

Thermal storage and water heaters

Purdue ME 597, Distributed Energy Resources

Kevin J. Kircher

Outline

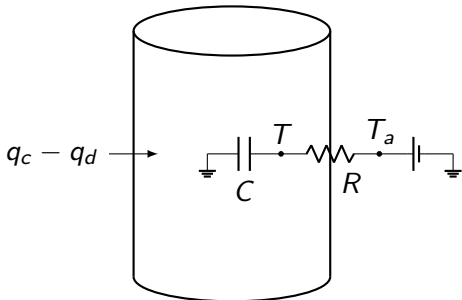
Three types of thermal storage

Domestic hot water use

Resistance water heaters

Heat-pump water heaters

Lumped sensible thermal storage



- C (kWh/°C) is thermal capacitance of storage medium
- R (°C/kW) is thermal resistance of tank wall
- q_c (kW) is charging thermal power
- q_d (kW) is discharging thermal power
- assumption: temperature is spatially uniform inside tank

Lumped sensible thermal storage (continued)

- suppose tank temperature satisfies $T \in [\underline{T}, \overline{T}]$
- choose an arbitrary reference temperature ('datum') T_{LS}
- define stored energy x (kWh) as zero when $T = T_{LS}$:

$$x = C(T - T_{LS}) \in [C(\underline{T} - T_{LS}), C(\overline{T} - T_{LS})]$$

- from conservation of energy,

$$\frac{dx}{dt} =$$

=

=

Lumped sensible thermal storage in a 'box of rocks'

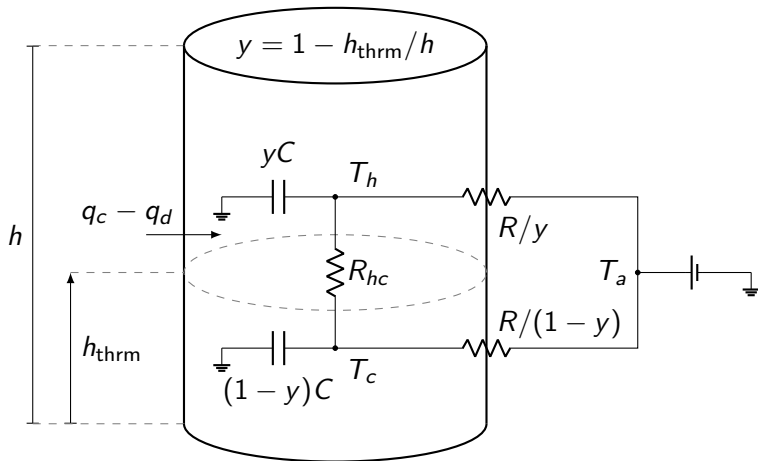


Lumped sensible thermal storage in a 'box of rocks'



Steffes: Comfort Plus Forced Air

Stratified sensible thermal storage



- hot column sits on cold column, separated by thermocline
- T_h and T_c are constant; charging moves thermocline down

Stratified sensible thermal storage (continued)

- define column energies relative to (arbitrary) T_{SS} :

$$x_h = yC(T_h - T_{SS}), \quad x_c = (1 - y)C(T_c - T_{SS})$$

- then total energy is

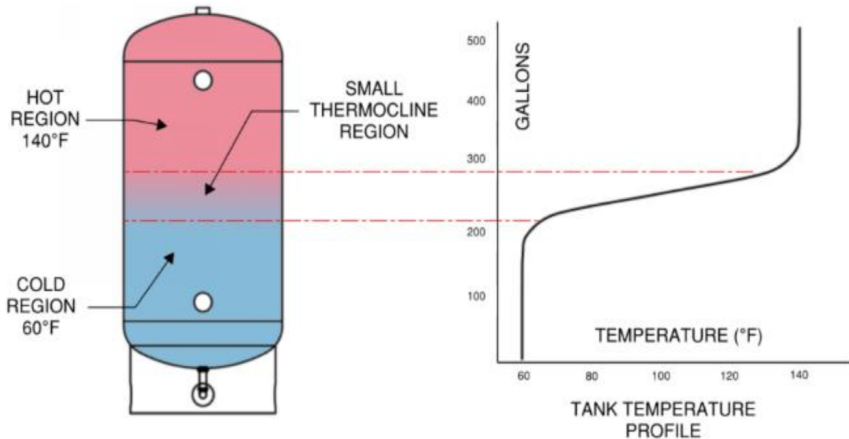
$$\begin{aligned} x &= x_h + x_c = \dots = C[y(T_h - T_c) + T_c - T_{SS}] \\ &\in [C(T_c - T_{SS}), C(T_h - T_{SS})] \end{aligned}$$

- from conservation of energy,

$$\frac{dx}{dt} =$$

$$= \dots =$$

Stratified sensible thermal storage in hot water



Bonneville Power Administration (2022): [Improving thermal energy storage to reduce installation costs for heat pump water heating systems](#)

Latent thermal storage

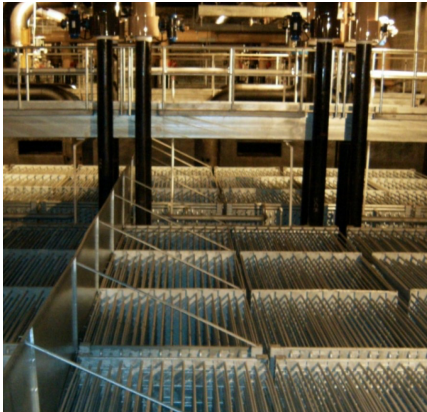
- tank contains M kg of material that freezes and melts
- latent heat of fusion L (kWh/kg), melting point T_m ($^{\circ}\text{C}$)
- liquid mass $m_{\ell} \in [0, M]$ (kg) increases as tank charges
- define energy relative to (arbitrary) T_L :

$$x = C(T_m - T_L) + Lm_{\ell} \in [C(T_m - T_L), C(T_m - T_L) + LM]$$

- from conservation of energy,

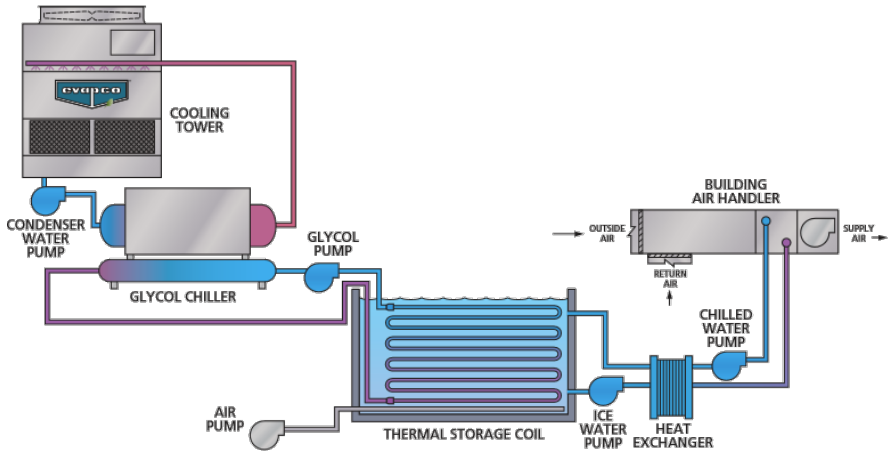
$$\frac{dx}{dt} =$$

Latent thermal storage in ice



Evapco: [Extra-Pak Ice Coils](#)

Latent thermal storage in ice



Evapco: [Extra-Pak Ice Coils](#)

Summary

- all three storage types follow 'heat battery' model:

$$\frac{dx}{dt} =$$

- in discrete time with $a = e^{-\Delta t/\tau}$,

$$x(k+1) =$$

	Lumped sensible	Stratified sensible	Latent
x	$C(T - T_{LS})$	$C[y(T_h - T_c) + T_c - T_{SS}]$	$C(T_m - T_L) + Lm_\ell$
\bar{x}	$C(\bar{T} - T_{LS})$	$C(T_h - T_{SS})$	$C(T_m - T_L) + LM$
\underline{x}	$C(\underline{T} - T_{LS})$	$C(T_c - T_{SS})$	$C(T_m - T_L)$
τ	RC	RC	∞
w	$(T_a - T_{LS})/R - q_d$	$(T_a - T_{SS})/R - q_d$	$(T_a - T_m)/R - q_d$

One model can capture switches between storage types

- example: suppose tank goes stratified \rightarrow lumped as it heats
- goal: choose T_{LS} , T_{SS} , \bar{T} so model switches smoothly
- to get $w_{LS} = w_{SS}$, need $T_{LS} = T_{SS}$
(might as well choose $T_{LS} = T_{SS} = T_a$ so $w_{LS} = w_{SS} = -q_d$)
- to get $\underline{x}_{LS} = \bar{x}_{SS}$, need $\bar{T} = T_h$
- with these choices, one model captures full operating range:

$$\frac{dx}{dt} = -\frac{x}{\tau} + q_c - q_d$$
$$C(T_c - T_a) \leq x \leq C(\bar{T} - T_a)$$

- stratified \leftrightarrow lumped switch happens when $x = C(T_h - T_a)$

Charging and discharging efficiencies

- discharging typically takes little energy (just a pump or fan)
- charging typically uses a heat pump or resistance heat
- heat pump COPs depend on tank and ambient temperatures
- for resistance, $\text{COP} \approx 1$

Typical parameter values

- thermal capacitance is $C = \rho cV$
- thermal resistance is $R = 1/(UA)$
- typically, $U \approx 0.0005$ to 0.001 kW/(°C m²)
- for liquid water at 50 °C,
 - ◊ $\rho = 988$ kg/m³
 - ◊ $c = 4.17$ kJ/(°C kg) = 0.00116 kWh/(°C kg)
 - ◊ $\rho c = 1.15$ kWh/(°C m³)
 - ◊ $L = 334$ kJ/kg = 0.09 kWh/kg
- for a cylinder with radius r and height h ,
 - ◊ $V = \pi r^2 h$
 - ◊ $A = 2\pi r h + 2\pi r^2 = 2V(1/r + 1/h)$

How much energy does a domestic water heater store?

- stratified sensible thermal storage capacity:

$$\bar{x} - \underline{x} = C(T_h - T_c)$$

- typical $V \approx 0.2$ to 0.3 m^3 , so $C \approx 0.25$ to $0.35 \text{ kWh}/^\circ\text{C}$
- typically, $T_c \approx 15 \text{ }^\circ\text{C}$
- below $\sim 49 \text{ }^\circ\text{C}$, legionella bacteria can grow
- above $\sim 60 \text{ }^\circ\text{C}$, water can scald
- with $T_h \approx 51 \text{ }^\circ\text{C}$, energy capacity ≈ 9 to 13 kWh

Outline

Three types of thermal storage

Domestic hot water use

Resistance water heaters

Heat-pump water heaters

Typical hot water volume draws

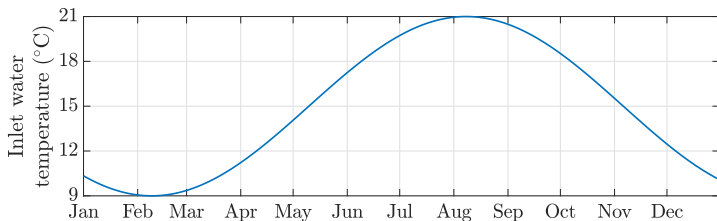
- typical household draws $\sim(16n + 4)$ gal of hot water per day
 - ◊ $\sim 16n$ gal for one shower or bath apiece from n occupants
 - ◊ ~ 4 gal for everything else (like washing dishes and clothes)
- typical shower lasts ~ 8 min, draws volumetric flow rate

$$\dot{V} \approx 2 \text{ gal/min} = 0.45 \text{ m}^3/\text{h}$$

Typical hot water thermal power draws

- thermal power associated with \dot{V} is $\rho c \dot{V} (T - T_{\text{in}})$
- inlet water temperature on day d of the year ($d = 1$ on Jan 1):

$$T_{\text{in}} \approx 15 \text{ }^\circ\text{C} + (6 \text{ }^\circ\text{C}) \sin(d - 130)$$



- with $T \approx 51 \text{ }^\circ\text{C}$, $T_{\text{in}} \approx 15 \text{ }^\circ\text{C}$, typical shower draws

$$\sim (1.15 \text{ kWh}/(^\circ\text{C m}^3)) (0.45 \text{ m}^3/\text{h}) (36^\circ\text{C}) \approx 19 \text{ kW}$$

Most heat withdrawals do not depend on tank temperature

- typical shower or bath has constant mixed water flow \dot{m}^*
- typical bather wants constant mixed water temperature T^*
- so

\Leftrightarrow

\Leftrightarrow

\Rightarrow heat withdrawal q_d is independent of tank temperature T
(if T decreases, bather can mix in more hot water, less cold)

Simulating hot water draws

- showers and baths dominate hot water use
- so generate n showers per day, each at ~ 19 kW for ~ 10 min
- and place them at random plausible times

Outline

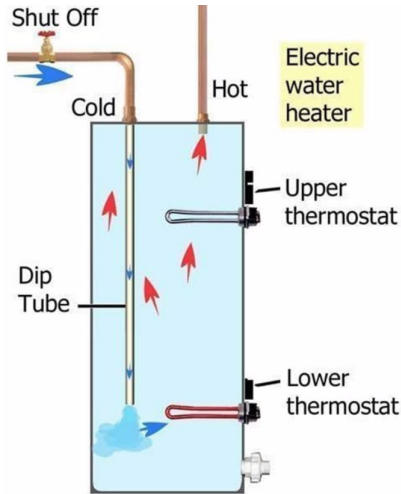
Three types of thermal storage

Domestic hot water use

Resistance water heaters

Heat-pump water heaters

Electric resistance water heaters

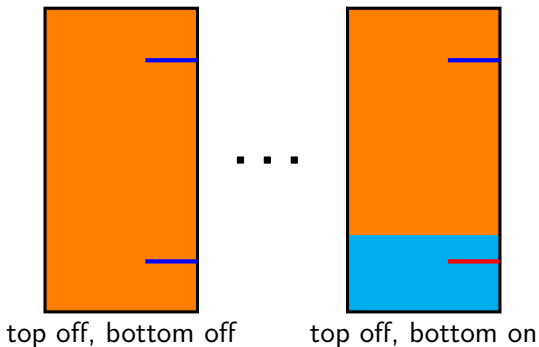


- typically designed to maintain stratification
- hot water drawn from top, cold added to bottom

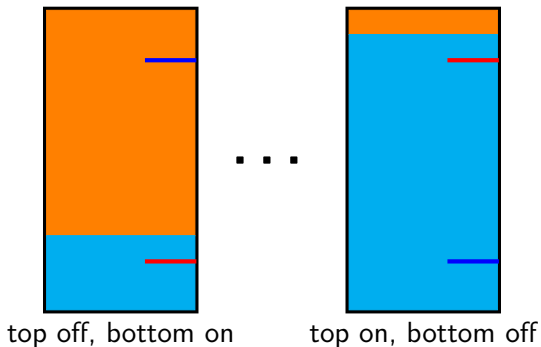
Resistance-only water heater controls

- water heater controls vary by manufacturer
- one approach:
 - ◇ one resistor near bottom, one near top
 - ◇ each resistor has a water temperature sensor
 - ◇ run bottom resistor if bottom temperature drops below setpoint
 - ◇ switch to top resistor if top temperature drops below setpoint
 - ◇ when top temperature returns to setpoint, switch to bottom
 - ◇ when bottom temperature returns to setpoint, turn bottom off

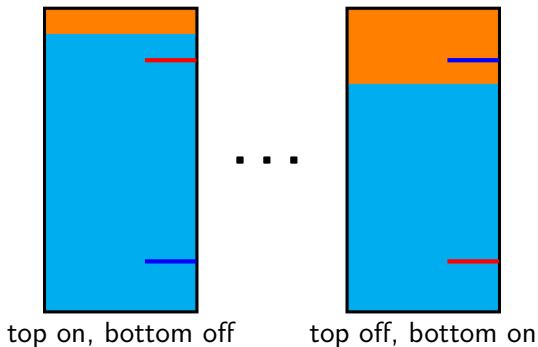
Resistance-only water heater controls (continued)



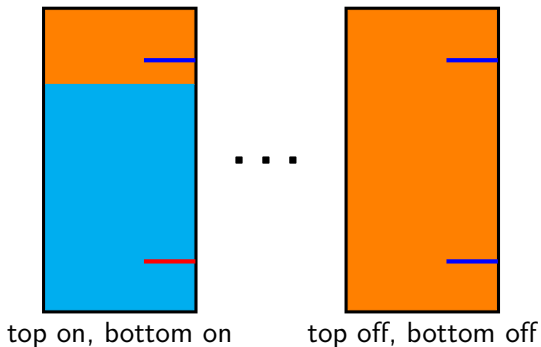
Resistance-only water heater controls (continued)



Resistance-only water heater controls (continued)



Resistance-only water heater controls (continued)



Simulating resistance-only water heater controls

- anytime tank is not fully charged, exactly one resistor runs

⇒ model:

- ◇ try to place $x(k+1) = ax(k) + (1-a)\tau(q_c(k) + w(k))$ at \bar{x}
- ◇ but saturate $q_c(k)$ at capacity limits $[0, \bar{p}_r]$

$$q_c(k) =$$

$$p(k) =$$

Outline

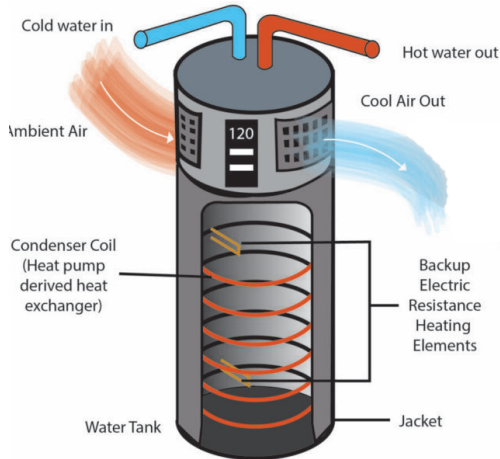
Three types of thermal storage

Domestic hot water use

Resistance water heaters

Heat-pump water heaters

Heat-pump water heaters (and hybrids)



- most HPWHs are hybrids with resistance backup
- heat pump COP depends on temperatures of water, air

Simulating heat-pump-only water heater controls

- anytime tank is not fully charged, heat pump runs

⇒ try to place $x(k+1)$ at \bar{x}

- but saturate $q_c(k)$ at capacity limits $[0, \eta(k)\bar{p}]$

$$q_c(k) =$$

$$p(k) =$$

Hybrid water heater controls

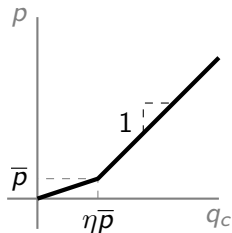
- anytime tank is not fully charged, heat pump runs
- if charge drops below a threshold x_r , resistor also runs

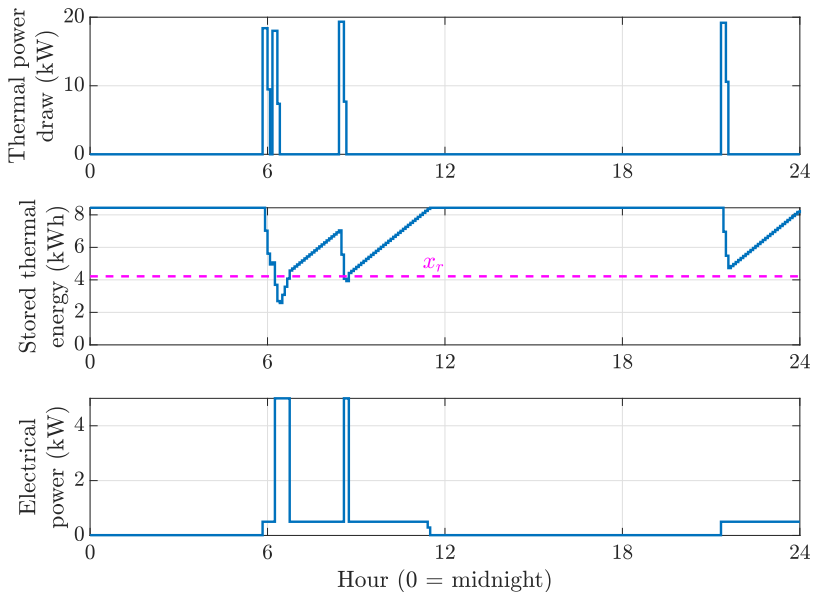
⇒ try to place $x(k+1)$ at \bar{x} , but

- ◇ if $x(k) \geq x_r$, same as heat-pump-only case
- ◇ if $x(k) < x_r$, saturate $q_c(k)$ at combined limits $[0, \eta(k)\bar{p} + \bar{p}_r]$

$$q_c(k) =$$

$$p(k) =$$





4 occupants, 50 gal (0.19 m³) hybrid HPWH