Homework 6: Optimization

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Directions:

- Students may work individually or in groups, but each student must upload their own solutions to Gradescope by **11:59 PM ET on Thursday, April 10**. Students should finish the reading (see Problem 3) before the class discussion on **Thursday, April 17**.
- Use any outside resources you want, but **cite your sources**. (If you really want to learn the material, I recommend seriously attempting the problems yourself before looking for outside help.)
- The TA will grade each problem or subproblem quickly on a three-tier scale:
  - No credit for a solution that's mostly unreadable or missing.
  - Half credit for a serious attempt that's not easy to read or is substantially incorrect.
  - Full credit for a solution that's clearly readable and nearly or completely correct.

Problems:

1. (60%) Consider the function $f : \mathbf{R}^2 \to \mathbf{R}$ defined by
$$f(x) = e^{x_1+3x_2-0.1} + e^{x_1-3x_2-0.1} + e^{-x_1-0.1}.$$
   (a) (15%) Use composition rules from the 'Convex sets and functions' slides to show that f is convex.
   (b) (15%) Find the (unique) global minimizer $x^\star$ of f by solving $\nabla f(x^\star) = 0$ for $x^\star$.
   (c) (30%) Download the files in the Github repository gradient-descent. Fill in the missing code in the script `gradientDescent`, using the line search method from the 'Solving convex optimization problems' slides. Your code should minimize $f(x)$ given the initial guess $x(1) = (-2.5, 0.5)$ and the initial step size $\alpha(1) = 1$. Show your code and figures 1 and 2 here.

2. (40%) Install CVX for Matlab or import CVXPY for Python. Use CVX or CVXPY to find a battery power profile that maximizes 'buy low, sell high' revenue with an electricity price that is 0.25 \$/kWh between 2 PM and 9 PM and 0.15 \$/kWh otherwise. Use the following input data: One-day planning horizon starting at midnight, fifteen-minute time steps, 1,600 hour self-dissipation time constant, 95% charging and discharging efficiencies, 13.5 kWh of energy capacity, minimum charge state of 20%, initial charge state of 50%, charging and discharging power capacities of 5 kW. Constrain the final charge state to equal the initial charge state. Turn in your code, the total revenue earned by the battery over the day, and a figure containing three vertical subplots: (1) electricity price vs. time, (2) electrical charging/discharging power vs. time, (3) stored chemical energy vs. time.

3. (Due **Thursday, April 17**) Listen to The Carbon Copy podcast episode "Virtual power plants: The 'sandwich' for the grid". Read pages 1–55 of the Department of Energy's Pathways to commercial liftoff: Virtual power plants report and pages 2–15 of the 2025 update. (Jennifer Downing, the podcast guest, led the 2023 report.) As you listen and read, do the following to prepare for in-class discussion:
   - Write down any terms you don't recognize or fully understand. Do your best to find the meaning of each of these terms through Internet searches.
   - Write down at least two questions or topics that you'd like to discuss in class.