Solar energy

Purdue ME 597, Distributed Energy Resources

Kevin J. Kircher

Outline

The solar resource

The sun's position in the sky

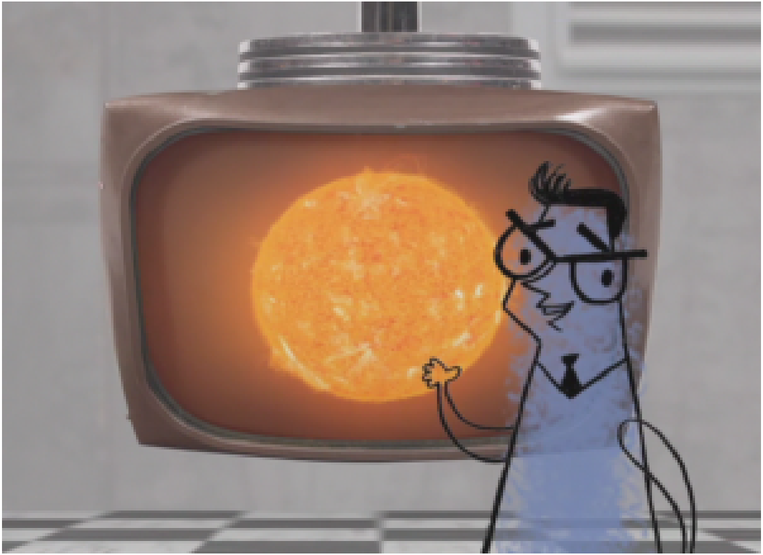
Incident irradiance on surfaces

Solar photovoltaics

The sun is (?) a mass of incandescent gas

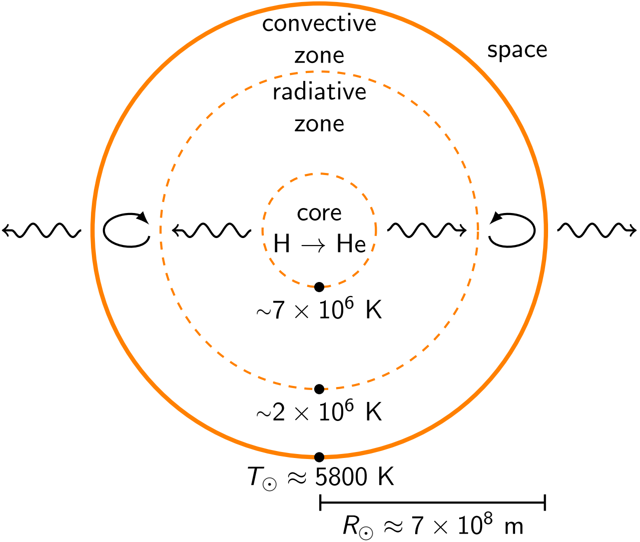
They Might Be Giants (1993): Why Does the Sun Shine?

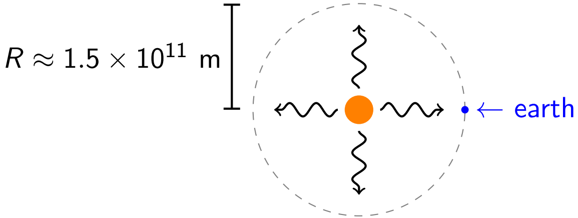
The sun is a miasma of incandescent **plasma**



They Might Be Giants (2009): Why Does the Sun Really Shine?

The sun


convective
zone
space
radiative
zone
core
H → He
~7 × 10^6 K
~2 × 10^6 K
$T_\odot \approx 5800$ K
$R_\odot \approx 7 \times 10^8$ m



- as a black body at $T_\odot \approx 5800$ K, sun's surface radiates

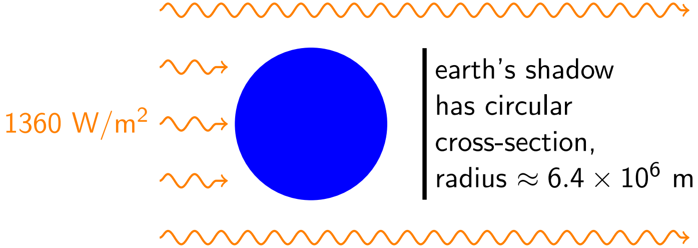
$$\sigma A_\odot T_\odot^4 = 4\sigma\pi R_\odot^2 T_\odot^4 \approx 4 \times 10^{26} \text{ W}$$

- at earth's distance from sun, irradiance is

$$S_0 := \frac{\sigma A_\odot T_\odot^4}{4\pi R^2} \approx \frac{4 \times 10^{26} \text{ W}}{4\pi (1.5 \times 10^{11} \text{ m})^2} \approx 1360 \text{ W/m}^2$$

- **solar constant** S_0 is irradiance at top of earth's atmosphere

How much solar power 'hits' the earth?



- solar power incident on earth's upper atmosphere:

$$(1360 \text{ W/m}^2)\pi(6.4 \times 10^6 \text{ m})^2 \approx 1.7 \times 10^{17} \text{ W}$$

- $\sim$10,000 times humanity's $\sim 1.8 \times 10^{13}$ W used for all purposes

(Almost) all energy on earth is (?) solar energy

- hydro $\leftarrow$ solar (via the water cycle)
- wind $\leftarrow$ solar (via hot air rising)
- biomass $\leftarrow$ solar (via photosynthesis)
- fossil fuels $\leftarrow$ biomass (via rotting/cooking underground) $\leftarrow$ solar

but

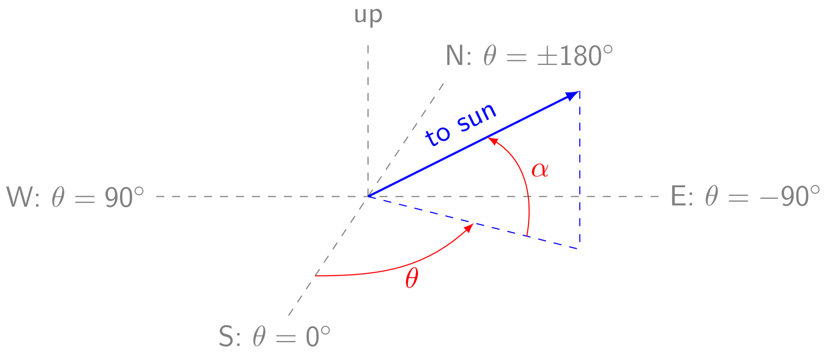
- nuclear $\not\leftarrow$ solar
- geothermal $\leftarrow$ nuclear (via reactions underground)

and solar $\leftarrow$ nuclear

so really, all (?) energy on earth is **nuclear** energy

Outline

Elevation and azimuth angles



- origin is at observer on earth's surface
- observer sees sun at **elevation** α and **azimuth** θ
- $\alpha = 0^\circ$ at sunrise/sunset
- $\theta = 0^\circ$ if sun is due south, increases clockwise

Elevation and azimuth formulas

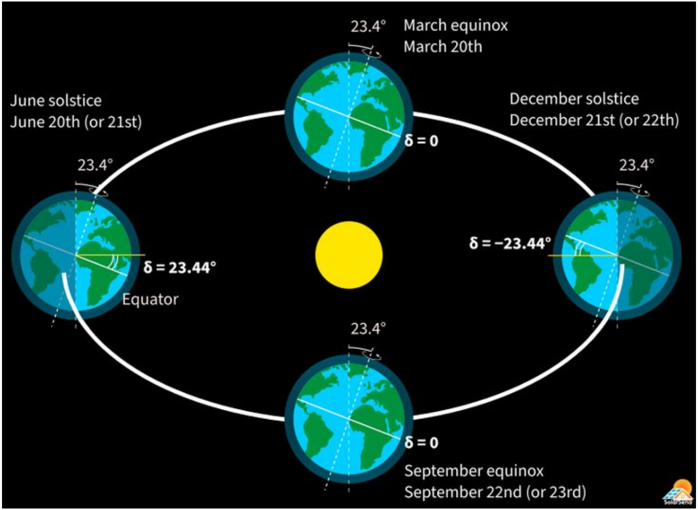
$$\alpha =$$

$$\theta =$$

- ϕ_0 is observer's **latitude**
- δ and λ are **declination** and **hour** angles
- acos returns values in $[0^\circ, 90^\circ]$; atan2 in $[-180^\circ, 180^\circ]$

Zhang (2021): A solar azimuth formula based on the subsolar point and atan2

Declination angle



$\delta \approx$, $\gamma =$, day # d is

- the empirical **equation of time**,

$$\tau \approx \qquad ,$$

  converts Greenwich Mean Time t_{gm} to local **solar time** (solar time, $t_{gm} + \tau$, equals 12 h when sun is highest)

- the hour angle,

$$\lambda = \qquad ,$$

  is defined such that
  - λ equals observer's **longitude** λ_0 when sun is highest
  - λ increases by 360° every 24 h

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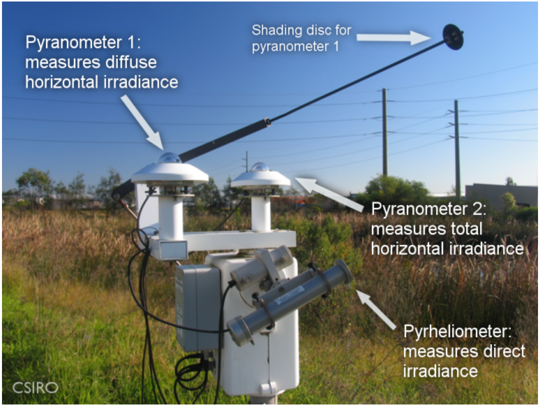
Incident irradiance on surfaces

Solar photovoltaics

Sunlight at earth's surface

- on a clear day, $\sim$75% of S_0 reaches earth's surface
  - ⋄ $\sim$70% transmitted through atmosphere (**beam**)
  - ⋄ $\sim$5% scattered to earth by air, dust, water vapor (**diffuse**)
  - ⋄ $\sim$20% absorbed by atmosphere
  - ⋄ $\sim$5% scattered back to space
- as cloud cover increases,
  - ⋄ less of S_0 reaches surface (as little as $\sim$10%)
  - ⋄ beam % of total surface sunlight falls, diffuse % rises

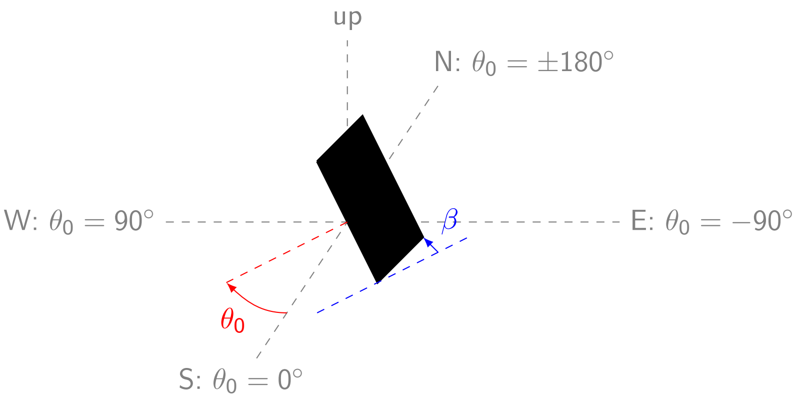
Sunlight measurements



- weather data services often provide one or more of
  - ◇ beam irradiance $S_{\mathrm{b}}^{\perp}$ on surface ⊥ to sunbeam
  - ◇ diffuse irradiance S_{d}^{-} on horizontal surface
  - ◇ total irradiance S_{tot}^{-} on horizontal surface
  - ◇ beam irradiance S_{b}^{-} on horizontal surface

CSIRO: Perfect day for solar power

Surface tilt and azimuth



- surface **tilt** β and **azimuth** θ_0 define its orientation
- $\beta = 0°$ for horizontal surfaces, $90°$ for vertical
- θ_0 follows same convention as sun's azimuth θ

Irradiance on arbitrarily oriented surface

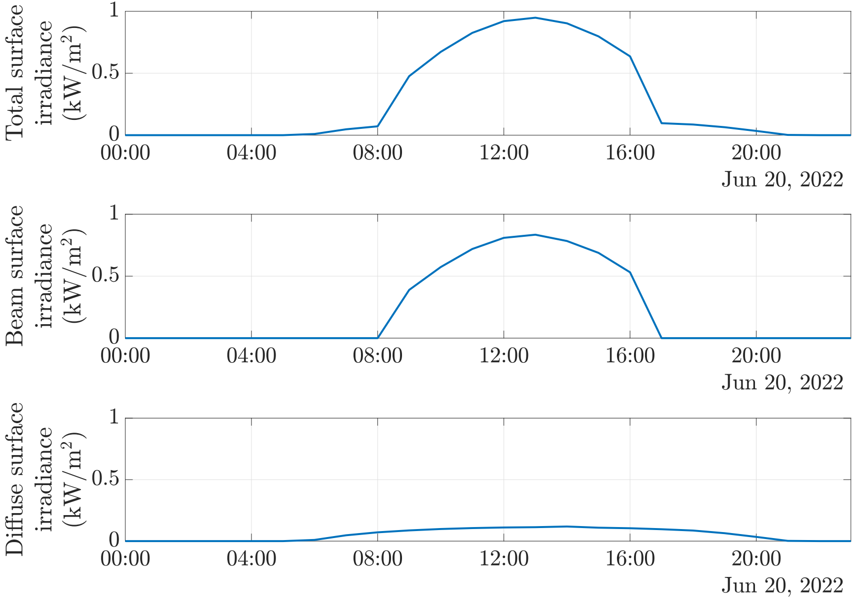
$$S_{\text{b}} = \begin{cases} \\ \end{cases}$$

$$S_{\text{d}} \approx$$

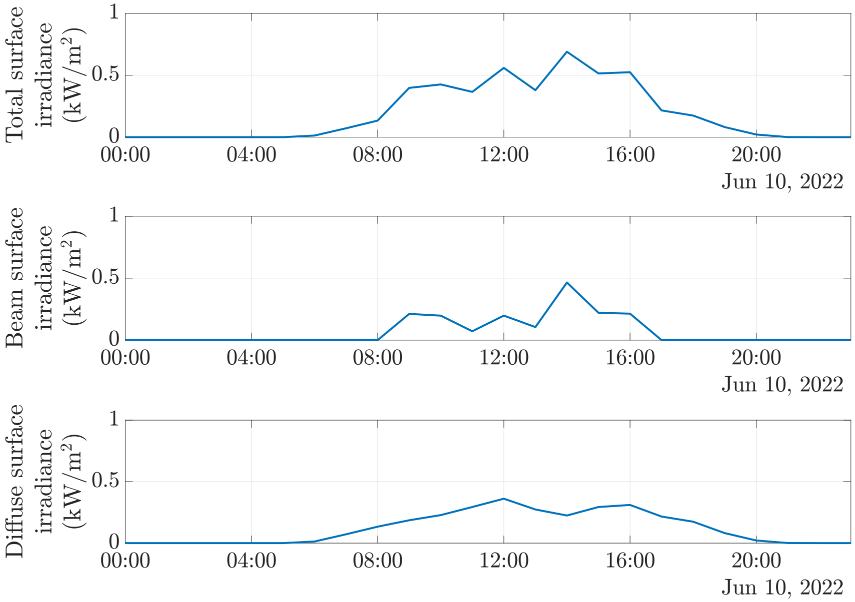
$$S_{\text{tot}} =$$

- S_{b} and S_{tot} are beam and total irradiance on surface
- $\alpha < 0^\circ$ means sun is down
- $90^\circ < |\theta - \theta_0| < 270^\circ$ means sun is behind surface

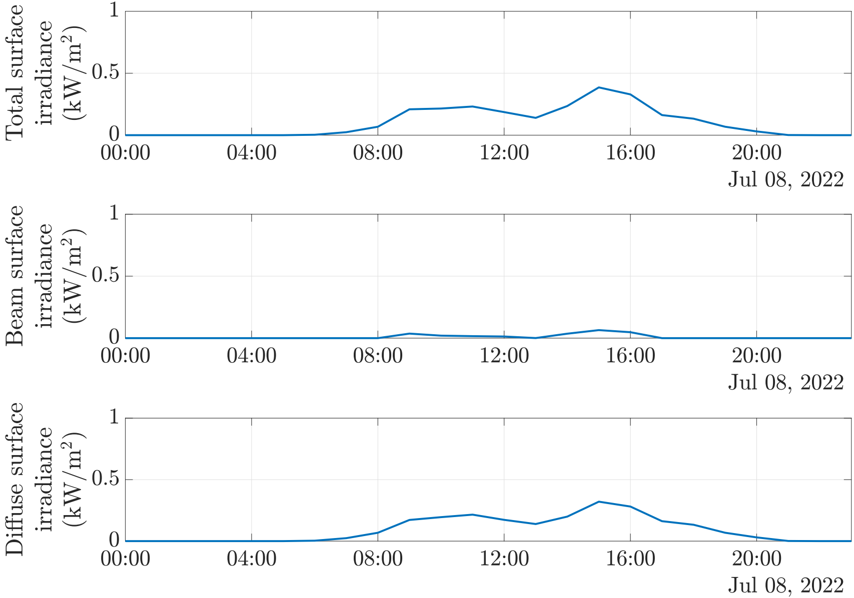
Clear summer day with $\beta = \phi_0$, $\theta_0 = 0^\circ$



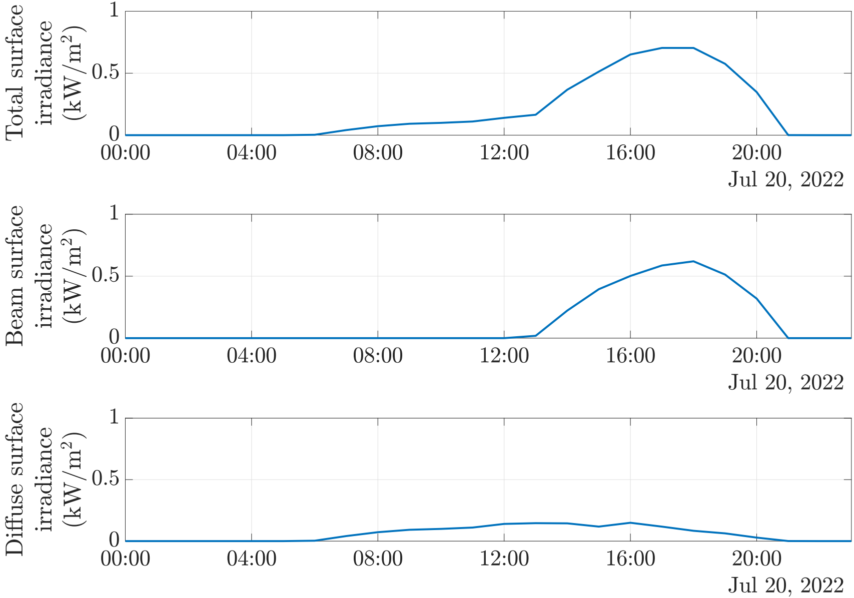
Partly cloudy summer day with $\beta = \phi_0$, $\theta_0 = 0°$



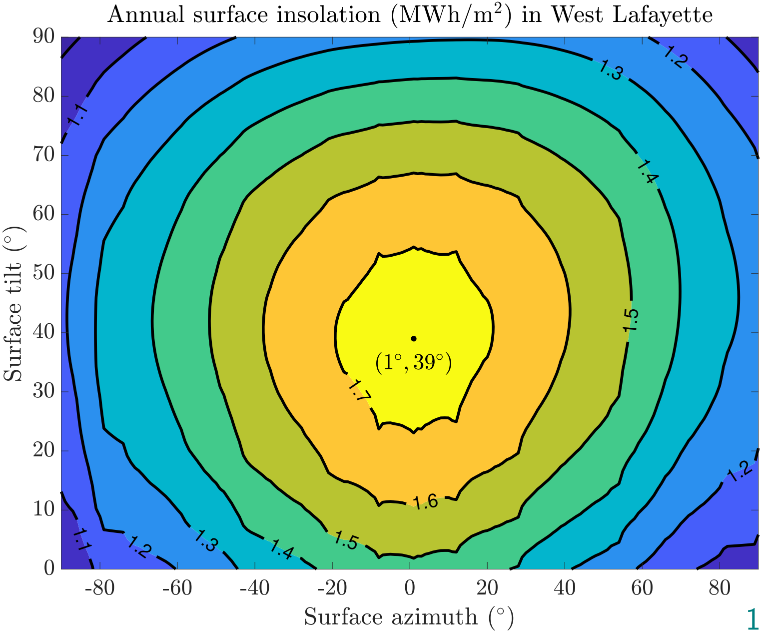
Cloudy summer day with $\beta = \phi_0$, $\theta_0 = 0^\circ$



Clear summer day with $\beta = \theta_0 = 90°$



$\beta \approx \phi_0$ and $\theta_0 \approx 0^\circ$ maximize annual incident energy



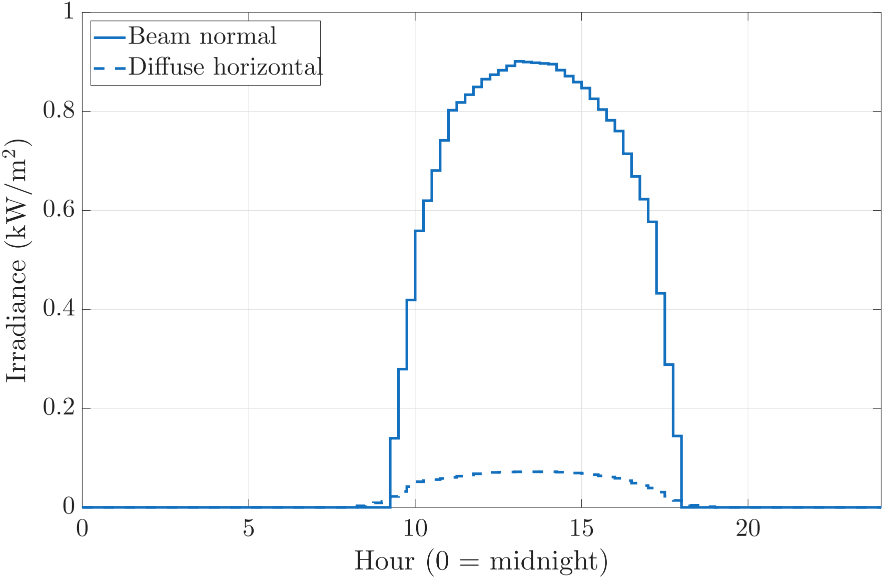
Sunlight and windows

- most windows are vertical: $\beta = 90^\circ$
- with $\sin(\beta) = 1$ and $\cos(\beta) = 0$, incident irradiance reduces to

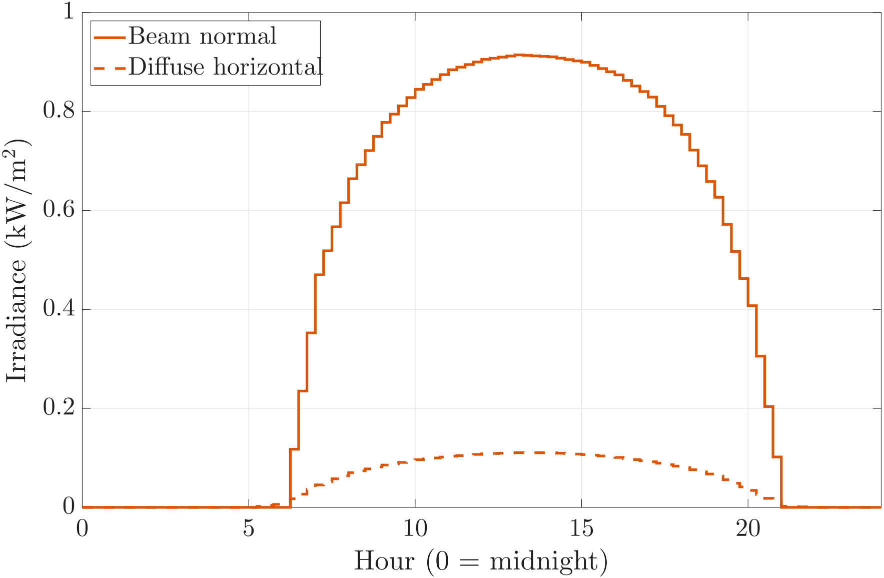
$$S_{\text{tot}} \approx \left\{ \right.$$

- a window of area A transmits solar power cAS_{tot}
- $c \in [0, 1]$ is the window's **solar heat gain coefficient** (typically, $c \approx 0.25$ to 0.8)
- can simulate shading (from trees, blinds, ...) by adjusting c

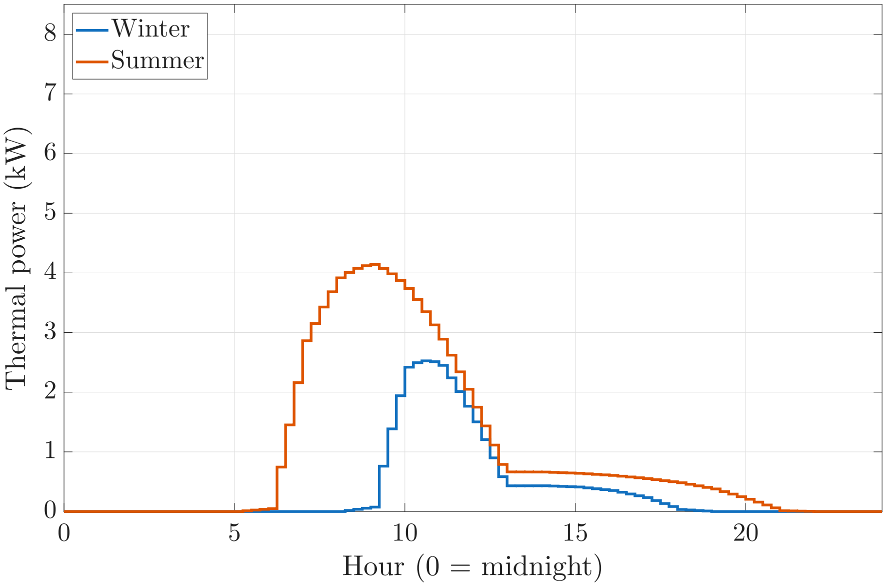
Winter irradiance data for window examples



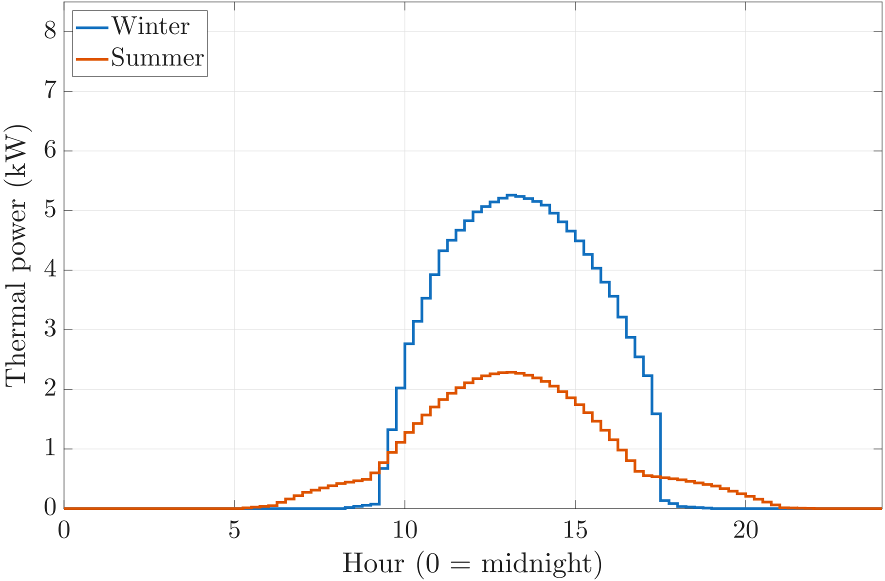
Summer irradiance data for window examples



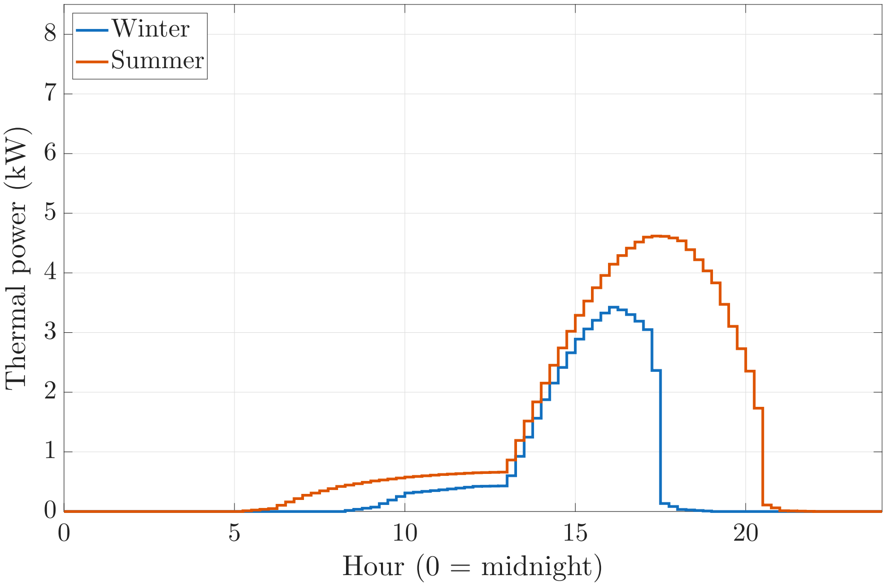
Solar thermal power through **east**-facing windows



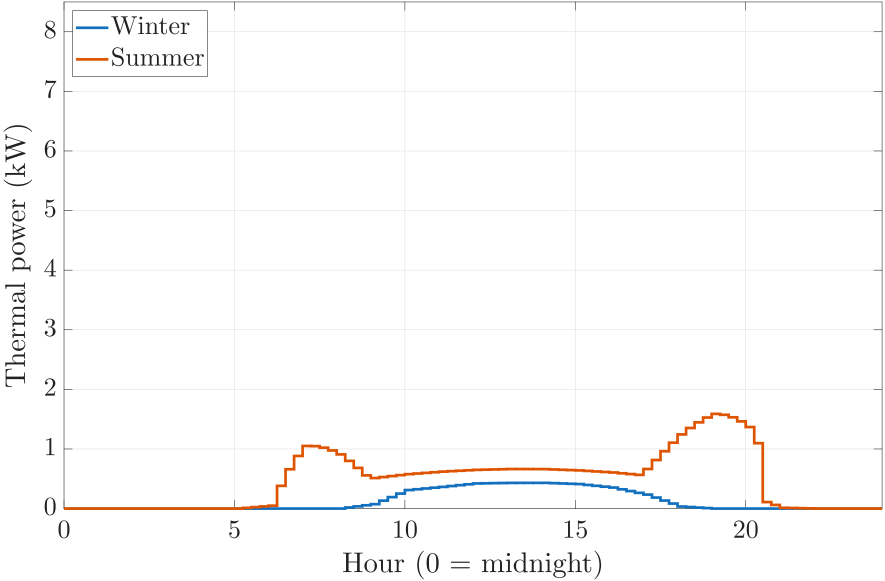
Solar thermal power through **south**-facing windows



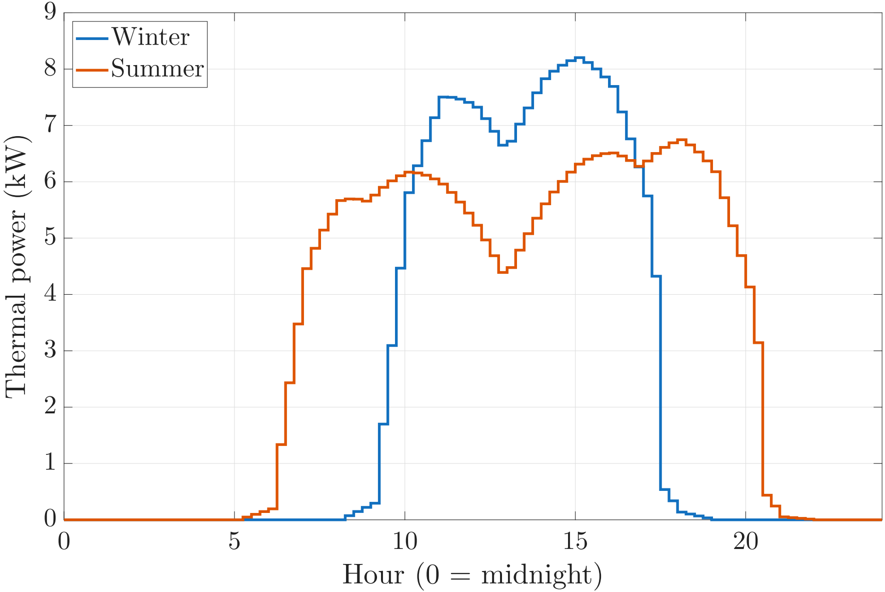
Solar thermal power through **west**-facing windows



Solar thermal power through **north**-facing windows

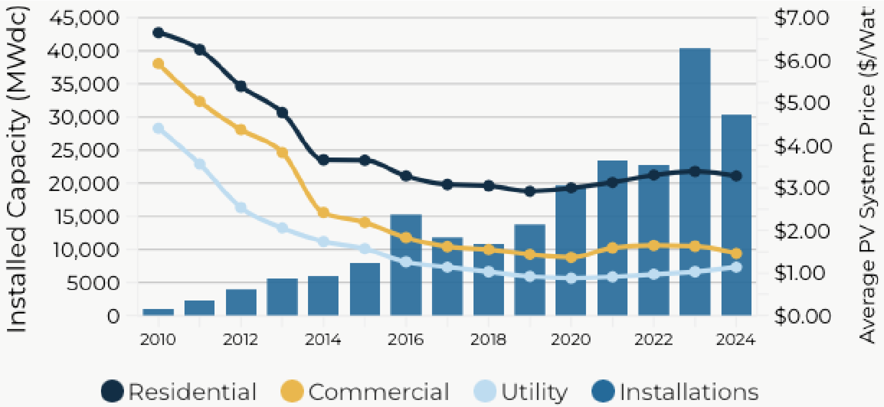


Solar thermal power through **all** windows



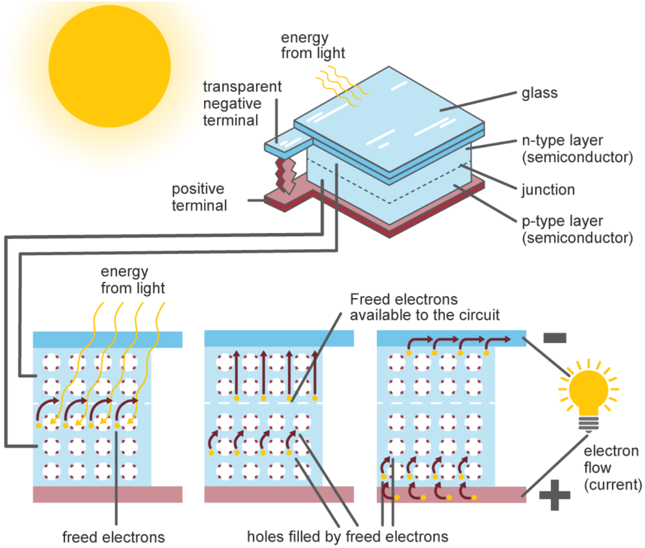
Outline

US costs and installations of solar photovoltaics



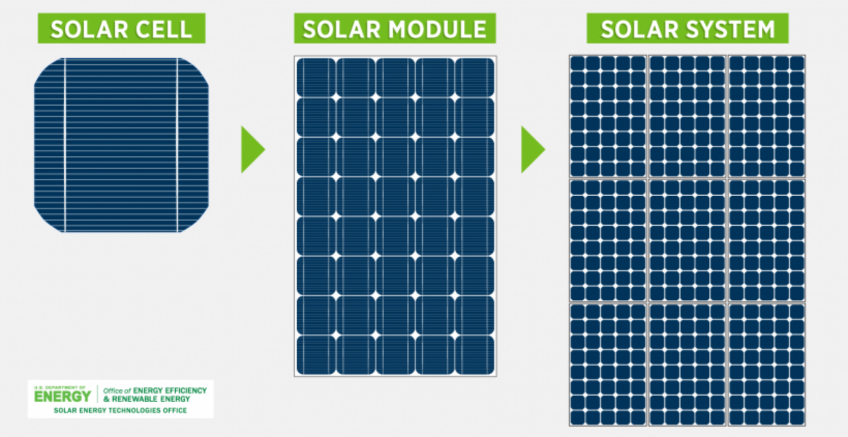
Solar Energy Industries Association (2024): Solar Industry Research Data

Solar photovoltaic cells



US Energy Information Administration: Photovoltaics and electricity

Solar photovoltaic modules and arrays



- typical cell size: ~6" × 6", ~0.023 m^2, ~4.2 W peak
- typical module size: ~5' × 3', ~1.4 m^2, ~250 W peak

US Department of Energy: PV Cells 101

Solar photovoltaic efficiency

$$\eta = \frac{\text{electric power output}}{\text{radiative power input}}$$

- whole-system efficiency includes cells, conductors, inverter, ...
- for typical solar arrays, $\eta \approx 18$ to 22%
- Shockley-Queisser limit: $\eta \leq {\sim}33\%$ for any single-junction cell

Efficiency and temperature

- efficiency scales ∼linearly with cell temperature T_c

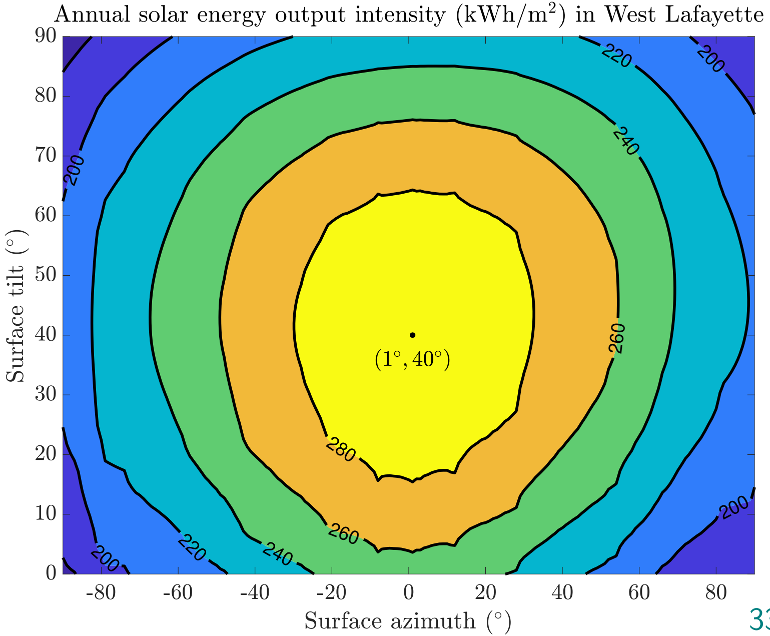
$$\eta \approx$$

- $\tilde{\eta}$ is efficiency at rated cell temperature $\tilde{T} \approx 25\ ^\circ\mathrm{C}$
- $T_0 \approx 270\ ^\circ\mathrm{C}$ is cell temperature at which generation stops
- cell temperature scales ∼linearly with outdoor temperature T_a

$$T_c \approx$$

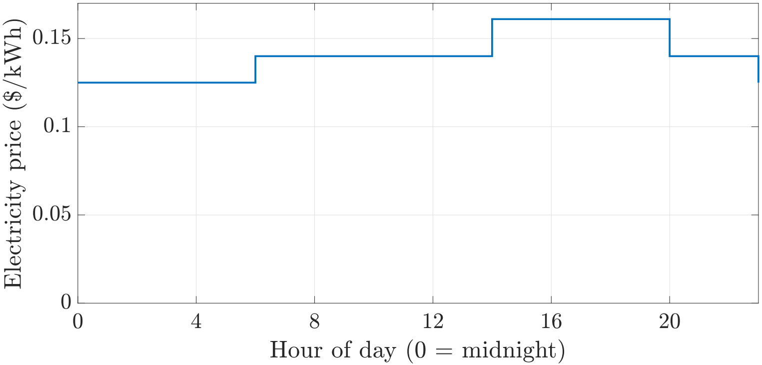
Dubey (2012): Temperature dependent photovoltaic efficiency; PV Education

$\beta \approx \phi_0$ and $\theta_0 \approx 0^\circ$ maximize annual energy output

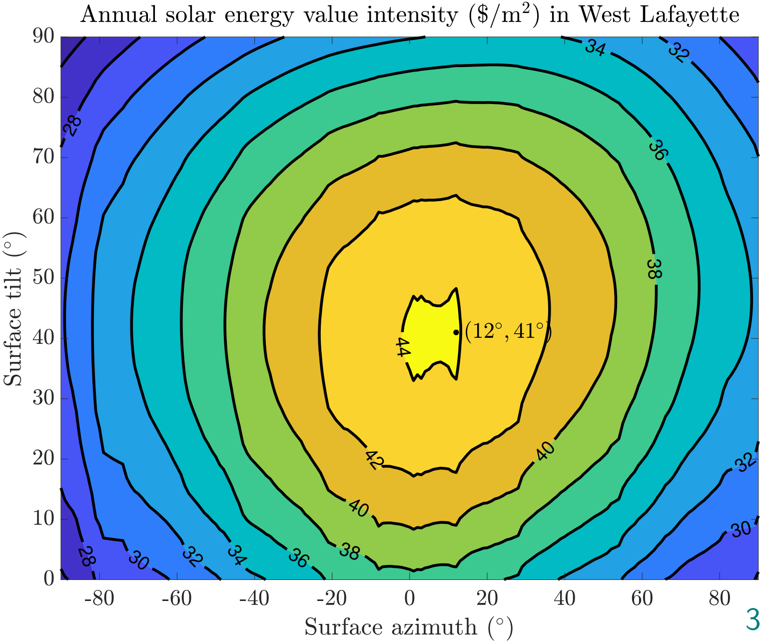


Time-varying electricity prices

- most people buy electricity at a constant price
- some see 'time-of-use' prices with 2 or 3 tiers
- a few see hourly prices tied to wholesale markets



Time-of-use pricing shifts optimal panel orientation west



Net metering

- sometimes, rooftop solar supply exceeds building demand
- some utilities buy excess power at their electricity sale price
- others pay a lower price; some pay nothing at all