

# Modeling summary: ~Everything's a battery

Purdue ME 597, Distributed Energy Resources

Kevin J. Kircher

# Outline

Batteries and electric vehicles

Passive thermal storage

Active thermal storage

One model to rule them all

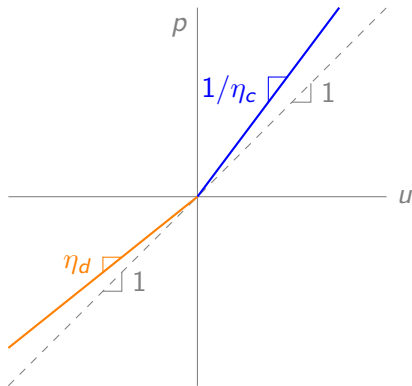
Other DERs

# Stationary battery: Definitions

- state  $x(k)$  (kWh): stored chemical energy
- action  $u(k)$  (kW): chemical charging power
- output  $p(k)$  (kW): electrical charging power
- chemical energy capacities  $\underline{x}, \bar{x}$  (kWh)
- chemical power capacities  $\underline{u} = -\bar{p}_d/\eta_d, \bar{u} = \eta_c\bar{p}_c$  (kW)
- self-dissipation time constant  $\tau$  (h)
- dynamics parameters  $a = e^{-\Delta t/\tau}, b = (1 - a)\tau$

# Stationary battery: Model

- dynamics:  $x(k+1) = ax(k) + bu(k)$
- state constraints:  $\underline{x} \leq x(k) \leq \bar{x}$
- action constraints:  $\underline{u} \leq u(k) \leq \bar{u}$
- output equation:  $p(k) = \max\{\eta_d u(k), u(k)/\eta_c\}$



# Electric vehicle: Definitions

like stationary battery, but with

- chemical power capacities

$$(\underline{u}(k), \bar{u}(k)) = \begin{cases} (-\bar{p}_d/\eta_d, \eta_c \bar{p}_c) & \text{if EV is plugged in at time } k \\ (0, 0) & \text{otherwise} \end{cases}$$

( $\bar{p}_d = 0$  for EVs with one-way charging)

- disturbance  $w(k) = -\frac{\alpha(k)d(k)}{\Delta t}$

like stationary battery, but with

- dynamics  $x(k+1) = ax(k) + b(u(k) + w(k))$
- time-varying action constraints:

$$\underline{u}(k) \leq u(k) \leq \bar{u}(k)$$

# Outline

Batteries and electric vehicles

Passive thermal storage

Active thermal storage

One model to rule them all

Other DERs

# Indoor air/shallow thermal mass: Definitions

- state  $x(k) = C(T(k) - \hat{T}(k))$ : stored thermal energy
- action  $u(k) = q_c(k) - \hat{q}_c(k)$ : thermal power perturbation
- output  $p(k) = \frac{q_c(k)}{\eta(k)}$ : electrical charging power
- thermal energy capacities:

$$\underline{x}(k) = C(\underline{T}(k) - \hat{T}(k)), \quad \bar{x}(k) = C(\bar{T}(k) - \hat{T}(k))$$

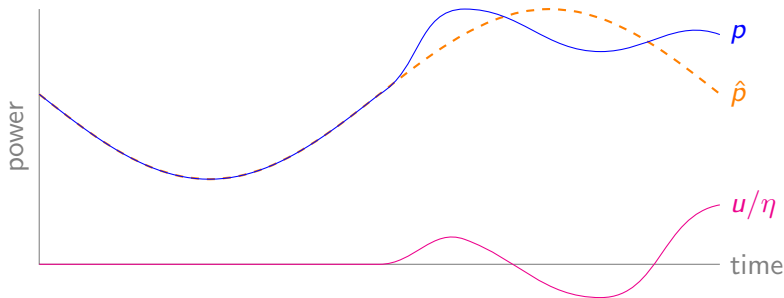
- thermal power capacities:

$$\underline{u}(k) = \underline{q}_c(k) - \hat{q}_c(k), \quad \bar{u}(k) = \bar{q}_c(k) - \hat{q}_c(k)$$

- self-dissipation time constant  $\tau = RC$
- dynamics parameters  $a = e^{-\Delta t/\tau}$ ,  $b = (1 - a)\tau$

# Indoor air/shallow thermal mass: Model

- dynamics:  $x(k + 1) = ax(k) + bu(k)$
- state constraints:  $\underline{x}(k) \leq x(k) \leq \bar{x}(k)$
- action constraints:  $\underline{u}(k) \leq u(k) \leq \bar{u}(k)$
- output equation:  $p(k) = \hat{p}(k) + u(k)/\eta(k)$



## Deep thermal mass: Definitions

- state  $x(k) = C_m(T_m(k) - \hat{T}_m(k))$ : stored thermal energy
- action  $u(k) = \frac{T(k) - \hat{T}(k)}{R_m}$ : thermal power perturbation
- output  $p(k) = \frac{q_c(k)}{\eta(k)}$ : electrical charging power
- thermal energy capacities:

$$\underline{x}(k) = C_m(\underline{T}_m(k) - \hat{T}_m(k)), \quad \bar{x}(k) = C_m(\bar{T}_m(k) - \hat{T}_m(k))$$

- thermal power capacities:

$$\underline{u}(k) = \frac{\underline{T}(k) - \hat{T}(k)}{R_m}, \quad \bar{u}(k) = \frac{\bar{T}(k) - \hat{T}(k)}{R_m}$$

- self-dissipation time constant  $\tau = R_m C_m$
- dynamics parameters  $a = e^{-\Delta t/\tau}$ ,  $b = (1 - a)\tau$

like indoor air, but with state-dependent output equation

$$p(k) = \hat{p}(k) + \frac{1}{\eta(k)} \left[ \left( 1 + \frac{R_m}{R_{\text{out}}} \right) u(k) - \frac{x(k)}{\tau} \right]$$

## Deep thermal mass output equation

- in an approximate 2R2C model with  $dT(t)/dt \approx 0$ ,

$$q_c(k) = \frac{T(k) - T_m(k)}{R_m} + \frac{T(k) - T_{\text{out}}(k)}{R_{\text{out}}} - q_e(k) \quad (1)$$

- similarly, under baseline operation,

$$\hat{q}_c(k) = \frac{\hat{T}(k) - \hat{T}_m(k)}{R_m} + \frac{\hat{T}(k) - T_{\text{out}}(k)}{R_{\text{out}}} - q_e(k) \quad (2)$$

- subtracting (2) from (1) gives

$$\begin{aligned} q_c(k) - \hat{q}_c(k) &= \left( \frac{1}{R_m} + \frac{1}{R_{\text{out}}} \right) (T(k) - \hat{T}(k)) - \frac{T_m(k) - \hat{T}_m(k)}{R_m} \\ &= \left( \frac{1}{R_m} + \frac{1}{R_{\text{out}}} \right) R_m u(k) - \frac{x(k)}{R_m C_m} \\ &= \left( 1 + \frac{R_m}{R_{\text{out}}} \right) u(k) - \frac{x(k)}{\tau} \end{aligned}$$

# Outline

Batteries and electric vehicles

Passive thermal storage

Active thermal storage

One model to rule them all

Other DERs

## Lumped sensible thermal storage: Definitions

- state  $x(k) = C(T(k) - T_{LS})$ : stored thermal energy
- action  $u(k) = q_c(k)$ : thermal charging power
- output  $p(k)$ : electrical charging power
- thermal energy capacities:

$$\underline{x}(k) = C(\underline{T}(k) - T_{LS}), \quad \bar{x}(k) = C(\bar{T}(k) - T_{LS})$$

- thermal power capacities:

$$\underline{u}(k) = \underline{q}_c(k), \quad \bar{u}(k) = \bar{q}_c(k)$$

- self-dissipation time constant  $\tau = RC$
- dynamics parameters  $a = e^{-\Delta t/\tau}$ ,  $b = (1 - a)\tau$
- disturbance  $w(k) = (T_a(k) - T_{LS})/R - q_d(k)$

# Lumped sensible thermal storage: Model

- dynamics:  $x(k+1) = ax(k) + b(u(k) + w(k))$
- state constraints:  $\underline{x}(k) \leq x(k) \leq \bar{x}(k)$
- action constraints:  $\underline{u}(k) \leq u(k) \leq \bar{u}(k)$
- output equation:  $p(k) = u(k)/\eta(k)$

## Stratified sensible thermal storage: Definitions

- state  $x(k) = C[y(k)(T_h - T_c) + T_c - T_{SS}]$
- action  $u(k) = q_c(k)$ : thermal charging power
- output  $p(k)$ : electrical charging power
- thermal energy capacities:

$$\underline{x} = C(T_c - T_{SS}), \bar{x} = C(T_h - T_{SS})$$

- thermal power capacities:

$$\underline{u}(k) = \underline{q}_c(k), \bar{u}(k) = \bar{q}_c(k)$$

- self-dissipation time constant  $\tau = RC$
- dynamics parameters  $a = e^{-\Delta t/\tau}$ ,  $b = (1 - a)\tau$
- disturbance  $w(k) = (T_a(k) - T_{SS})/R - q_d(k)$

## Stratified sensible thermal storage: Model

like lumped sensible, but with time-invariant state constraints:

$$\underline{x} \leq x(k) \leq \bar{x}$$

## Latent thermal storage: Definitions

- state  $x(k) = C(T_m - T_L) + Lm_\ell(k)$
- action  $u(k) = q_c(k)$ : thermal charging power
- output  $p(k)$ : electrical charging power
- thermal energy capacities:

$$\underline{x} = C(T_m - T_L), \quad \bar{x} = C(T_m - T_L) + LM$$

- thermal power capacities:

$$\underline{u}(k) = \underline{q}_c(k), \quad \bar{u}(k) = \bar{q}_c(k)$$

- self-dissipation time constant  $\tau = \infty$
- dynamics parameters  $a = 1$ ,  $b = \Delta t$
- disturbance  $w(k) = (T_a(k) - T_m)/R - q_d(k)$

# Latent thermal storage dynamics parameters

- for latent thermal storage,  $a = 1$  and  $b = \Delta t$
- these are limits of  $a = e^{-\Delta t/\tau}$  and  $b = (1 - a)\tau$  as  $\tau \rightarrow \infty$ :
  - ◊  $e^{-\Delta t/\tau} \rightarrow 1$  as  $\tau \rightarrow \infty$
  - ◊ letting  $\alpha = 1/\tau$  and using L'Hôpital's rule,

$$\begin{aligned}\lim_{\tau \rightarrow \infty} \left(1 - e^{-\Delta t/\tau}\right) \tau &= \lim_{\alpha \rightarrow 0} \frac{1 - e^{-\alpha \Delta t}}{\alpha} \\ &= \lim_{\alpha \rightarrow 0} \frac{\frac{d}{d\alpha}(1 - e^{-\alpha \Delta t})}{\frac{d}{d\alpha}(\alpha)} \\ &= \lim_{\alpha \rightarrow 0} \Delta t e^{-\alpha \Delta t} \\ &= \Delta t\end{aligned}$$

# Latent thermal storage: Model

like stratified sensible

# Outline

Batteries and electric vehicles

Passive thermal storage

Active thermal storage

**One model to rule them all**

Other DERs

# Unified model

- linear time-invariant dynamics:

$$x(k+1) = ax(k) + b(u(k) + w(k))$$

- linear time-varying state constraints:

$$\underline{x}(k) \leq x(k) \leq \bar{x}(k)$$

- linear time-varying action constraints:

$$\underline{u}(k) \leq u(k) \leq \bar{u}(k)$$

- nonlinear time-varying state-dependent output equation:

$$p(k) = g(k, x(k), u(k))$$

# Dynamics parameters

- for everything but latent thermal storage,

$$a = e^{-\Delta t/\tau}, \quad b = (1 - a)\tau$$

- for latent thermal storage,  $a = 1$ ,  $b = \Delta t$   
(the limits of  $e^{-\Delta t/\tau}$  and  $(1 - e^{-\Delta t/\tau})\tau$  as  $\tau \rightarrow \infty$ )

# Disturbance

zero for

- stationary batteries
- indoor air/shallow thermal mass
- deep thermal mass

nonzero in general for

- electric vehicles
- lumped sensible thermal storage
- stratified sensible thermal storage
- latent thermal storage

time-invariant for

- stationary batteries
- electric vehicles
- stratified sensible thermal storage
- latent thermal storage

time-varying for

- indoor air/shallow thermal mass
- deep thermal mass
- lumped sensible thermal storage

time-invariant for

- stationary batteries

time-varying for

- electric vehicles
- indoor air/shallow thermal mass
- deep thermal mass
- lumped sensible thermal storage
- stratified sensible thermal storage
- latent thermal storage

# Output equation

nonlinear, time-invariant, state-independent for

- stationary batteries
- electric vehicles

linear, time-varying, state-independent for

- indoor air/shallow thermal mass
- lumped sensible thermal storage
- stratified sensible thermal storage
- latent thermal storage

linear, time-varying, state-dependent for

- deep thermal mass

# Outline

Batteries and electric vehicles

Passive thermal storage

Active thermal storage

One model to rule them all

Other DERs

# Refrigerators and freezers



- there are ~200 million refrigerators and freezers in the US
- they all have some thermal mass and temperature flexibility
- at ~500 W each, that's ~100 GW of flexible capacity (for comparison, total US generation capacity is ~1.3 TW)
- can model them as thermal circuits, just like buildings

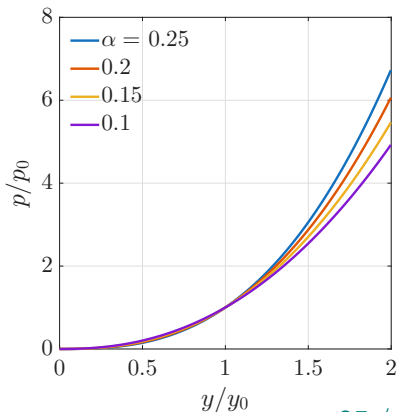
## Deferrable loads

- some appliances just need to run before a deadline
- for example, a dishwasher might
  - ◇ get loaded at ~8 PM
  - ◇ need to finish by ~6 AM
  - ◇ take ~1 to 3 hours to run
  - ◇ draw ~1.5 to 2 kW while running
- there are ~80 million dishwashers in the US
- at ~1.5 kW each, that's ~120 GW of flexible capacity
- clothes washers and dryers may have similar flexibility

# Wind turbines (typically taller = more economical)

- wind turbine power output  $p$  scales like the product of
  - ◇ area  $\sim \pi y^2$  swept by blades, where  $y$  is turbine height
  - ◇ cube of wind speed  $v$
- wind speed scales like  $y^\alpha$  with wind shear exponent  $\alpha \approx 0.1$  to  $0.25$

$$p \propto y^2 v^3, \quad v \propto y^\alpha \implies p \propto y^{2+3\alpha}$$



# The world's biggest wind turbine has 150 m blades



a Boeing 747 is 71 m long

---

Offshore Magazine: [China tests 'world's largest' offshore wind turbine](#)

# Simple wind power simulation

- get wind speed  $\tilde{v}$  at whatever height  $\tilde{y}$  weather data report
- estimate wind speed  $v$  at turbine height  $y$  by  $v \approx \tilde{v}(y/\tilde{y})^{0.2}$
- get turbine's cut-in and cut-out speeds,  $\underline{v}$  and  $\bar{v}$
- if  $\underline{v} \leq v \leq \bar{v}$ ,
  - ◊ get turbine's rated power  $p^*$  at rated wind speed  $v^*$
  - ◊ estimate power  $p$  at wind speed  $v$  by  $p \approx p^*(v/v^*)^3$
- otherwise, set  $p = 0$

# Things that use fuel

- some things burn fuel for heat or work
  - ◇ heaters fueled by wood, methane, propane, heating oil, or hydrogen
  - ◇ diesel generators
  - ◇ combined heat and power ('cogeneration' or 'trigeneration')
- others use fuel without combustion
  - ◇ methane fuel cells
  - ◇ hydrogen fuel cells
- potentially useful for backup, stabilizing microgrids, ...
- can model (roughly) via "output = efficiency × input"

# Combined heat and power

