

Batteries and electric vehicles

Purdue ME 597, Distributed Energy Resources

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Outline

Modeling stationary batteries

Stationary battery model parameters

Modeling electric vehicles

Simple EV charging policies

Battery basics

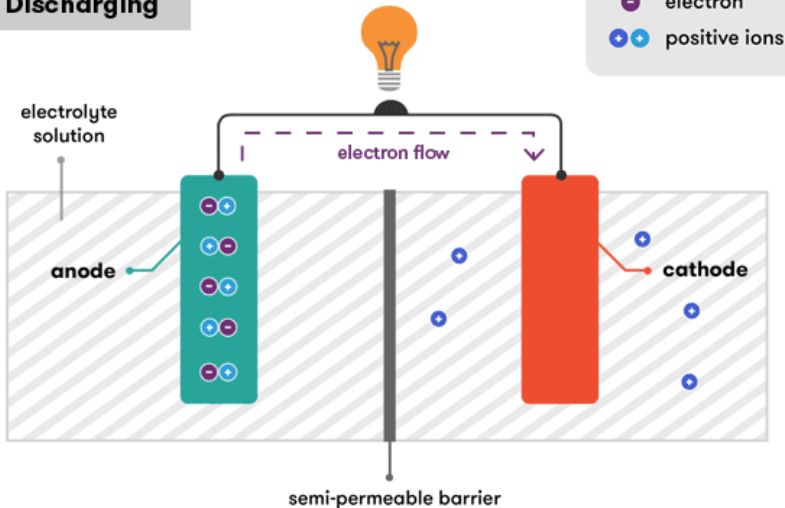
- batteries
 - ◇ convert between electrical and chemical potential energy
 - ◇ are also called electrochemical energy storage
- most batteries today are lithium-ion
 - ◇ personal electronics (phones, laptops, tablets, headphones, ...)
 - ◇ electric vehicles (cars, trucks, bikes, scooters, skateboards, ...)
 - ◇ stationary applications (home batteries, grid-scale storage, ...)
 - ★ invented 1980s, commercialized 1991, Nobel Prize 2019

Battery components

- **anode** emits electrons and ions during discharge (typically graphite in lithium-ion batteries)
- **cathode** absorbs electrons and ions during discharge (lithium/metal oxides)
- **electrolyte** allows movement of ions but not electrons (polymer gels or mixes of salts, solvents, additives)
- **separator** blocks electrons, prevents anode-cathode contact (plastics such as polyethylene or polypropylene)

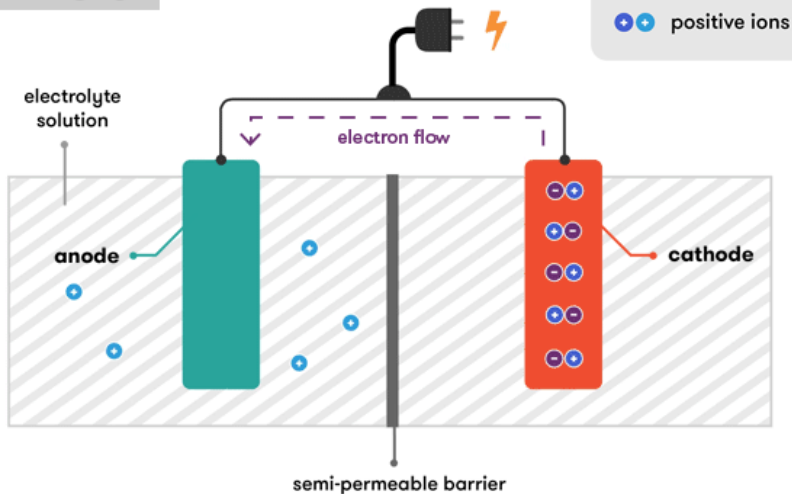
Discharging a battery

Discharging



Charging a battery

Charging



Australian Academy of Science: [How a battery works](#)



- 1** Minimize exposure to high temperatures in storage or use.
- 2** Minimize exposure to low temperatures, especially when charging.

TEMPERATURE



- 3** Minimize time spent at 100% charge.
- 4** Minimize time spent at 0% charge.

STATE OF CHARGE



- 5** Avoid using fast charging unless needed.
- 6** Avoid discharging devices more quickly than is needed.

CURRENT



- 7** Avoid use or storage in high moisture environments.
- 8** Avoid mechanical damage.
- 9** Follow manufacturer's calibration instructions.

OTHER

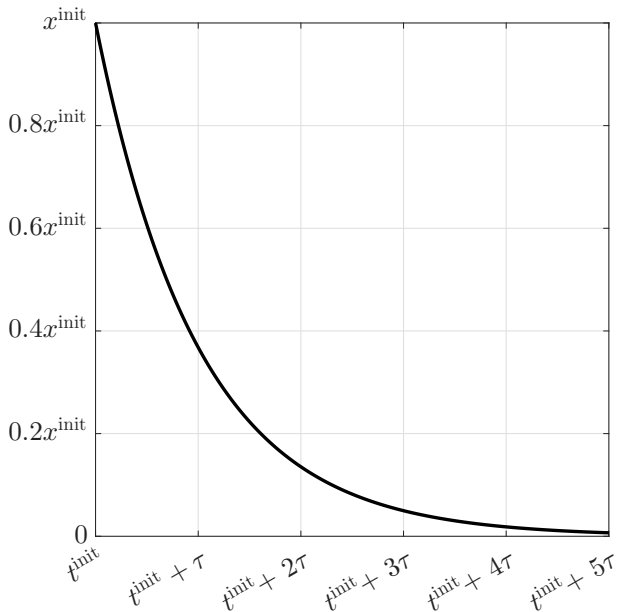
A simple battery model

- a simple model of a battery's energy dynamics is

$$\frac{dx(t)}{dt} = -\frac{x(t)}{\tau} + p^{\text{chem}}(t)$$

- $x(t) \in \mathbf{R}$ (kWh) is the stored chemical potential energy
- $\tau > 0$ (h) is the self-dissipation time constant
(for an ideal battery with no self-dissipation, $\frac{dx(t)}{dt} = p^{\text{chem}}(t)$)
- $p^{\text{chem}}(t) \in \mathbf{R}$ (kW) is the chemical charging power
(or discharging if $p^{\text{chem}}(t) < 0$)

Energy evolution with $p^{\text{chem}} = 0$

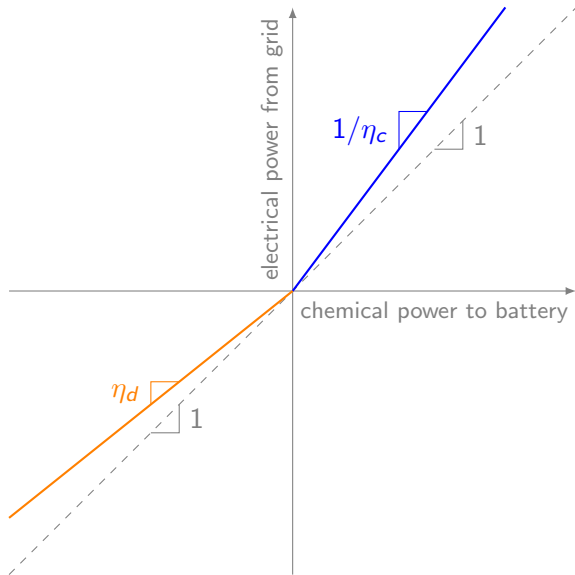


- the electrical charging (or discharging) power is

$$\begin{aligned} p(t) &= \begin{cases} p^{\text{chem}}(t)/\eta_c & \text{if } p^{\text{chem}}(t) \geq 0 \\ \eta_d p^{\text{chem}}(t) & \text{if } p^{\text{chem}}(t) < 0 \end{cases} \\ &= \max \left\{ p^{\text{chem}}(t)/\eta_c, \eta_d p^{\text{chem}}(t) \right\} \end{aligned}$$

- $\eta_c, \eta_d \in (0, 1]$ are the charging and discharging efficiencies

Chemical and electrical power (continued)



Discrete-time battery model

- with uniform time step Δt and piecewise constant $p^{\text{chem}}(t)$,

$$x(k+1) = ax(k) + (1-a)\tau p^{\text{chem}}(k),$$

where

$$a = e^{-\Delta t/\tau}$$

Battery constraints

- batteries have energy and power constraints:

$$\begin{aligned}0 &\leq x(k) \leq \bar{x} \\ -\bar{p}_d &\leq p(k) \leq \bar{p}_c\end{aligned}$$

- $\bar{x} \geq 0$ (kWh) is the chemical energy capacity
- $\bar{p}_c \geq 0$ (kW) is the electrical charging power capacity
- $\bar{p}_d \geq 0$ (kW) is the electrical discharging power capacity
- in terms of p^{chem} , the power constraints are

$$-\frac{\bar{p}_d}{\eta_d} \leq p^{\text{chem}}(k) \leq \eta_c \bar{p}_c$$

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Stationary battery parameters

the stationary battery model has six parameters:

1. chemical energy capacity $\bar{x} \geq 0$ (kWh)
2. electrical charging power capacity $\bar{p}_c \geq 0$ (kW)
3. electrical discharging power capacity $\bar{p}_d \geq 0$ (kW)
4. self-dissipation time constant $\tau > 0$ (h)
5. charging efficiency $\eta_c \in (0, 1]$
6. discharging efficiency $\eta_d \in (0, 1]$

Typical values of the self-dissipation time constant τ

- unused, a typical battery might lose ~ 1 to 3% energy per day
- if $p^{\text{chem}}(t) = 0$ for all t and $x(0) = x_0$, then $x(t) = e^{-t/\tau} x_0$
- after one day, $t = 24$ h and $x(t)/x_0 \approx 0.97$ to 0.99
- so a typical time constant is

$$\tau = \frac{-t}{\ln(x(t)/x_0)} = \frac{-24 \text{ h}}{\ln(0.97 \text{ to } 0.99)} \approx 800 \text{ to } 2400 \text{ h}$$

Typical values of the energy and power capacities

- choosing appropriate capacities is a design problem
- sometimes, hardware limitations impose constraints
- for example, stationary lithium-ion batteries typically have
 - ◇ similar charging and discharging power capacities ($\bar{p}_c \approx \bar{p}_d$)
 - ◇ one to four hours of storage ($\bar{x}/\bar{p}_c \approx 1$ to 4 h)
- also, a building's wiring may limit current or voltage:

$$p_c = IV \approx (15 \text{ to } 25 \text{ A})(240 \text{ V}) = 3.6 \text{ to } 6 \text{ kW}$$

- one popular home battery has
 - ◇ $\bar{p}_c = \bar{p}_d = 5 \text{ kW}$
 - ◇ $\bar{x} = 13.5 \text{ kWh}$ (so $\bar{x}/\bar{p}_c = 2.7 \text{ h}$)

Round-trip efficiency

- consider a battery with no self-dissipation ($\tau = \infty$)
- if we charge at electrical power $p_{\text{in}} > 0$ at time k , then

$$x(k+1) = x(k) + \Delta t \eta_c p_{\text{in}}$$

- suppose we then return the battery to its initial state $x(k)$:

$$x(k+2) = x(k)$$

$$\iff x(k+1) - \Delta t p_{\text{out}} / \eta_d = x(k)$$

$$\iff x(k) + \Delta t \eta_c p_{\text{in}} - \Delta t p_{\text{out}} / \eta_d = x(k)$$

$$\iff \eta_c p_{\text{in}} = p_{\text{out}} / \eta_d$$

- so we get electrical power $p_{\text{out}} = \eta_c \eta_d p_{\text{in}}$ back out
- the product $\eta_c \eta_d$ is called the **round-trip efficiency**

Typical values of the charging and discharging efficiencies

- a typical battery has round-trip efficiency $\eta_c \eta_d \approx 0.9$
- usually, charging and discharging efficiencies are similar
- so $\eta_c \approx \eta_d \approx \sqrt{0.9} \approx 0.95$

Summary: Instantiating a stationary battery model

- set $\tau \approx 800$ to 2400 h
- set $\eta_c \approx \eta_d \approx 0.95$
- either
 - ◇ choose an appropriate energy capacity \bar{x} for the application
 - ◇ set $\bar{p}_c \approx \bar{p}_d \approx \bar{x}/(1 \text{ to } 4 \text{ h})$
- or
 - ◇ set $\bar{p}_c \approx \bar{p}_d = IV$ with appropriate I, V for building's wiring
 - ◇ set $\bar{x} \approx (1 \text{ to } 4 \text{ h})\bar{p}_c$

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EV battery dynamics

- for modeling purposes, EVs are just batteries that move
- they have the same energy dynamics as stationary batteries:

$$x(k+1) = ax(k) + (1-a)\tau p^{\text{chem}}(k)$$

- if an EV drives $d(k)$ km between t_k and $t_{k+1} = t_k + \Delta t$, then

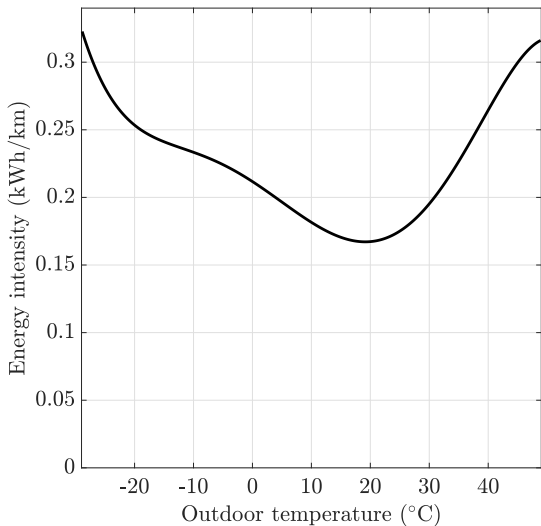
$$p^{\text{chem}}(k) = -\frac{\alpha(k)d(k)}{\Delta t}$$

- $\alpha(k)$ (kWh/km) is the energy intensity of driving

The energy intensity of driving an EV

- the energy intensity $\alpha(k)$ (kWh/km) depends on the vehicle's
 - ◇ speed
 - ◇ weight
 - ◇ shape
 - ◇ drivetrain efficiencyas well as the
 - ◇ terrain (hills)
 - ◇ weather (cabin heating/cooling, battery thermal management)
- a typical EV energy intensity is 0.15 to 0.4 kWh/km
- ★ electric bikes use ~ 0.005 kWh/km

EVs use more energy when it's cold or hot outside



Yuksel and Michalek (2015): *Effects of Regional Temperature on Electric Vehicle Efficiency, Range, and Emissions in the United States*

An energy intensity model

$$\alpha(k) = c_0 + c_1 T(k) + \dots + c_5 T(k)^5$$

- $\alpha(k)$ is the energy intensity at time k , in kWh/km
- $T(k)$ is the outdoor temperature at time k , in $^{\circ}\mathbf{F}$
- the coefficients (in the corresponding units) are

c_0	0.3950
c_1	-0.0022
c_2	9.1978×10^{-5}
c_3	-3.9249×10^{-6}
c_4	5.2918×10^{-8}
c_5	-2.0659×10^{-10}

Yuksel and Michalek (2015): *Effects of Regional Temperature on Electric Vehicle Efficiency, Range, and Emissions in the United States*

EV charging and discharging constraints

- when unplugged, EVs
 - ◊ can't charge
 - ◊ functionally, have no discharge power limit (400+ kW)
- when plugged in,
 - ◊ most EVs can charge but not discharge
 - ◊ EVs with bidirectional charging can do both
- define the indicator variable

$$z(k) = \begin{cases} 1 & \text{if the EV is plugged in over time step } k \\ 0 & \text{otherwise} \end{cases}$$

- then the charging and discharging constraints become

$$\begin{cases} -\bar{p}_d \leq p(k) \leq \bar{p}_c & \text{if } z(k) = 1 \\ p(k) \leq 0 & \text{if } z(k) = 0 \end{cases}$$

with $\bar{p}_d = 0$ for EVs without bidirectional charging

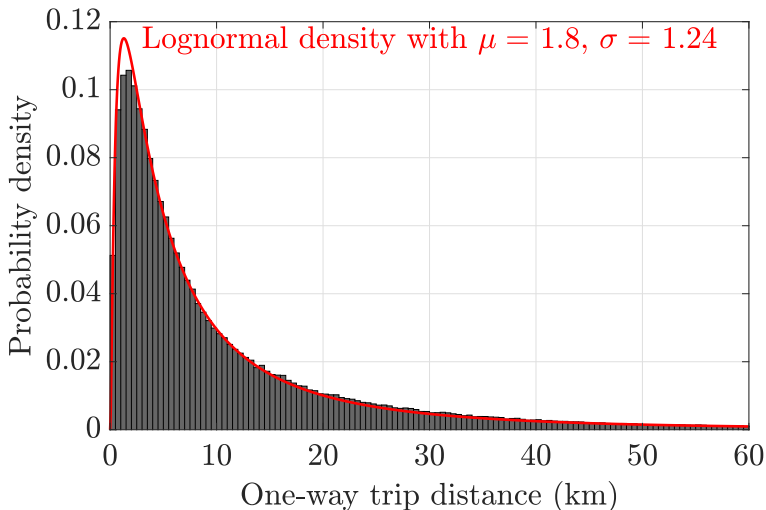
Typical EV parameter values

- τ , η_c , and η_d are typically similar to stationary batteries
- typical values of \bar{x} are 55, 80, 100, 130, or 200 kWh
- for Level 1 charging (~ 120 V/15 A), $\bar{p}_c \approx 1.8$ kW
- for Level 2 charging (~ 240 V/48 A), $\bar{p}_c \approx 11.5$ kW
- for EVs without bidirectional charging, $\bar{p}_d = 0$
- for EVs with bidirectional charging, $\bar{p}_d \approx \bar{p}_c$

Typical US driving patterns

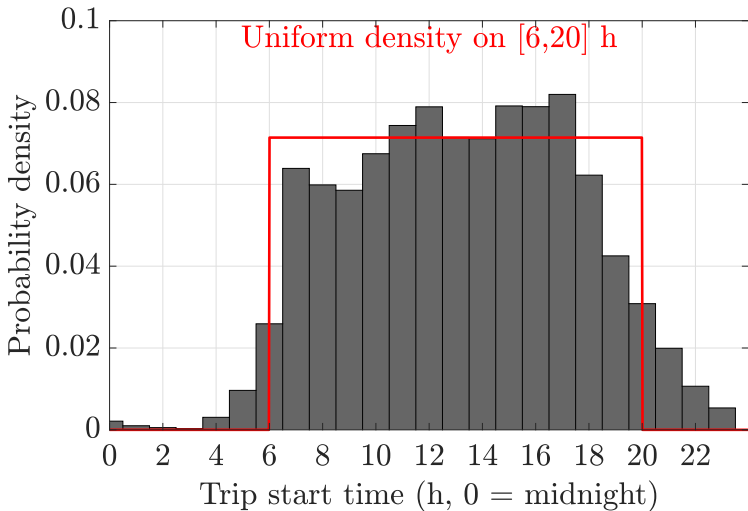
- the average household has 1.9 private vehicles (cars, pickup trucks, SUVs, or vans)
- the average private vehicle
 - ◇ drives 41.6 km (26 miles) per day
 - ◇ drives 12.6 km (7.9 miles) per trip (one way, not round trip)
 - ◇ so takes about 3.3 trips per day
- most trips happen between 6 AM and 8 PM

One-way trip distance distribution for private vehicles



US DOT (2017): *National Household Travel Survey*

Trip start time distribution for private vehicles



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1. When plugged in, charge at maximum until full

- if $z(k) = 1$ (meaning the EV is plugged in), set

$$p^{\text{chem}}(k) = \min \left\{ \eta_c \bar{p}_c, \frac{\bar{x} - ax(k)}{(1-a)\tau} \right\}$$

- if the battery is nearly full, this places $x(k+1)$ at \bar{x} :

$$x(k+1) = ax(k) + (1-a)\tau \frac{\bar{x} - ax(k)}{(1-a)\tau} = \bar{x}$$

2. When energy gets low, charge at maximum until full

- define the minimum acceptable energy \underline{x} (kWh)
- initialize $y(k)$, an indicator of 'charging mode,' at $y(k - 1)$
- if $z(k) = 0$ or $x(k) = \bar{x}$, set $y(k) = 0$
- if $z(k) = 1$ and $x(k) < \underline{x}$, set $y(k) = 1$
- if $y(k) = 1$, set

$$p^{\text{chem}}(k) = \min \left\{ \eta_c \bar{p}_c, \frac{\bar{x} - ax(k)}{(1 - a)\tau} \right\}$$

3. When energy gets low, charge steadily to meet deadline

- suppose the user wants energy x^* stored by deadline $k^* > k$
- charging at constant power p_0^{chem} from time k to k^* gives

$$x(k+1) = ax(k) + (1-a)\tau p_0^{\text{chem}}$$

$$\begin{aligned}x(k+2) &= ax(k+1) + (1-a)\tau p_0^{\text{chem}} \\ &= a[ax(k) + (1-a)\tau p_0^{\text{chem}}] + (1-a)\tau p_0^{\text{chem}} \\ &= a^2x(k) + (1+a)(1-a)\tau p_0^{\text{chem}}\end{aligned}$$

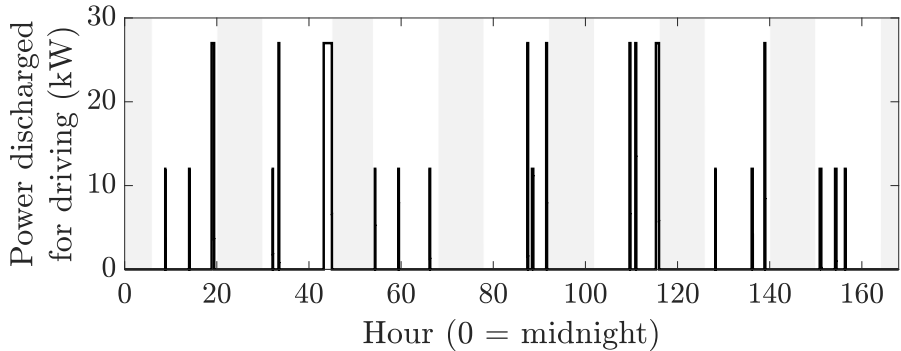
\vdots

$$x(k^*) = a^{k^*-k}x(k) + (1+a+\dots+a^{k^*-k-1})(1-a)\tau p_0^{\text{chem}}$$

- so if $y(k) = 1$, set

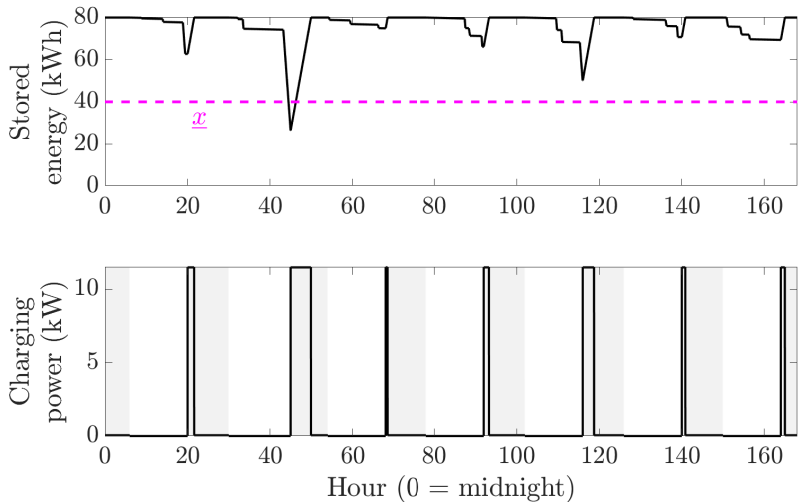
$$p^{\text{chem}}(k) = \min \left\{ \eta_c \bar{p}_c, \frac{x^* - a^{k^*-k}x(k)}{(1+a+\dots+a^{k^*-k-1})(1-a)\tau} \right\}$$

Example

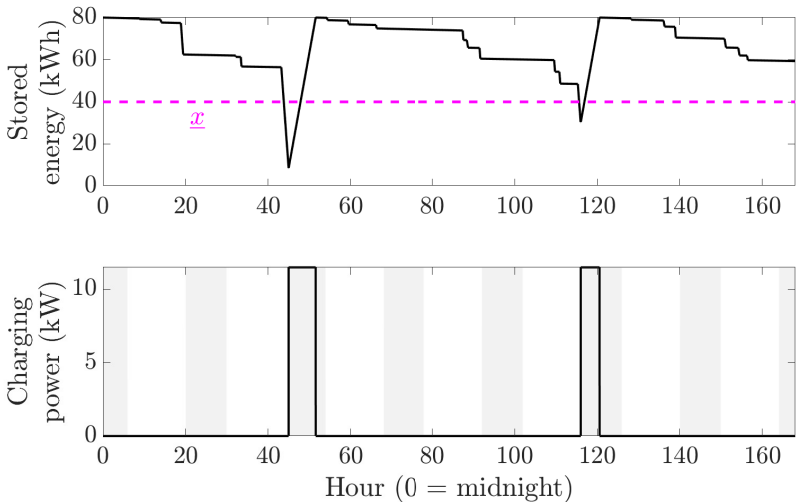


- EV with 80 kWh battery over 7 days, $\alpha = 0.3$ kWh/km
- EV is plugged in during shaded periods
- short trips are at 40 km/h, long trips at 90

Policy 1 tops off battery every night



Policy 2 charges full blast when $x < \underline{x}$



Policy 3 spreads charging out over whole night

