

# Heating, ventilation, and air conditioning

Purdue ME 597, Distributed Energy Resources

Kevin J. Kircher

# Outline

Heating with electricity

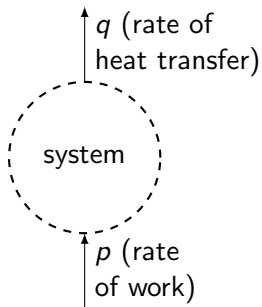
Air conditioning

Sizing heating and cooling equipment

Common HVAC system configurations

Thermal distribution models

# Electric resistance heating

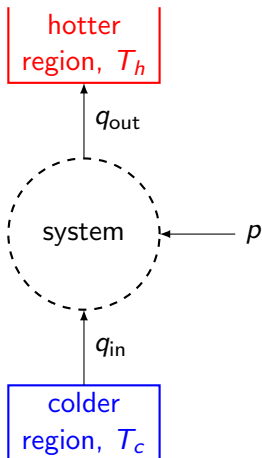


- (steady-state) 1st law:  $p = q$
- coefficient of performance:

$$\eta = \frac{\text{heat transfer output}}{\text{net work input}} = \frac{q}{p} = 1$$

- Joule heating:  $p = I^2 R$
- dirt-cheap to install, lasts  $\sim$ forever
- but inefficient/expensive to run

# Heat pump thermodynamic cycles



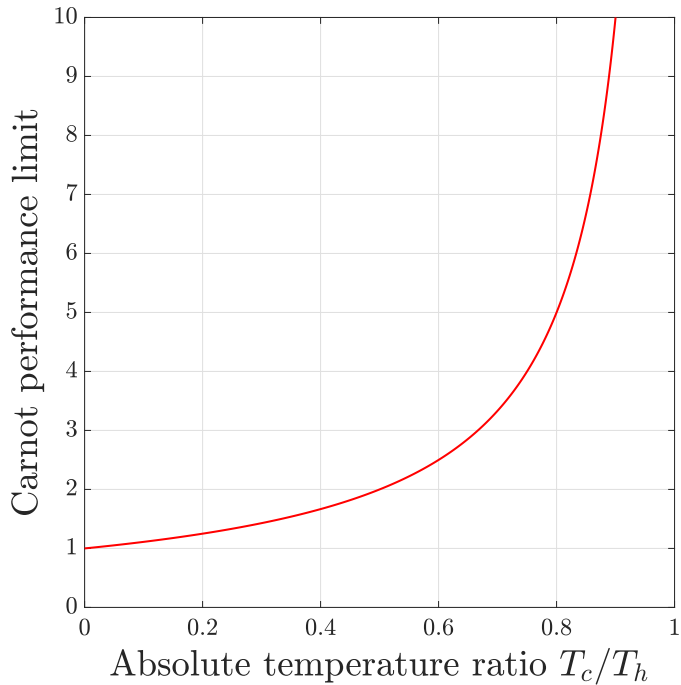
- 1st law:  $q_{in} + p = q_{out}$
- coefficient of performance:

$$\begin{aligned}\eta &= \frac{\text{heat transfer output}}{\text{work input}} \\ &= \frac{q_{out}}{p} = \frac{q_{out}}{q_{out} - q_{in}} \\ &= \frac{1}{1 - q_{in}/q_{out}}\end{aligned}$$

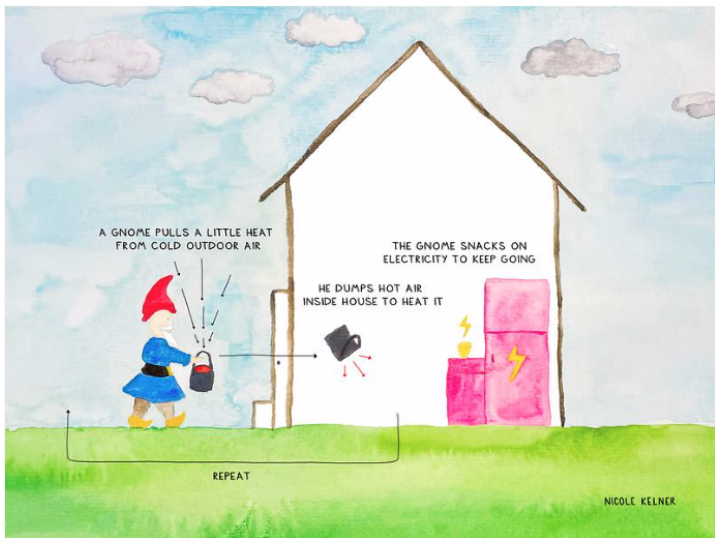
- Carnot performance limit:

$$\eta \leq \frac{1}{1 - T_c/T_h}$$

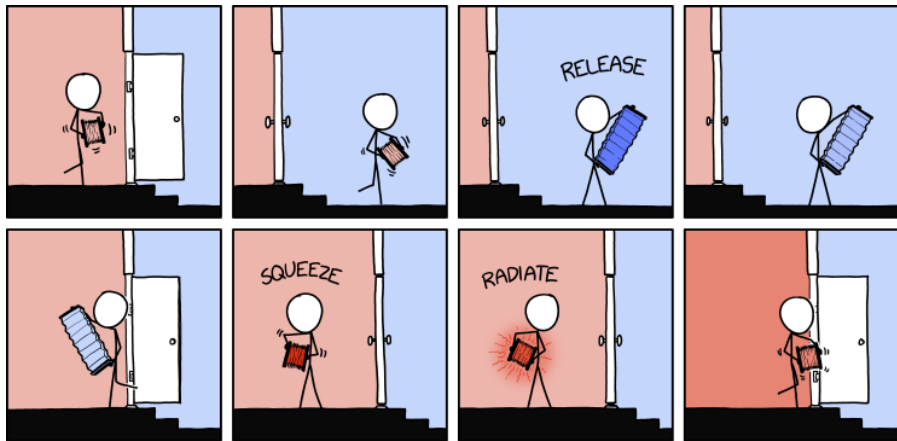
with  $T_c$ ,  $T_h$  in Kelvin



# How to pump heat

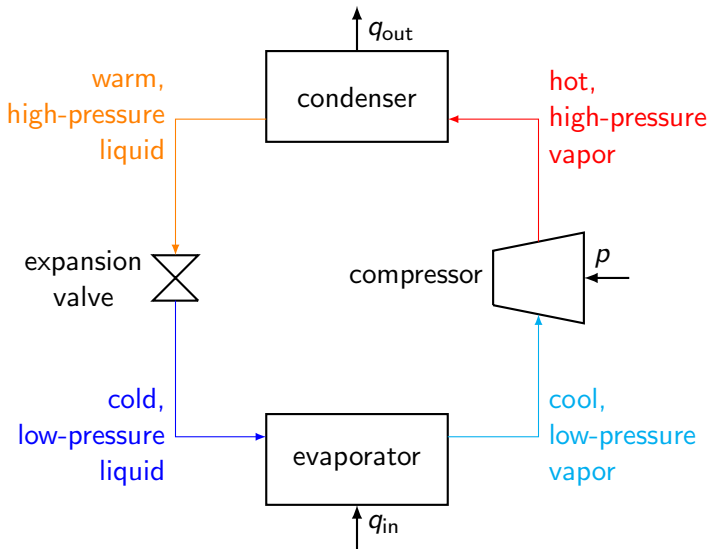


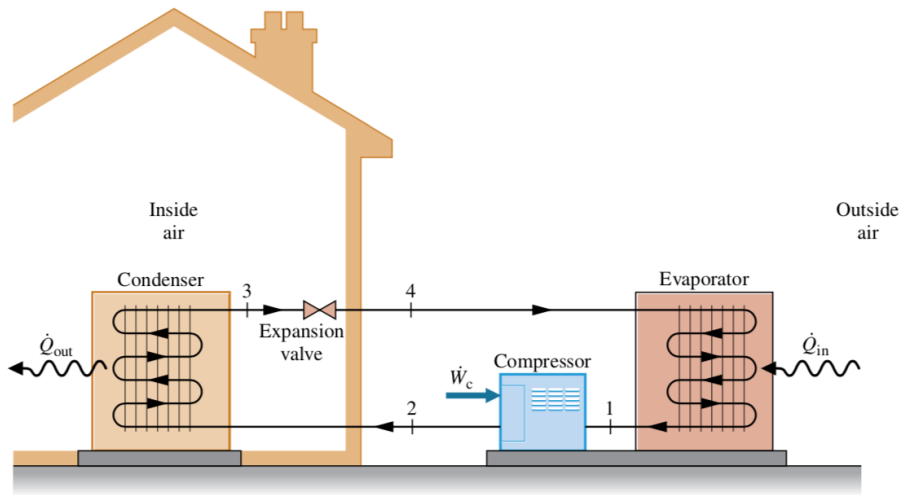
# How to pump heat



MANUAL HEAT PUMPS ARE SUCH A PAIN.

# How to pump heat (vapor compression cycle)

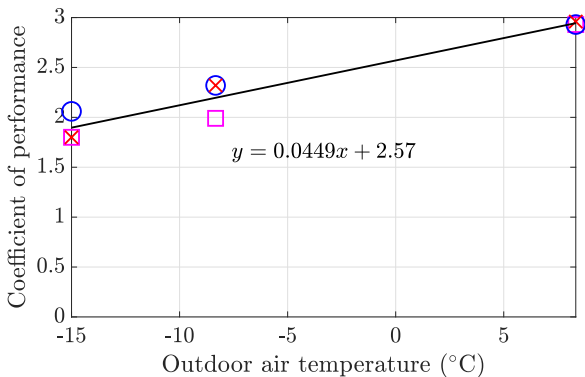




Moran (2018): *Fundamentals of Engineering Thermodynamics*

# Fitting real heat pump COPs to manufacturer data

- real heat pumps COPs don't come close to Carnot limit
- NEEP [collects](#) manufacturer-reported steady-state COP data



central ducted units from 3 manufacturers, 21 °C indoor air

# Simple heat pump simulation

- assume  $\sim$ constant indoor temperature
- model COP as a function  $\sim$ only of outdoor temperature  $\theta$
- fit COP curve  $\eta : \mathbf{R} \rightarrow \mathbf{R}$  to manufacturer data, such as

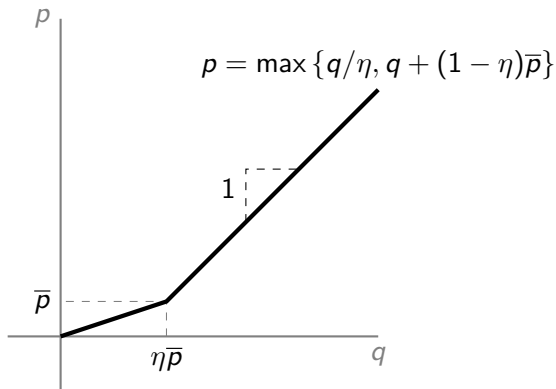
$$\eta(\theta) = \max \{1, 0.0449\theta + 2.57\}$$

(most heat pumps switch to resistance [COP 1] at low  $\theta$ )

- read off cold-weather compressor power limit  $\bar{p}$  from data
- simulate building with constraint  $q(t) \in [0, \eta(\theta(t))\bar{p}]$
- set input electric power to  $p(t) = q(t)/\eta(\theta(t))$

# Heat pumps with resistance backup

- heat pumps are expensive to install but cheap to run
- resistance is cheap to install but expensive to run
- hybrid systems pair heat pumps with resistance backup



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**Air conditioning**

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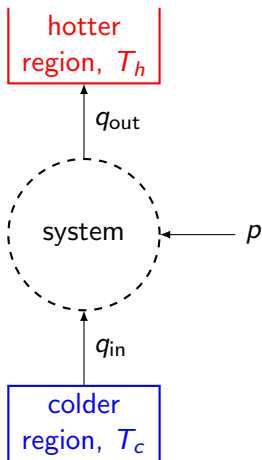
# Air conditioners are just one-way heat pumps

- most heat pumps can run in reverse to cool and dehumidify  
⇒ lower up-front cost than heater + (one-way) air conditioner

# Heat pump vocabulary

heat source	heat sink	device name
refrigerator air	kitchen air	refrigerator
freezer air	kitchen air	freezer
outdoor air	indoor air	air-source heat pump (ASHP) (or air-to-air heat pump)
indoor air	outdoor air	air conditioner or ASHP
outdoor ground	indoor air	ground-source heat pump (or geothermal heat pump)
outdoor air	indoor water	heat-pump water heater
indoor air	indoor water	heat-pump water heater
indoor water	outdoor air	chiller
outdoor water	indoor air	water-source heat pump

# Refrigeration thermodynamic cycles



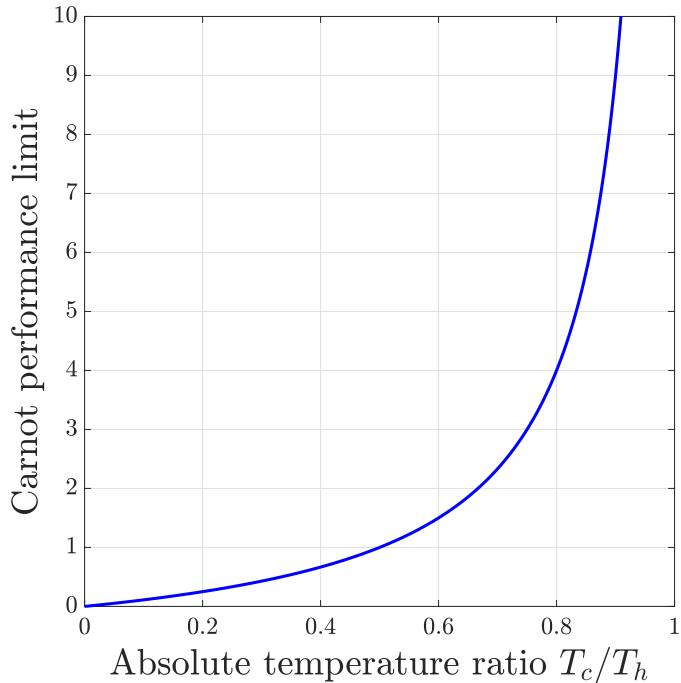
- 1st law:  $q_{in} + p = q_{out}$
- coefficient of performance:

$$\begin{aligned}\eta &= \frac{\text{heat transfer input}}{\text{work input}} \\ &= \frac{q_{in}}{p} = \frac{q_{in}}{q_{out} - q_{in}} \\ &= \frac{q_{in}/q_{out}}{1 - q_{in}/q_{out}}\end{aligned}$$

- Carnot performance limit:

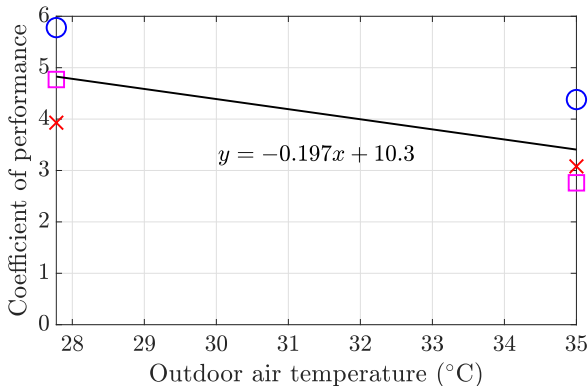
$$\eta \leq \frac{T_c/T_h}{1 - T_c/T_h}$$

with  $T_c$ ,  $T_h$  in Kelvin



# Real air conditioner COPs

NEEP [database](#) also has cooling COP data



central ducted units from 3 manufacturers, 21 °C indoor air

# Dehumidification

- air conditioners
  - ◊ reduce indoor air temperature (**sensible** load)
  - ◊ condense water out of indoor air (**latent** load)
- total load = sensible load + latent load
- **sensible heat ratio**  $s$  is ratio of sensible load to total load
- building simulations may produce sensible load  $q(t)$  only
- to account for dehumidification, estimate  $s$  and set

$$p(t) = \frac{q_{\text{tot}}(t)}{\eta(\theta(t))} = \frac{q(t)}{s\eta(\theta(t))}$$

- in reality,  $s$  depends on weather, building, occupant behavior
- first cut: set  $s \approx 70$  to 95% for humid to dry climates

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## Sizing for heating

- estimate overall indoor-outdoor thermal resistance  $R$
- get design outdoor temperature  $\theta^{\text{des}}$
- set design indoor temperature  $T^{\text{des}}$  to occupant preference
- pick plausible  $q_e^{\text{des}}$  for  $\sim 4$  AM
- size to steady-state heat load in design conditions:

$$\bar{p}_h = \frac{r}{\eta(\theta^{\text{des}})} \left( \frac{T^{\text{des}} - \theta^{\text{des}}}{R} - q_e^{\text{des}} \right)$$

- oversize ratio  $r \approx 1.2$  to  $1.5$ , typically

## Sizing for cooling

- like heating, but

$$\bar{p}_c = \frac{r}{s\eta(\theta^{\text{des}})} \left( \frac{\theta^{\text{des}} - T^{\text{des}}}{R} + q_e^{\text{des}} \right)$$

- $q_e^{\text{des}}$  should be plausible for sunny afternoon

## Sizing two-way heat pumps

- calculate  $\bar{p}_h$  and  $\bar{p}_c$  for heating and cooling design conditions
- if  $\bar{p}_h \leq \bar{p}_c$ , set  $\bar{p} = \bar{p}_c$  (size for cooling)
- if  $\bar{p}_h > \bar{p}_c$ , options:
  1. set  $\bar{p} = \bar{p}_h$  (size for heating)
  2. set  $\bar{p} = \bar{p}_c$  and add backup  $\geq \eta(\theta^{\text{des}})(\bar{p}_h - \bar{p})$
  3. get biggest available unit and add backup  $\geq \eta(\theta^{\text{des}})(\bar{p}_h - \bar{p})$
- backup heat could be
  - ◇ another heat pump
  - ◇ resistance
  - ◇ heat storage
  - ◇ wood
  - ◇ propane
  - ◇ heating oil
  - ◇ natural gas

## Sizing example for a house in Lafayette

	$\theta^{\text{des}}$ ( $^{\circ}\text{C}$ )	$T^{\text{des}}$ ( $^{\circ}\text{C}$ )	$q_e^{\text{des}}$ (kW)	$\eta(\theta^{\text{des}})$
heating	-16	21	1	1.8
cooling	32	24	4	4

- input parameters:  $R = 3$   $^{\circ}\text{C}/\text{kW}$ ,  $r = 1.3$ ,  $s = 0.8$
- sizing results:  $\bar{p}_h = 8.2$  kW,  $\bar{p}_c = 2.7$  kW
- biggest available residential heat pumps have  $\bar{p} \approx 7.5$  kW

$\implies$  need some form of backup heat

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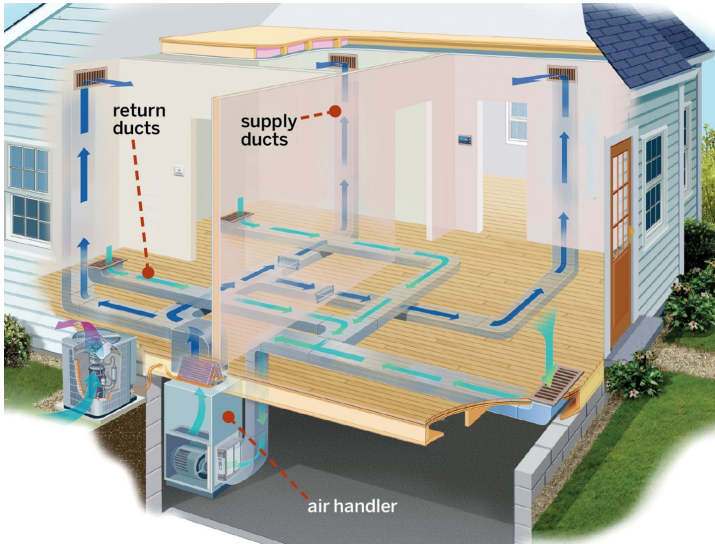
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# Central ducted residential systems



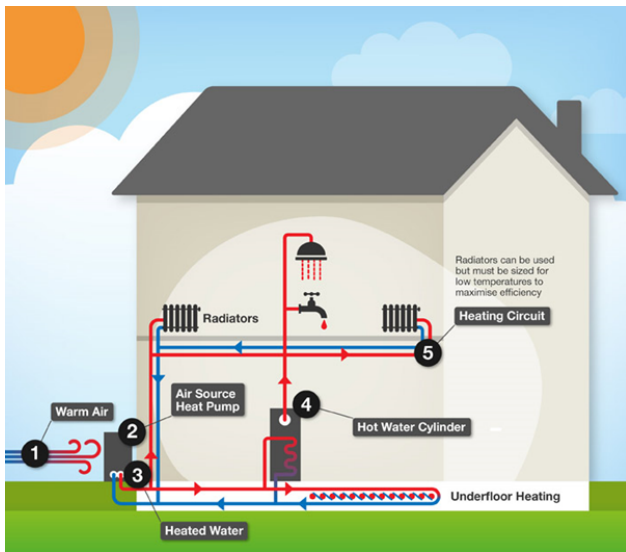
This Old House: [Central air conditioning](#)

# Ductless mini-split residential systems

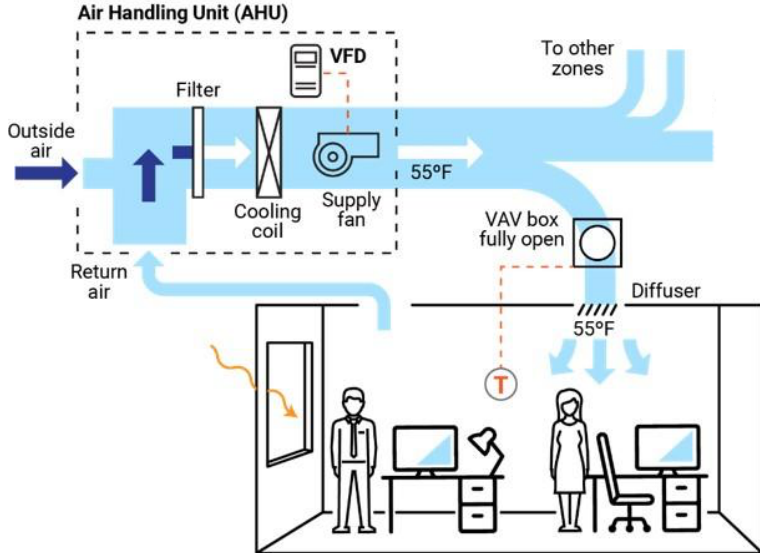


New Hampshire Electric Co-Op: [Ductless mini-split heat pumps](#)

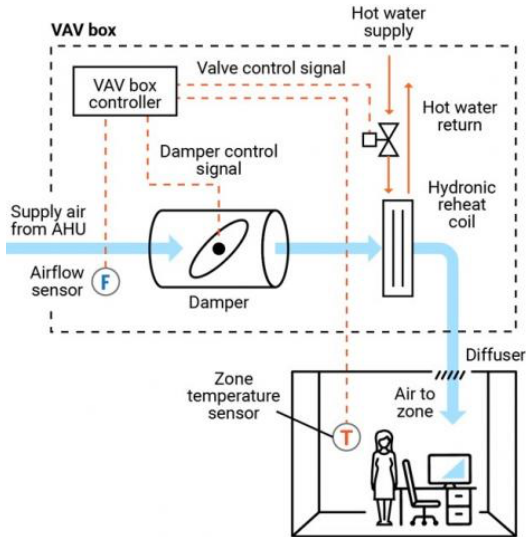
# Hydronic residential systems



# Variable air volume commercial systems



# Variable air volume boxes



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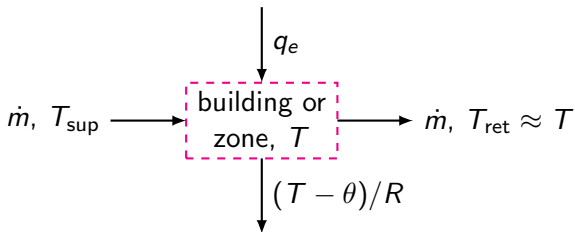
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## Forced-air heat transfer

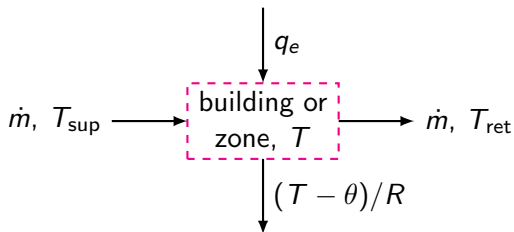


- $\dot{m}(t)$  (kg/s) is mass flow rate of supply air
- power balance:

$$C \frac{dT(t)}{dt} = \frac{\theta(t) - T(t)}{R} + \underbrace{\dot{m}(t)c_p(T_{\text{sup}}(t) - T(t))}_{q_c(t)} + q_e(t)$$

- $c_p = 1 \text{ kJ}/(\text{kg}^\circ\text{C})$  is specific heat of air at constant pressure

# Hydronic heat transfer

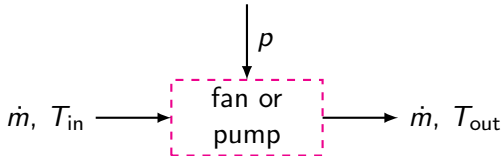


- $\dot{m}(t)$  (kg/s) is mass flow rate of supply water
- power balance:

$$C \frac{dT(t)}{dt} = \frac{\theta(t) - T(t)}{R} + \underbrace{\dot{m}(t)c(T_{\text{sup}}(t) - T_{\text{ret}}(t))}_{q_c(t)} + q_e(t)$$

- $c = 4.2 \text{ kJ}/(\text{kg}^\circ\text{C})$  is specific heat of water

# Fans and pumps



- (rough) fan power balance:

$$p(t) \approx \dot{m}(t)c_p(T_{out}(t) - T_{in}(t))$$

- pump:

$$p(t) \approx \dot{m}(t) \left[ c(T_{out}(t) - T_{in}(t)) + \frac{P_{out}(t) - P_{in}(t)}{\rho} \right]$$

- $P_{in}(t)$ ,  $P_{out}(t)$  (kPa) are inlet, outlet pressures
- $\rho = 1000 \text{ kg/m}^3$  is density of water

# Pump and fan affinity laws

- in theory, pumps and fans follow the affinity law

$$p(t) = \alpha \dot{m}(t)^3$$

where  $\alpha = p_{\text{rated}} / \dot{m}_{\text{rated}}^3$

- in practice, usually fit a model to  $(\dot{m}, p)$  data, such as

$$p(t) = \beta_0 + \beta_1 \dot{m}(t) + \beta_2 \dot{m}(t)^2 + \beta_3 \dot{m}(t)^3$$