

Homework 3: Solar energy

Kevin J. Kircher, Purdue ME 597

Directions:

- Students may work individually or in groups, but each student must upload their own solutions to [Gradescope](#) by **11:59 PM ET on Monday, February 16**.
- Use any outside resources you want, but **cite your sources**, including if and how you use any AI tools. (If you want to learn the material, I recommend trying the problems yourself before looking for outside help. This lets us identify the things we don't fully understand so we can figure them out.)
- The TA will grade each problem or subproblem quickly on a three-tier scale:
 - No credit for a solution that's mostly unreadable or missing.
 - Half credit for a serious attempt that's not easy to read or is substantially incorrect.
 - Full credit for a solution that's clearly readable and nearly or completely correct.

Problems:

1. (10%) At what local time will the sun set in West Lafayette on March 8? Show your work. *Hint: What is the sun's elevation angle at sunset?*
2. (15%) How many km^2 of land would be needed to power US light-duty vehicles if they were all
 - (a) combustion vehicles powered by corn ethanol?
 - (b) electric vehicles powered by solar photovoltaics?

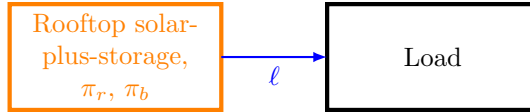
Assume the following:

- Each year, US light-duty vehicles drive ~ 2.9 trillion miles and burn ~ 120 billion gallons of gasoline.
- Ethanol's energy density is $\sim 67\%$ of gasoline's.
- The US produces ~ 15.5 billion gallons per year of corn ethanol from ~ 35 million acres of farmland.
- On average over the US, solar photovoltaic farms generate ~ 400 MWh per year per acre of land.
- The energy intensity of driving a typical US electric vehicle is ~ 0.3 kWh/km.
- Between a solar farm and a vehicle, $\sim 10\%$ of electricity is lost in transmission and distribution.

For each scenario, find a [state](#) with about the same land area as your answer. What percent of the total 7.7 million km^2 of land area in the US would be needed in each scenario? Does this analysis make any questionable assumptions or leave out any potentially important factors? If so, explain.

3. This question compares the overall capital costs of installing rooftop solar photovoltaics with energy storage ('solar-plus-storage') and utility-scale solar-plus-storage. To explore this question, imagine adding new load to a power grid that currently has just enough generation, transmission, and distribution capacity to meet load at the grid's peak times. Adding new load pushes the grid's peak load beyond its existing capacity by an amount ℓ (W).

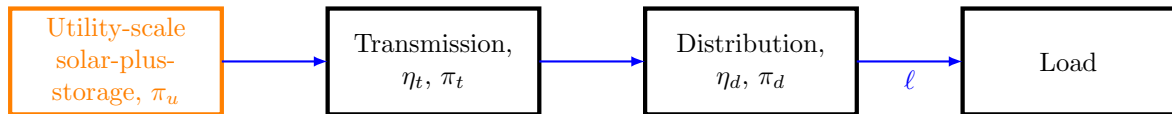
In the rooftop configuration pictured below, new rooftop solar-plus-storage directly meets the new load ℓ at grid peak times.



The rooftop configuration costs $\pi_r \ell$, where π_r (\$/W) is the installed cost of rooftop solar-plus-storage, normalized by its capacity to produce power at grid peak times. The rooftop configuration also adds value $\pi_b \ell$ by providing the capability to power potentially vital services during power outages. The price π_b (\$/W) is the value of 1 Watt of emergency backup capacity. The total cost to society of the rooftop configuration is therefore

$$\underbrace{\pi_r \ell}_{\text{solar-plus-storage cost}} - \underbrace{\pi_b \ell}_{\text{blackout value}} = (\pi_r - \pi_b) \ell.$$

In the utility-scale configuration pictured below, new utility-scale solar-plus-storage indirectly meets the new load ℓ by way of a transmission network and a distribution network.



- (a) (10%) Show that the utility-scale configuration costs

$$\left(\frac{\pi_u + \pi_t}{\eta_t} + \pi_d \right) \frac{\ell}{\eta_d},$$

where π_u (\$/W) is the installed cost of utility-scale solar-plus-storage, normalized by the system's capacity to produce power at system peak times, π_t (\$/W) is the price of new transmission capacity, π_d (\$/W) is the price of new distribution capacity, η_t is the transmission network efficiency, and η_d is the distribution network efficiency.

- (b) (10%) Find an expression for the break-even rooftop solar-plus-storage price π_r^* , defined such that if $\pi_r = \pi_r^*$, then the costs of the rooftop and utility-scale configurations are equal.
- (c) (10%) Assume $\eta_t = 97\%$, $\eta_d = 96\%$, $\pi_t = 0.5$ \$/W, $\pi_d = 1$ \$/W, and $\pi_b = 0.55$ \$/W (roughly the price of a backup generator [~ 20 kW for ~ 11 k]). Plot π_r^* vs. π_u , with π_u ranging from 0 to 5 \$/W. Mark the region where the rooftop configuration costs less than the utility-scale configuration. (*Hint: What does it mean if $\pi_r < \pi_r^*$?*) What do you take away from this plot? Are there assumptions or input data in this problem that, if changed, would significantly change your take-away(s)? Explain.

4. Download the Matlab or Python files in the Github repository [solar](#) and open the script `simulateSolar`.

- (a) (15%) Fill in the missing code in the function `solarAngles`, which should calculate the solar elevation and azimuth angles (in degrees) at a given location (specified by the latitude and longitude angles) over a given time span (specified as a `datetime` object). Show your code here.
- (b) (15%) Fill in the missing code in the function `surfaceIrradiance`, which should calculate the total, beam, and diffuse irradiance on a surface at a given orientation (specified by its azimuth and tilt angles) given solar angles and irradiance measurements from a standard weather station. Show your code here.
- (c) (5%) In the `solar power supply` section of `simulateSolar`, fill in the code to calculate `solarEta`, the temperature-dependent solar array efficiency. Show your code here.

- (d) (5%) In the `electricity costs` section of `simulateSolar`, fill in the code to calculate `c3`, the annual electricity cost with reduced net metering. Report your numerical results for the electricity costs with one-to-one net metering, with reduced net metering, and without solar.
5. (5%, graded for completion, either no credit or full credit.) Write down the names of your project team members and the topic or topics your project will focus on.