at capacity limits $[0, \overline{p}_r]$

$$\begin{aligned} q_c(k) &= \max\left\{0, \min\left\{\overline{p}_r, \frac{\overline{x} - ax(k)}{(1-a)\tau} - w(k)\right\}\right\} \\ p(k) &= q_c(k) \end{aligned}$$

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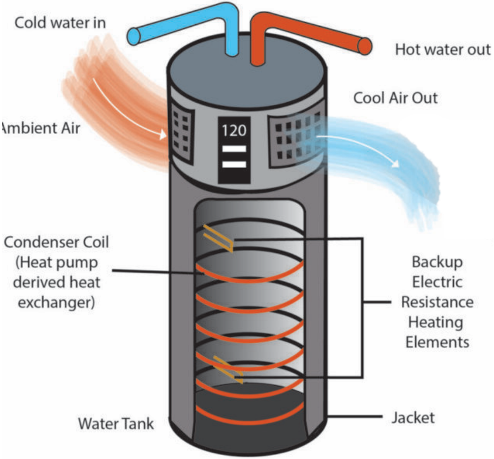
Heat-pump water heaters (and hybrids)



- most HPWHs are hybrids with resistance backup
- heat pump COP depends on temperatures of water, air

TRC Solutions: Heat pump water heater technology

Simulating heat-pump-only water heater controls

- anytime tank is not fully charged, heat pump runs

$\Longrightarrow$ try to place $x(k+1)$ at $\overline{x}$

- but saturate $q_c(k)$ at capacity limits $[0, \eta(k)\overline{p}]$

$$\begin{aligned} q_c(k) &= \max\left\{0, \min\left\{\eta(k)\overline{p}, \frac{\overline{x} - ax(k)}{(1-a)\tau} - w(k)\right\}\right\} \\ p(k) &= q_c(k)/\eta(k) \end{aligned}$$

Hybrid water heater controls

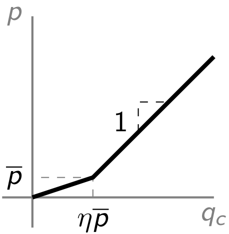
- anytime tank is not fully charged, heat pump runs
- if charge drops below a threshold x_r, resistor also runs

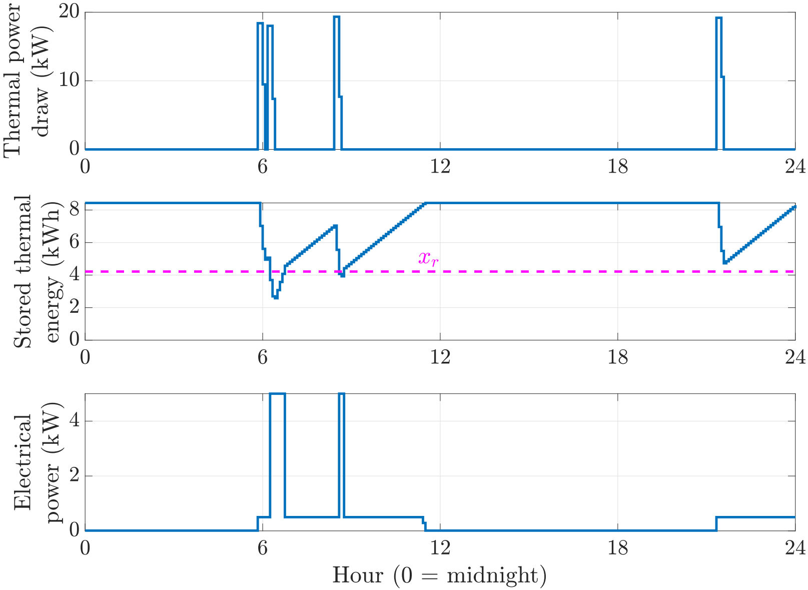
$\Longrightarrow$ try to place $x(k+1)$ at $\overline{x}$, but

  - ◇ if $x(k) \geq x_r$, same as heat-pump-only case
  - ◇ if $x(k) < x_r$, saturate $q_c(k)$ at combined limits $[0, \eta(k)\overline{p} + \overline{p}_r]$

$$q_c(k) = \max\left\{0, \min\left\{\eta(k)\overline{p} + \overline{p}_r, \frac{\overline{x} - ax(k)}{(1-a)\tau} - w(k)\right\}\right\}$$

$$p(k) = \max\left\{\frac{q_c(k)}{\eta(k)}, q_c(k) + (1-\eta(k))\overline{p}\right\}$$





4 occupants, 50 gal (0.19 m^3) hybrid HPWH