

# Homework 6: Optimization

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## Directions:

- Students may work individually or in groups, but each student must upload their own solutions to [Gradescope](#) by **11:59 PM ET on Friday, April 24**. Students should finish the reading (see Problem 3) before the class discussion on **Thursday, April 30**.

Use any outside resources you want, but **cite your sources**, including if and how you use any AI tools. (If you want to learn the material, I recommend trying the problems yourself before looking for outside help. This lets us identify the things we don't fully understand so we can figure them out.)

- The TA will grade each problem or subproblem quickly on a three-tier scale:
  - No credit for a solution that's mostly unreadable or missing.
  - Half credit for a serious attempt that's not easy to read or is substantially incorrect.
  - Full credit for a solution that's clearly readable and nearly or completely correct.

## Problems:

1. (60%) Consider the function  $f: \mathbf{R}^2 \rightarrow \mathbf{R}$  defined by

$$f(x) = e^{x_1+3x_2-0.1} + e^{x_1-3x_2-0.1} + e^{-x_1-0.1}.$$

- (a) (15%) Use composition rules from the 'Convex sets and functions' slides to show that  $f$  is convex.
  - (b) (15%) Find the (unique) global minimizer  $x^*$  of  $f$  by solving  $\nabla f(x^*) = 0$  for  $x^*$ .
  - (c) (30%) Download the files in the Github repository [gradient-descent](#). Fill in the missing code in the script `gradientDescent`, using the line search method from the 'Solving convex optimization problems' slides. Your code should minimize  $f(x)$  given the initial guess  $x(1) = (-2.5, 0.5)$  and the initial step size  $\alpha(1) = 1$ . Show your code and figures 1 and 2 here.
2. (40%) Install [CVX](#) for Matlab or import [CVXPY](#) for Python. Use CVX or CVXPY to find a battery power profile that maximizes 'buy low, sell high' revenue with an electricity price that is 0.25 \$/kWh between 2 PM and 9 PM and 0.15 \$/kWh otherwise. Use the following input data: One-day planning horizon starting at midnight, fifteen-minute time steps, 1,600 hour self-dissipation time constant, 95% charging and discharging efficiencies, 13.5 kWh of energy capacity, minimum charge state of 20%, initial charge state of 50%, charging and discharging power capacities of 5 kW. Constrain the final charge state to equal the initial charge state. Turn in your code, the total revenue earned by the battery over the day, and a figure containing three vertical subplots: (1) electricity price vs. time, (2) electrical charging/discharging power vs. time, (3) stored chemical energy vs. time.
  3. (Due **Thursday, April 30**) Listen to The Carbon Copy podcast episode "[Virtual power plants: The 'sandwich' for the grid](#)". Read pages 1–55 of the Department of Energy's [Pathways to commercial liftoff: Virtual power plants](#) report and pages 2–15 of the [2025 update](#). (Jennifer Downing, the podcast guest, led the 2023 report.) As you listen and read, do the following to prepare for in-class discussion:

- Write down any terms you don't recognize or fully understand. Do your best to find the meaning of each of these terms.
- Write down at least two questions or topics that you'd like to discuss in class.