

# Homework 2: Batteries and electric vehicles

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## Directions:

- Students may work individually or in groups, but each student must upload their own solutions to [Gradescope](#) by **11:59 PM ET on Monday, February 9**.
- Use any outside resources you want, but **cite your sources**, including if and how you use any AI tools. (If you want to learn the material, I recommend trying the problems yourself before looking for outside help. This lets us identify the things we don't fully understand so we can figure them out.)
- The TA will grade each problem or subproblem quickly on a three-tier scale:
  - No credit for a solution that's mostly unreadable or missing.
  - Half credit for a serious attempt that's not easy to read or is substantially incorrect.
  - Full credit for a solution that's clearly readable and nearly or completely correct.

## Problems:

1. (Refer to 'Linear dynamical systems' lecture slides.)

- (a) (10%) Show that with uniform time step  $\Delta t$  and piecewise constant  $p^{\text{chem}}(t)$ , the continuous-time battery model

$$\frac{dx(t)}{dt} = -\frac{x(t)}{\tau} + p^{\text{chem}}(t)$$

can be written in discrete time as

$$x(k+1) = ax(k) + (1-a)\tau p^{\text{chem}}(k), \quad (1)$$

where  $a = e^{-\Delta t/\tau}$ . *Hint: You can start from a highlighted equation in the slides; you do not have to derive the highlighted equation from scratch.*

- (b) (15%) In the special case of a battery with no self-dissipation, the continuous-time model simplifies to

$$\frac{dx(t)}{dt} = p^{\text{chem}}(t).$$

Show that with uniform time step  $\Delta t$  and piecewise constant  $p^{\text{chem}}(t)$ , a discrete-time version of this model is

$$x(k+1) = x(k) + \Delta t p^{\text{chem}}(k). \quad (2)$$

Show that (1)  $\rightarrow$  (2) as  $\tau \rightarrow \infty$ . *Hint: Set  $\alpha = 1/\tau$  and show that  $\lim_{\alpha \rightarrow 0} (1 - e^{-\alpha \Delta t})/\alpha = \Delta t$ .*

2. (Short calculations; take care with units.)

- (a) (5%) The charge state of a battery, initially at 80% of its energy capacity, drops to 50% after 30 days unplugged and unused. What is the battery's self-dissipation time constant?

- (b) (15%) Suppose an electric vehicle has an energy intensity of  $\alpha = 0.3$  kWh/km and a comparable gasoline vehicle gets  $\beta = 25$  miles per gallon. If burning one gallon of gasoline causes  $\gamma = 26$  pounds of CO<sub>2</sub> emissions (including upstream emissions associated with oil extraction and processing), what is the break-even CO<sub>2</sub> intensity of electricity  $\mu$  (in units of g/kWh) at which the two vehicles cause the same CO<sub>2</sub> emissions per unit distance driven? By what percent would the EV reduce CO<sub>2</sub> emissions from driving with the US-average CO<sub>2</sub> intensity of electricity, 345 g/kWh? With the average CO<sub>2</sub> intensity of electricity in your home [state](#) or [country](#)? What factors not considered here might complicate this analysis?
- (c) (10%) Suppose you commute two miles each way, five days per week, 50 weeks per year, in a place where electricity costs 0.15 \$/kWh and gasoline costs 3 \$/gallon. How much do you spend per year on energy for commuting if you
- ride an electric bike with an energy intensity of 5 Wh/km?
  - drive an electric car with an energy intensity of 0.3 kWh/km?
  - drive a gasoline car that get 30 miles per gallon?
- (d) (5%) Suppose that, in the setting of question 2c, you're considering buying the electric car or the gasoline car. You're willing to spend more on the electric car as long as the cost savings from commuting energy and reduced maintenance pay off the additional up-front cost within five years. Assume maintaining the gasoline car costs \$400 more per year than maintaining the electric car. How much more money can you spend up front on the electric car than the gasoline car?
3. (Refer to 'Batteries and electric vehicles' lecture slides.) Download the files in the Github repository [electric-vehicles](#). Fill in the missing code from the functions `simulatePolicy1` (10%), `simulatePolicy2` (10%), and `simulatePolicy3` (20%). Given the inputs in the `simulateEV` script, these functions should return trajectories of the EV battery's stored chemical energy and electrical charging power. Show the missing lines of code here. Show the graphs here that `simulateEV` draws in figures 1 through 4.