Homework 5: HVAC and thermal storage

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Directions:

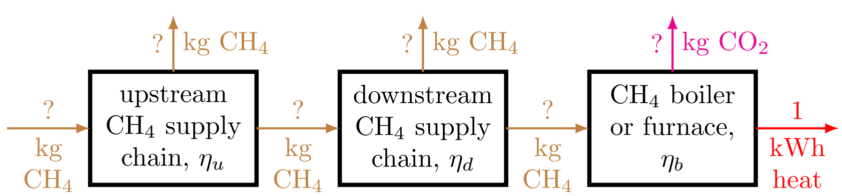
- Students may work individually or in groups, but each student must upload their own solutions to Gradescope by **11:59 PM ET on Friday, March 6**.
- Use any outside resources you want, but **cite your sources**, including if and how you use any AI tools. (If you want to learn the material, I recommend trying the problems yourself before looking for outside help. This lets us identify the things we don't fully understand so we can figure them out.)
- The TA will grade each problem or subproblem quickly on a three-tier scale:
  - No credit for a solution that's mostly unreadable or missing.
  - Half credit for a serious attempt that's not easy to read or is substantially incorrect.
  - Full credit for a solution that's clearly readable and nearly or completely correct.

Problems:

1. (10%) Fill in the missing steps (marked '...') in slide 6 of the 'Thermal storage and water heaters' lecture.

2. Methane (CH_4, the main component of natural gas) is both a fuel that releases CO_2 when burned and a potent greenhouse gas that, when accidentally leaked or intentionally vented, causes α times as much climate harm per unit mass as CO_2. (The parameter α is uncertain, but is likely between 30 and 90.) This question explores the influence of CH_4 emissions on comparisons of climate impacts from heating with natural gas vs. electricity.

   Useful input data: The energy density of CH_4 is about $\gamma = 15$ kWh/kg. Burning one kmol (16 kg) of CH_4 via the reaction $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$ produces one kmol (44 kg) of CO_2. Since $44/16 = 2.75$, burning one kg of CH_4 produces $\mu = 2.75$ kg of CO_2.

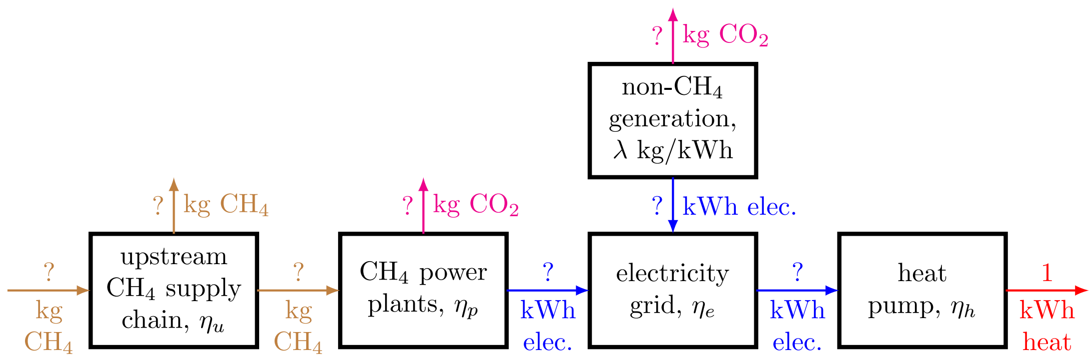


   (a) (10%) In the CH_4 heating system illustrated above, the efficiencies of the upstream CH_4 supply chain (extraction, gathering, processing, transmission, and storage), downstream CH_4 supply chain (distribution and use), and CH_4 boiler or furnace are η_u, η_d, $\eta_b \in (0, 1)$. Show that to produce 1 kWh of heat, the above system emits

$$\frac{1}{\gamma \eta_b}\left[\alpha\left(\frac{1}{\eta_u \eta_d} - 1\right) + \mu\right] \tag{1}$$

kg of CO_2-equivalent. Assume that in terms of harm done by climate change, 1 kg of CH_4 pollution is equivalent to α (a variable) kg of CO_2 pollution. *Hint: Start at the right and work your way left through the diagram, filling in the '?'s.*

(b) (10%) Suppose $\eta_u = 97\%$ and $\eta_d = 97.5\%$. With $\alpha = 30$, what percent of the CO_2-equivalent emissions in the CH_4 heating system comes from CH_4 leaks? With $\alpha = 90$?



(c) (15%) In the electric heating system illustrated above, the efficiencies of the CH_4 power plants and electricity grid are η_p, $\eta_e \in (0, 1)$. The heat pump's coefficient of performance is η_h. A share $\phi \in [0, 1]$ of electricity comes from non-CH_4 generators, while the remaining share $1 - \phi$ comes from CH_4 power plants. The non-CH_4 generators collectively emit λ kg of CO_2-equivalent per kWh generated. Show that to produce 1 kWh of heat, the above system emits

$$\frac{1}{\eta_e \eta_h}\left[\frac{1-\phi}{\gamma \eta_p}\left(\alpha\left[\frac{1}{\eta_u}-1\right]+\mu\right)+\lambda\phi\right] \tag{2}$$

kg of CO_2-equivalent.

(d) (15%) Write down an expression for the break-even heat pump coefficient of performance $\eta_h^\star$ that makes the climate impacts from the CH_4 heating system from part (a) equal the climate impacts from the electric heating system from part (c). (In other words, set expression (1) equal to expression (2) and solve for η_h.) Assuming $\eta_u = 97\%$, $\eta_d = 97.5\%$, $\eta_p = 50\%$, $\eta_e = 90\%$, $\eta_b = 90\%$, $\gamma = 15$ kWh/kg, $\mu = 2.75$, and $\lambda = 0.02$ kg/kWh, fill in the $\eta_h^\star$ column of the following table. What observations do you take away from this table?

ϕ	α	$\eta_h^\star$
0 (electricity entirely from CH_4)	30	
0	90	
0.4 (current U.S. share of clean electricity)	30	
0.4	90	
1 (entirely clean electricity)	30	
1	90	

3. Download the Matlab files in the Github repository water-heaters and open the script `simulateWH`.

(a) (15%) Fill in the missing code in the function `waterHeaterControl`, which should implement near-perfect setpoint tracking control (with saturation of the HVAC thermal power at equipment capacity limits if necessary) in any of three cases: a resistance-only water heater, a heat-pump-only water heater, or a hybrid water heater with a heat pump and resistance backup. Show your code here.

(b) (15%) The script `simulateWH` randomly generates plausible hot water draws. Run it until the stored energy drops below the resistance turn-on threshold x_r in the hybrid case. Show the plots from figures 1, 2, and 3 here.

4. (5%) Provide a link to your project pitch video. The video must be no more than ten minutes long and may feature only one student speaking (even for teams of two or more students). The video should (with slides) succinctly describe the problem you propose to investigate, summarize the background reading your team has done, and discuss your proposed solution approach and anticipated findings. You may record using Zoom (local recording), PowerPoint "Record/Export," or a screen recorder with Google Slides, and then share a view-only link via Google Drive, OneDrive, or an unlisted YouTube link. Please ensure the link is accessible to course staff without requesting access. Pitch videos that follow these instructions will receive full credit for this homework problem. The clarity of the pitch video will influence the final project grade to a small degree.