Modeling summary: $\sim$Everything's a battery

Purdue ME 597, Distributed Energy Resources

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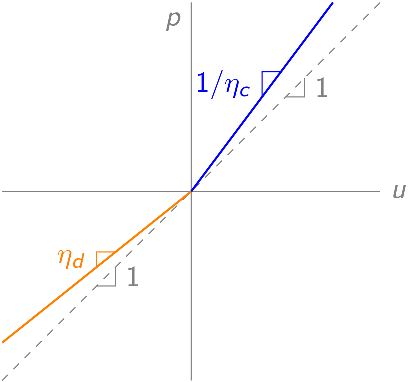
Outline

Stationary battery: Definitions

- state $x(k)$ (kWh): stored chemical energy
- action $u(k)$ (kW): chemical charging power
- output $p(k)$ (kW): electrical charging power
- chemical energy capacities $\underline{x}$, $\overline{x}$ (kWh)
- chemical power capacities $\underline{u} = -\overline{p}_d/\eta_d$, $\overline{u} = \eta_c\overline{p}_c$ (kW)
- self-dissipation time constant τ (h)
- dynamics parameters $a = e^{-\Delta t/\tau}$, $b = (1-a)\tau$

Stationary battery: Model

- dynamics: $x(k+1) = ax(k) + bu(k)$
- state constraints: $\underline{x} \leq x(k) \leq \overline{x}$
- action constraints: $\underline{u} \leq u(k) \leq \overline{u}$
- output equation: $p(k) = \max\{\eta_d u(k), u(k)/\eta_c\}$



like stationary battery, but with

- chemical power capacities

$$(\underline{u}(k), \overline{u}(k)) = \begin{cases} (-\overline{p}_d/\eta_d, \eta_c \overline{p}_c) & \text{if EV is plugged in at time } k \\ (0, 0) & \text{otherwise} \end{cases}$$

  ($\overline{p}_d = 0$ for EVs with one-way charging)

- disturbance $w(k) = -\frac{\alpha(k) d(k)}{\Delta t}$

Electric vehicle: Model

like stationary battery, but with

- dynamics $x(k+1) = ax(k) + b(u(k) + w(k))$
- time-varying action constraints:

$$\underline{u}(k) \leq u(k) \leq \overline{u}(k)$$

Outline

Indoor air/shallow thermal mass: Definitions

- state $x(k) = C(T(k) - \hat{T}(k))$: stored thermal energy
- action $u(k) = q_c(k) - \hat{q}_c(k)$: thermal power perturbation
- output $p(k) = \frac{q_c(k)}{\eta(k)}$: electrical charging power
- thermal energy capacities:

$$\underline{x}(k) = C(\underline{T}(k) - \hat{T}(k)),\ \overline{x}(k) = C(\overline{T}(k) - \hat{T}(k))$$

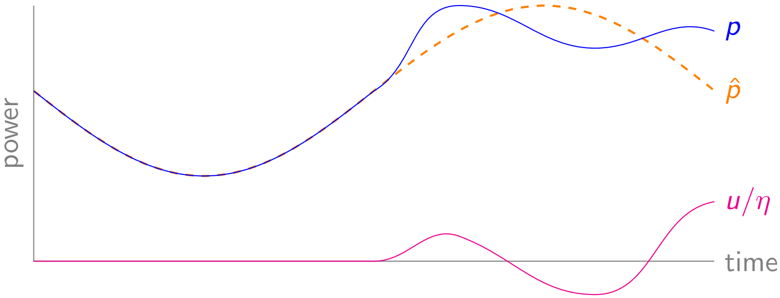
- thermal power capacities:

$$\underline{u}(k) = \underline{q}_c(k) - \hat{q}_c(k),\ \overline{u}(k) = \overline{q}_c(k) - \hat{q}_c(k)$$

- self-dissipation time constant $\tau = RC$
- dynamics parameters $a = e^{-\Delta t/\tau}$, $b = (1 - a)\tau$

Indoor air/shallow thermal mass: Model

- dynamics: $x(k+1) = ax(k) + bu(k)$
- state constraints: $\underline{x}(k) \leq x(k) \leq \overline{x}(k)$
- action constraints: $\underline{u}(k) \leq u(k) \leq \overline{u}(k)$
- output equation: $p(k) = \hat{p}(k) + u(k)/\eta(k)$



Deep thermal mass: Definitions

- state $x(k) = C_m(T_m(k) - \hat{T}_m(k))$: stored thermal energy
- action $u(k) = \frac{T(k) - \hat{T}(k)}{R_m}$: thermal power perturbation
- output $p(k) = \frac{q_c(k)}{\eta(k)}$: electrical charging power
- thermal energy capacities:

$$\underline{x}(k) = C_m(\underline{T}_m(k) - \hat{T}_m(k)),\ \overline{x}(k) = C_m(\overline{T}_m(k) - \hat{T}_m(k))$$

- thermal power capacities:

$$\underline{u}(k) = \frac{\underline{T}(k) - \hat{T}(k)}{R_m},\ \overline{u}(k) = \frac{\overline{T}(k) - \hat{T}(k)}{R_m}$$

- self-dissipation time constant $\tau = R_m C_m$
- dynamics parameters $a = e^{-\Delta t/\tau}$, $b = (1 - a)\tau$

Deep thermal mass: Model

like indoor air, but with state-dependent output equation

$$p(k) = \hat{p}(k) + \frac{1}{\eta(k)} \left[\left(1 + \frac{R_m}{R_{\text{out}}} \right) u(k) - \frac{x(k)}{\tau} \right]$$

Deep thermal mass output equation

- in an approximate 2R2C model with $\mathrm{d}T(t)/\mathrm{d}t \approx 0$,

$$q_c(k) = \frac{T(k) - T_m(k)}{R_m} + \frac{T(k) - T_{\text{out}}(k)}{R_{\text{out}}} - q_e(k) \qquad (1)$$

- similarly, under baseline operation,

$$\hat{q}_c(k) = \frac{\hat{T}(k) - \hat{T}_m(k)}{R_m} + \frac{\hat{T}(k) - T_{\text{out}}(k)}{R_{\text{out}}} - q_e(k) \qquad (2)$$

- subtracting (2) from (1) gives

$$\begin{aligned} q_c(k) - \hat{q}_c(k) &= \left(\frac{1}{R_m} + \frac{1}{R_{\text{out}}}\right)(T(k) - \hat{T}(k)) - \frac{T_m(k) - \hat{T}_m(k)}{R_m} \\ &= \left(\frac{1}{R_m} + \frac{1}{R_{\text{out}}}\right) R_m u(k) - \frac{x(k)}{R_m C_m} \\ &= \left(1 + \frac{R_m}{R_{\text{out}}}\right) u(k) - \frac{x(k)}{\tau} \end{aligned}$$

Outline

Lumped sensible thermal storage: Definitions

- state $x(k) = C(T(k) - T_{\mathrm{LS}})$: stored thermal energy
- action $u(k) = q_c(k)$: thermal charging power
- output $p(k)$: electrical charging power
- thermal energy capacities:

$$\underline{x}(k) = C(\underline{T}(k) - T_{\mathrm{LS}}),\ \overline{x}(k) = C(\overline{T}(k) - T_{\mathrm{LS}})$$

- thermal power capacities:

$$\underline{u}(k) = \underline{q}_c(k),\ \overline{u}(k) = \overline{q}_c(k)$$

- self-dissipation time constant $\tau = RC$
- dynamics parameters $a = e^{-\Delta t/\tau}$, $b = (1 - a)\tau$
- disturbance $w(k) = (T_a(k) - T_{\mathrm{LS}})/R - q_d(k)$

Lumped sensible thermal storage: Model

- dynamics: $x(k+1) = ax(k) + b(u(k) + w(k))$
- state constraints: $\underline{x}(k) \leq x(k) \leq \overline{x}(k)$
- action constraints: $\underline{u}(k) \leq u(k) \leq \overline{u}(k)$
- output equation: $p(k) = u(k)/\eta(k)$

Stratified sensible thermal storage: Definitions

- state $x(k) = C[y(k)(T_h - T_c) + T_c - T_{\mathrm{SS}}]$
- action $u(k) = q_c(k)$: thermal charging power
- output $p(k)$: electrical charging power
- thermal energy capacities:

$$\underline{x} = C(T_c - T_{\mathrm{SS}}),\ \overline{x} = C(T_h - T_{\mathrm{SS}})$$

- thermal power capacities:

$$\underline{u}(k) = \underline{q}_c(k),\ \overline{u}(k) = \overline{q}_c(k)$$

- self-dissipation time constant $\tau = RC$
- dynamics parameters $a = e^{-\Delta t/\tau}$, $b = (1-a)\tau$
- disturbance $w(k) = (T_a(k) - T_{\mathrm{SS}})/R - q_d(k)$

Stratified sensible thermal storage: Model

like lumped sensible, but with time-invariant state constraints:

$$\underline{x} \leq x(k) \leq \overline{x}$$

Latent thermal storage: Definitions

- state $x(k) = C(T_m - T_{\mathsf{L}}) + Lm_\ell(k)$
- action $u(k) = q_c(k)$: thermal charging power
- output $p(k)$: electrical charging power
- thermal energy capacities:

$$\underline{x} = C(T_m - T_{\mathsf{L}}),\ \overline{x} = C(T_m - T_{\mathsf{L}}) + LM$$

- thermal power capacities:

$$\underline{u}(k) = \underline{q}_c(k),\ \overline{u}(k) = \overline{q}_c(k)$$

- self-dissipation time constant $\tau = \infty$
- dynamics parameters $a = 1$, $b = \Delta t$
- disturbance $w(k) = (T_a(k) - T_m)/R - q_d(k)$

Latent thermal storage dynamics parameters

- for latent thermal storage, $a = 1$ and $b = \Delta t$
- these are limits of $a = e^{-\Delta t/\tau}$ and $b = (1 - a)\tau$ as $\tau \to \infty$:
  - $e^{-\Delta t/\tau} \to 1$ as $\tau \to \infty$
  - letting $\alpha = 1/\tau$ and using L'Hôpital's rule,

$$\begin{aligned}
\lim_{\tau \to \infty} \left(1 - e^{-\Delta t/\tau}\right)\tau &= \lim_{\alpha \to 0} \frac{1 - e^{-\alpha \Delta t}}{\alpha} \\
&= \lim_{\alpha \to 0} \frac{\frac{\mathrm{d}}{\mathrm{d}\alpha}(1 - e^{-\alpha \Delta t})}{\frac{\mathrm{d}}{\mathrm{d}\alpha}(\alpha)} \\
&= \lim_{\alpha \to 0} \Delta t e^{-\alpha \Delta t} \\
&= \Delta t
\end{aligned}$$

Latent thermal storage: Model

like stratified sensible

Outline

Unified model

- linear time-invariant dynamics:

$$x(k+1) = ax(k) + b(u(k) + w(k))$$

- linear time-varying state constraints:

$$\underline{x}(k) \leq x(k) \leq \overline{x}(k)$$

- linear time-varying action constraints:

$$\underline{u}(k) \leq u(k) \leq \overline{u}(k)$$

- nonlinear time-varying state-dependent output equation:

$$p(k) = g(k, x(k), u(k))$$

Dynamics parameters

- for everything but latent thermal storage,

$$a = e^{-\Delta t/\tau}, \ b = (1 - a)\tau$$

- for latent thermal storage, $a = 1$, $b = \Delta t$
  (the limits of $e^{-\Delta t/\tau}$ and $(1 - e^{-\Delta t/\tau})\tau$ as $\tau \to \infty$)

Disturbance

zero for

- stationary batteries
- indoor air/shallow thermal mass
- deep thermal mass

nonzero in general for

- electric vehicles
- lumped sensible thermal storage
- stratified sensible thermal storage
- latent thermal storage

State constraints

time-invariant for

- stationary batteries
- electric vehicles
- stratified sensible thermal storage
- latent thermal storage

time-varying for

- indoor air/shallow thermal mass
- deep thermal mass
- lumped sensible thermal storage

Action constraints

time-invariant for

- stationary batteries

time-varying for

- electric vehicles
- indoor air/shallow thermal mass
- deep thermal mass
- lumped sensible thermal storage
- stratified sensible thermal storage
- latent thermal storage

Output equation

nonlinear, time-invariant, state-independent for

- stationary batteries
- electric vehicles

linear, time-varying, state-independent for

- indoor air/shallow thermal mass
- lumped sensible thermal storage
- stratified sensible thermal storage
- latent thermal storage

linear, time-varying, state-dependent for

- deep thermal mass

Outline

Refrigerators and freezers

- there are ~200 million refrigerators and freezers in the US
- they all have some thermal mass and temperature flexibility
- at ~500 W each, that's ~100 GW of flexible capacity
  (for comparison, total US generation capacity is ~1.3 TW)
- can model them as thermal circuits, just like buildings

Whirlpool: Refrigerators

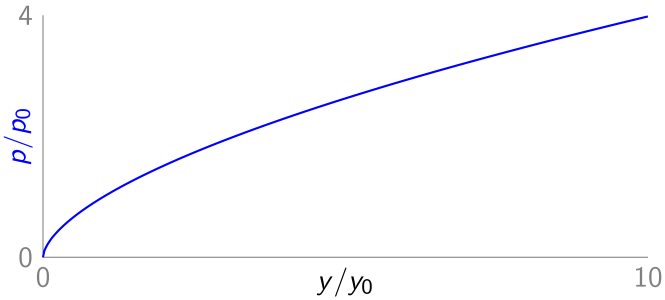
Deferrable loads

- some appliances just need to run before a deadline
- for example, a dishwasher might
  - get loaded at ~8 PM
  - need to finish by ~6 AM
  - take ~1 to 3 hours to run
  - draw ~1.5 to 2 kW while running
- there are ~80 million dishwashers in the US
- at ~1.5 kW each, that's ~120 GW of flexible capacity
- clothes washers and dryers may have similar flexibility

Small wind turbines

- wind turbine power output p scales like wind speed v cubed
- and wind speed scales like turbine height y to the $\alpha \approx 0.2$

$$\frac{p}{p_0} \approx \left(\frac{v}{v_0}\right)^3 \text{ and } \frac{v}{v_0} \approx \left(\frac{y}{y_0}\right)^\alpha \implies \frac{p}{p_0} \approx \left(\frac{y}{y_0}\right)^{3\alpha}$$



$\implies$ smaller turbines are typically less attractive economically

The world's biggest wind turbine has 130 m blades

a Boeing 747 is 71 m long

New Atlas: World's largest wind turbine is now fully operational and connected

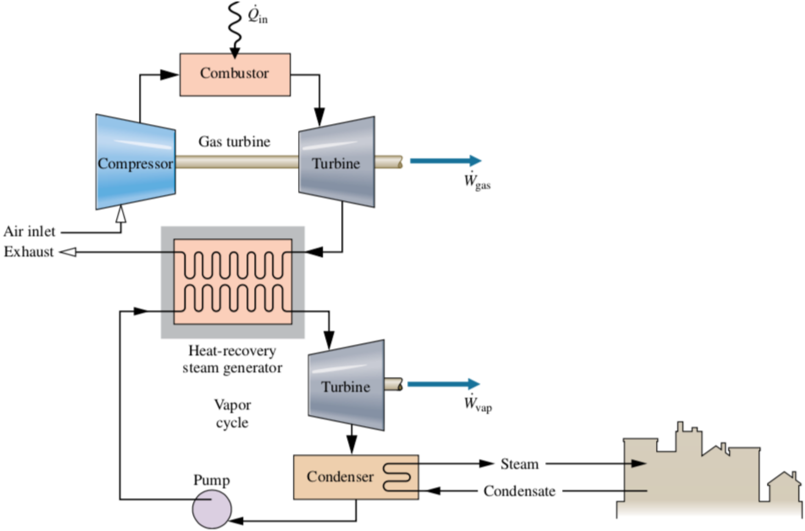
Simple wind power simulation

- get wind speed $\tilde{v}$ at whatever height $\tilde{y}$ weather data report
- estimate wind speed v at turbine height y by $v \approx \tilde{v}(y/\tilde{y})^{0.2}$
- get turbine's cut-in and cut-out speeds, $\underline{v}$ and $\overline{v}$
- if $\underline{v} \leq v \leq \overline{v}$,
    - ◇ get turbine's rated power p_0 at rated wind speed v_0
    - ◇ estimate power p at wind speed v by $p \approx p_0(v/v_0)^3$
- otherwise, set $p = 0$

Things that use fuel

- some things burn fuel for heat or work
  - heaters fueled by wood, methane, propane, or heating oil
  - diesel generators
  - combined heat and power ('cogeneration' or 'trigeneration')
- others use fuel without combustion
  - methane fuel cells
  - hydrogen fuel cells
- potentially useful for backup, stabilizing microgrids, ...
- can model (roughly) via "output $=$ efficiency $\times$ input"

Combined heat and power



Moran et al., *Fundamentals of Engineering Thermodynamics* (2018)