Thermal modeling of buildings: Part 2

Purdue ME 597, Distributed Energy Resources

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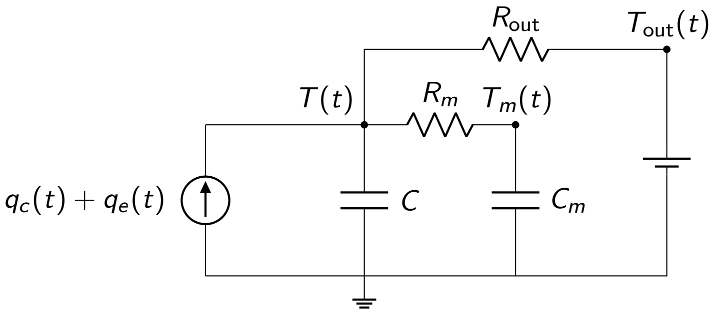
Outline

Higher-order thermal circuits

Simulating buildings

Buildings as thermal batteries

A 2R2C thermal circuit



$$C \frac{\mathrm{d}T(t)}{\mathrm{d}t} = \frac{T_m(t) - T(t)}{R_m} + \frac{T_{\text{out}}(t) - T(t)}{R_{\text{out}}} + q_c(t) + q_e(t)$$

$$C_m \frac{\mathrm{d}T_m(t)}{\mathrm{d}t} = \frac{T(t) - T_m(t)}{R_m}$$

Discrete-time dynamics

- continuous-time dynamics are

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} T(t) \\ T_m(t) \end{bmatrix} = \tilde{A}\begin{bmatrix} T(t) \\ T_m(t) \end{bmatrix} + \tilde{B}(q_c(t) + w(t))$$

  with

$$\tilde{A} = \begin{bmatrix} -(1/R_m + 1/R_{\text{out}})/C & 1/(R_m C) \\ 1/(R_m C_m) & -1/(R_m C_m) \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} 1/C \\ \end{bmatrix}, \; w(t) = q_e(t) + T_{\text{out}}(t)/R_{\text{out}}$$

- can exactly discretize this LDS via matrix exponential to

$$\begin{bmatrix} T(k+1) \\ T_m(k+1) \end{bmatrix} = A\begin{bmatrix} T(k) \\ T_m(k) \end{bmatrix} + B\left(q_c(k) + w(k)\right)$$

  where $A = e^{\Delta t \tilde{A}}$, $B = (A - I)\tilde{A}^{-1}\tilde{B}$ (can show $\tilde{A}^{-1}$ exists)

Thermal mass parameter values

- in empirical studies of real buildings, typically
  - $C_m \approx (8 \text{ to } 16)C$
  - $R_m \approx R/(4 \text{ to } 8)$
- check: $C_m/(0.3 \text{ kWh}/[\text{m}^3\ ^\circ\text{C}]) \approx$ equivalent volume of pine

Penman (1990): Second order system identification in the thermal response of a working school

Two-timing

- T typically changes much faster than T_m

$\implies$ air and mass dynamics define two characteristic time scales

- fast time scale: $T_m(t) \approx T_m(t_k)$ (a constant) for all $t \in [t_k, t_{k+1})$
- slow time scale: $\mathrm{d}T(t)/\mathrm{d}t \approx 0$ for all $t \in (t_k, t_{k+1})$

Fast dynamics: 1R1C with state $T(t)$

for all $t \in [t_k, t_{k+1})$,

$$
\begin{aligned}
T_m(t) &\approx T_m(t_k) =: T_m(k) \\
\implies C\frac{\mathrm{d}T(t)}{\mathrm{d}t} &\approx \frac{T_m(k) - T(t)}{R_m} + \frac{T_{\text{out}}(t) - T(t)}{R_{\text{out}}} + q_c(t) + q_e(t) \\
&= \cdots = \frac{R(T_m(k)/R_m + q_c(t) + w(t)) - T(t)}{R}
\end{aligned}
$$

with

$$R = \frac{R_m R_{\text{out}}}{R_m + R_{\text{out}}}, \;\; w(t) = \frac{T_{\text{out}}(t)}{R_{\text{out}}} + q_e(t)$$

so

$$T(k+1) \approx aT(k) + (1-a)R[T_m(k)/R_m + q_c(k) + w(k)]$$

with $a = \exp(-\Delta t/(RC))$

Slow dynamics: 1R1C with state $T_m(t)$

for all $t \in (t_k, t_{k+1})$,

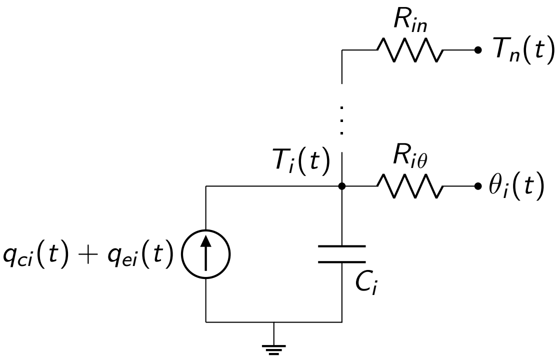
$$
\begin{aligned}
0 \approx \frac{\mathrm{d}T(t)}{\mathrm{d}t} &= \frac{T_m(t) - T(t)}{R_m} + \frac{T_{\text{out}}(t) - T(t)}{R_{\text{out}}} + q_c(t) + q_e(t) \\
\implies T(t) &\approx R\left(\frac{T_m(t)}{R_m} + q_c(t) + w(t)\right) \\
\implies C_m\frac{\mathrm{d}T_m(t)}{\mathrm{d}t} &= \frac{T(t) - T_m(t)}{R_m} \\
&\approx \cdots = \frac{R_{\text{out}}\left(q_c(t) + w(t)\right) - T_m(t)}{R_m + R_{\text{out}}}
\end{aligned}
$$

so

$$
T_m(k+1) \approx a_m T_m(k) + (1 - a_m)R_{\text{out}}(q_c(k) + w(k))
$$

with $a_m = \exp(-\Delta t/[(R_{\text{out}} + R_m)C_m])$

General mRnC thermal circuits



$$C_i \frac{\mathrm{d}T_i(t)}{\mathrm{d}t} = \frac{\theta_i(t) - T_i(t)}{R_{i\theta}} + \sum_{j=1}^{n} \frac{T_j(t) - T_i(t)}{R_{ij}} + q_{ci}(t) + q_{ei}(t)$$

General mRnC thermal circuits (continued)

- node 0 is a boundary node (such as the outdoor air)
- $1/R_{ij} = 0$ if no heat transfers between nodes i and j
- $q_{ci} = 0$ if equipment transfers no heat to or from node i
- can put mRnC model in matrix form and discretize like 2R2C

Outline

Perfect setpoint tracking control (1R1C)

- define indoor temperature setpoints $\hat{T}(k)$
- 1R1C discrete-time dynamics are

$$T(k+1) = aT(k) + (1-a)R(q_c(k) + w(k))$$

$\Longrightarrow$ to drive temperature from $T(k)$ to $T(k+1) = \hat{T}(k+1)$, set

$$\hat{q}_c(k) = \frac{\hat{T}(k+1) - aT(k)}{(1-a)R} - w(k)$$

Perfect setpoint tracking control (2R2C)

- 2R2C discrete-time dynamics are

$$\begin{bmatrix} T(k+1) \\ T_m(k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} T(k) \\ T_m(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} (q_c(k) + w(k))$$

$\Longrightarrow$ to drive temperature from $T(k)$ to $T(k+1) = \hat{T}(k+1)$, set

$$\hat{q}_c(k) = \frac{\hat{T}(k+1) - A_{11}T(k) - A_{12}T_m(k)}{B_1} - w(k)$$

- can treat arbitrary mRnC thermal circuits similarly

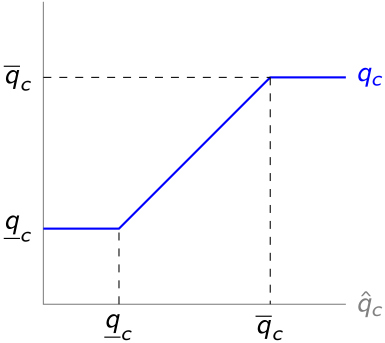
(Near-)perfect setpoint tracking with capacity limits

- heating and cooling systems have capacity constraints

$$\underline{q}_c(k) \leq q_c(k) \leq \overline{q}_c(k)$$

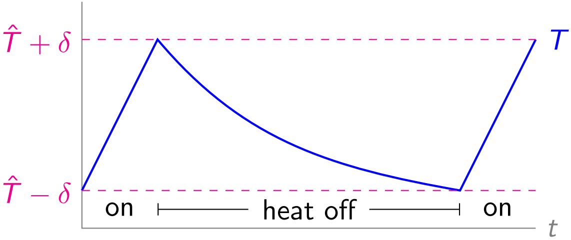
- to respect capacity constraints, saturate $\hat{q}_c(k)$:

$$q_c(k) = \max\left\{\underline{q}_c(k), \min\left\{\overline{q}_c(k), \hat{q}_c(k)\right\}\right\}$$



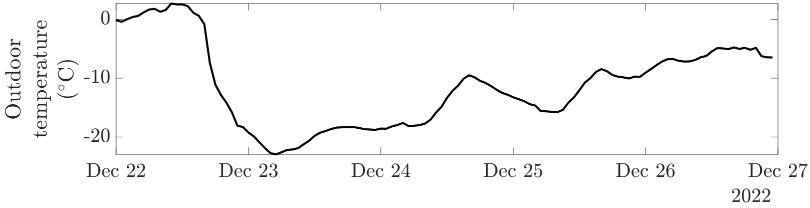
Thermostatic control

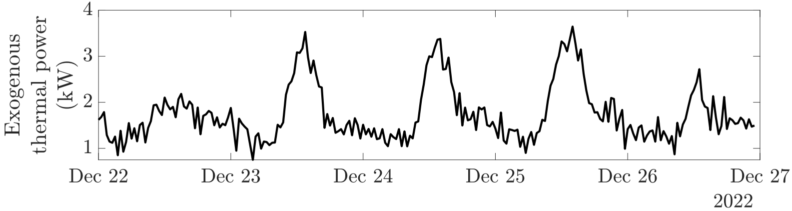
- many heating and cooling systems operate in an on/off fashion
- these systems typically use thermostatic control
- heating example with action $u(k) \in \{0, 1\}$, deadband δ (°C):
  - ◇ initialize $u(k) = u(k-1)$
  - ◇ if $T(k) > \hat{T}(k) + \delta$, set $u(k) = 0$
  - ◇ if $T(k) < \hat{T}(k) - \delta$, set $u(k) = 1$
  - ◇ set $q_c(k) = \underline{q}_c(k) + u(k)\left(\overline{q}_c(k) - \underline{q}_c(k)\right)$
  - ◇ increment k, update $T(k)$, and repeat



Example: Exact 2R2C vs. two-timing

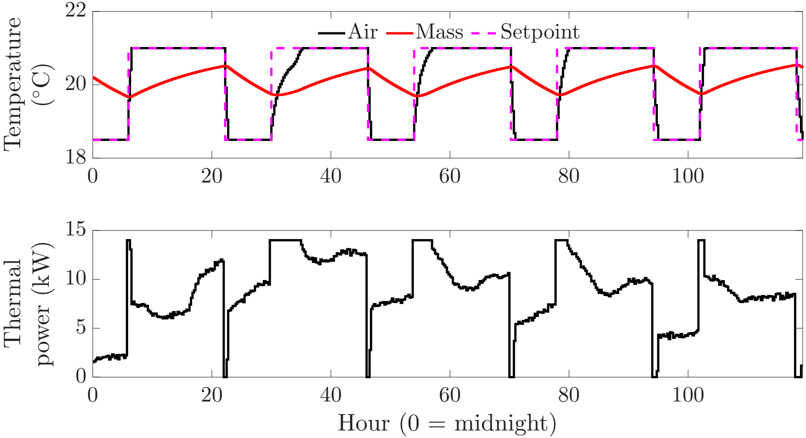
- $N = 2$ story house with $A_f = 200$ m^2 total floor area
- simulated over 5 very cold days from 2022 in West Lafayette





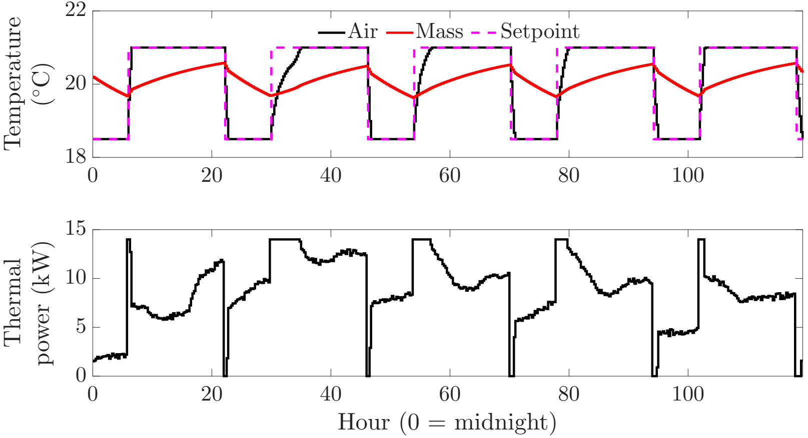
Exact 2R2C simulation results

- lower indoor air temperature setpoint overnight
- control tries to track setpoint but saturates at capacity limits



Approximate (two-timing) 2R2C simulation results

- 0.002 °C air temperature approximation MAE, 0.08 °C for mass
- 0.1 kW thermal power MAE, 4 kWh (0.4%) for thermal energy



Outline

Higher-order thermal circuits

Simulating buildings

Buildings as thermal batteries

Reminder: Batteries

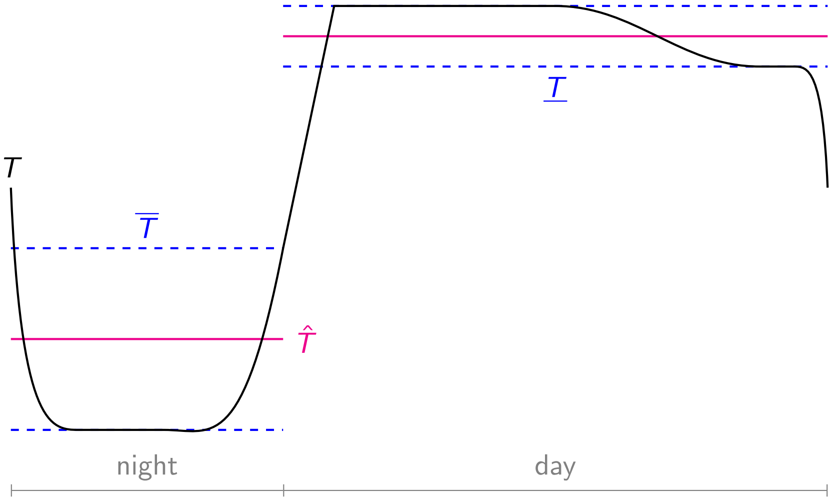
one battery model is

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -\frac{x(t)}{\tau} + p^{\text{chem}}(t)$$
$$\underline{x} \leq x(t) \leq \overline{x}$$
$$-\overline{p}_d^{\text{chem}} \leq p^{\text{chem}}(t) \leq \overline{p}_c^{\text{chem}}$$

Indoor air as a thermal battery

- at fast time scales, $C\frac{\mathrm{d}T(t)}{\mathrm{d}t} = \frac{\theta(t)-T(t)}{R} + q_c(t) + q_e(t)$
- define nominal $\hat{T}$, $\hat{q}_c$ such that $C\frac{\mathrm{d}\hat{T}(t)}{\mathrm{d}t} = \frac{\theta(t)-\hat{T}(t)}{R} + \hat{q}_c(t) + q_e(t)$
- then thermal energy $x(t) = C(T(t) - \hat{T}(t))$ satisfies

$$\begin{aligned}\frac{\mathrm{d}x(t)}{\mathrm{d}t} &= C\left(\frac{\mathrm{d}T(t)}{\mathrm{d}t} - \frac{\mathrm{d}\hat{T}(t)}{\mathrm{d}t}\right)\\ &= \frac{\hat{T}(t) - T(t)}{R} + q_c(t) - \hat{q}_c(t)\\ &= -\underbrace{\frac{x(t)}{RC}}_{x(t)/\tau} + \underbrace{q_c(t) - \hat{q}_c(t)}_{p^{\mathrm{thrm}}(t)}\end{aligned}$$


T
$\overline{T}$
$\underline{T}$
$\hat{T}$
night
day

Indoor air as a thermal battery: Energy capacity

- suppose indoor air temperature has comfort constraints

$$\underline{T}(t) \leq T(t) \leq \overline{T}(t)$$

- then thermal energy $x(t) = C(T(t) - \hat{T}(t))$ must satisfy

$$\underbrace{C(\underline{T}(t) - \hat{T}(t))}_{\underline{x}(t)} \leq x(t) \leq \underbrace{C(\overline{T}(t) - \hat{T}(t))}_{\overline{x}(t)}$$

- for a house with $C \approx 2.5$ kWh/°C, $\overline{T}(t) - \underline{T}(t) \approx 2$ °C,

$$\overline{x}(t) - \underline{x}(t) \approx 5 \text{ kWh}$$

Indoor air as a thermal battery: Power capacities

- heating and cooling systems have capacity constraints

$$\underline{q}_c(t) \leq q_c(t) \leq \overline{q}_c(t)$$

- so $p^{\text{thrm}}(t) = q_c(t) - \hat{q}_c(t)$ must satisfy

$$\underbrace{\underline{q}_c(t) - \hat{q}_c(t)}_{-\overline{p}_d^{\text{thrm}}(t)} \leq p^{\text{thrm}}(t) \leq \underbrace{\overline{q}_c(t) - \hat{q}_c(t)}_{\overline{p}_c^{\text{thrm}}(t)}$$

- for a typical US house,

$$\overline{p}_c^{\text{thrm}} + \overline{p}_d^{\text{thrm}} = \overline{q}_c(t) - \underline{q}_c(t) \approx 10 \text{ to } 15 \text{ kW}$$

Thermal mass as a thermal battery

- at slow time scales, $C_m \frac{\mathrm{d}T_m(t)}{\mathrm{d}t} = \frac{T(t) - T_m(t)}{R_m}$
- define nominal $\hat{T}$, $\hat{T}_m$ with $C_m \frac{\mathrm{d}\hat{T}_m(t)}{\mathrm{d}t} = \frac{\hat{T}(t) - \hat{T}_m(t)}{R_m}$
- then thermal energy $x_m(t) = C_m(T_m(t) - \hat{T}_m(t))$ satisfies

$$
\begin{aligned}
\frac{\mathrm{d}x_m(t)}{\mathrm{d}t} &= C_m \left(\frac{\mathrm{d}T_m(t)}{\mathrm{d}t} - \frac{\mathrm{d}\hat{T}_m(t)}{\mathrm{d}t} \right) \\
&= \frac{T(t) - \hat{T}(t) - (T_m(t) - \hat{T}_m(t))}{R_m} \\
&= -\underbrace{\frac{x_m(t)}{R_m C_m}}_{x_m(t)/\tau_m} + \underbrace{\frac{T(t) - \hat{T}(t)}{R_m}}_{p_m^{\mathrm{thrm}}(t)}
\end{aligned}
$$

Thermal mass as a thermal battery: Energy capacity

- suppose thermal mass temperature must satisfy

$$\underline{T}_m(t) \leq T_m(t) \leq \overline{T}_m(t)$$

- then thermal energy $x_m(t) = C(T_m(t) - \hat{T}_m(t))$ must satisfy

$$\underbrace{C_m(\underline{T}_m(t) - \hat{T}_m(t))}_{\underline{x}_m(t)} \leq x_m(t) \leq \underbrace{C_m(\overline{T}_m(t) - \hat{T}_m(t))}_{\overline{x}_m(t)}$$

- for a house with $C_m \approx 25$ kWh/°C, $\overline{T}_m(t) - \underline{T}_m(t) \approx 2$ °C,

$$\overline{x}_m(t) - \underline{x}_m(t) \approx 50 \text{ kWh}$$

Thermal mass as a thermal battery: Power capacities

- indoor air temperature has constraints

$$\underline{T}(t) \leq T(t) \leq \overline{T}(t)$$

- so $p_m^{\text{thrm}}(t) = (T(t) - \hat{T}(t))/R_m$ must satisfy

$$\underbrace{\frac{\underline{T}(t) - \hat{T}(t)}{R_m}}_{-\overline{p}_d^{\text{thrm}}(t)} \leq p_m^{\text{thrm}}(t) \leq \underbrace{\frac{\overline{T}(t) - \hat{T}(t)}{R_m}}_{\overline{p}_c^{\text{thrm}}(t)}$$

- for a house with $R_m \approx 0.5\ ^\circ\text{C/kW}$, $\overline{T}(t) - \underline{T}(t) \approx 2\ ^\circ\text{C}$,

$$\overline{p}_c^{\text{thrm}} + \overline{p}_d^{\text{thrm}} \approx 4\ \text{kW}$$

Summary: Buildings as thermal batteries

- can view a 2R2C model of a building as two thermal batteries
- indoor air and 'shallow' thermal mass has
  - time constant RC
  - thermal energy capacity $C(\overline{T}(t) - \underline{T}(t))$
  - thermal charging power capacity $\overline{q}_c(t) - \hat{q}_c(t)$
  - thermal discharging power capacity $\hat{q}_c(t) - \underline{q}_c(t)$
- 'deep' thermal mass has
  - time constant $R_m C_m$
  - thermal energy capacity $C_m(\overline{T}_m(t) - \underline{T}_m(t))$
  - thermal charging power capacity $(\overline{T}(t) - \hat{T}(t))/R_m$
  - thermal discharging power capacity $(\hat{T}(t) - \underline{T}(t))/R_m$

Comparing thermal storage and batteries

- a battery converts 1 kWh electrical back to ~1 kWh electrical
- a heat pump converts 1 kWh electrical to ~3 kWh thermal (more on heat pumps next lecture)
- so 1 kWh in a battery is 'worth' ~3 kWh thermal storage